Semidefinite and Cone Programming Bibliography/Comments

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Key words: ?????

Abstract

This online technical report presents abstracts (short outlines) of papers related to semidefinite programming. The papers are grouped by subject. This is not an exhaustive list by any means. Many of my own papers deal with SDP but may not be included here yet, though they are in the bibliography list. My papers are available at http://orion.math.uwaterloo.ca:80/~hwolkowi/henry/reports/ABSTRACTS.html

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1 THEORY

1.1 Background

1.2 Cones

1.2.1 Self-Dual Cones

1. In [107], Homogeneous and facially homogeneous self-dual cones, by BÉLLISSARD, J. and IOCHUM, B. and LIMA, R.,

   abstract: They show that any self-dual cone in a real finite dimensional Hilbert space is homogeneous iff it is facially homogeneous in the sense of A. Connes. They develop a spectral decomposition theory for these cones which is the analogue of the usual one for self-adjoint operators on a finite-dimensional Hilbert space.

   Included is a description of Vinberg's main result [987] that every indecomposable homogeneous self-dual cone is unitarily equivalent to a cone in one of 5 classes.

2. In [755], On cone-invariant linear matrix inequalities, by Parrilo, Pablo A. and Khatri, Sven,

   An exact solution for a special clas of cone-preserving linear matrix inequalities (LMIs) is developed. Bu using a generalized version of the classical Perron-Frobenius theorem, the optimal value is shown to be equal to the spectral readius of an associated linear operator.

1.2.2 Invariance

1. In "Positive definite preserving linear transformations on symmetric matrix spaces", by Authors: Huynh Dinh Tuan, Tran Thi Nha Trang, Doan The Hieu, (Submitted on 7 Aug 2010)

   Abstract: Base on some simple facts of Hadamard product, characterizations of positive definite preserving linear transformations on real symmetric matrix spaces with an additional assumption "rank$T(E_{ii}) = 1, i = 1, 2, ..., n$" or "$T(A) > 0 \rightarrow A > 0$", were given.

   Cite as: arXiv:1008.1347v1 [math.RA]

1.2.3 Properties of the Semidefinite Cone

1. The classical unpublished report [148], "Joint positiveness of matrices" by F. Bohnenblust: discusses joint positive definiteness and the geometry of the cone of semidefinite matrices.

2. In [194], "Finding a positive semidefinite interval for a parametric matrix", by Caron and Gould:

   Summary: "Let C and E be symmetric (n, n)-matrices such that C is positive semidefinite and E is of rank one or two. This paper is concerned with finding real numbers $\underline{t} \leq 0$ and $\bar{t} \geq 0$ such that $C(t) = C+tE$ is positive semidefinite if and only if $t \in [\underline{t}, \bar{t}\,]$. Explicit expressions for $\underline{t}$ and $\bar{t}$ are derived, and a method for computing $\underline{t}$ and $\bar{t}$ is presented along with preliminary numerical experience."

3. In [412], Characterization of positive definite and semidefinite matrices via quadratic programming duality, by Han, S.-P.(1-WI-R); Mangasarian, O. L.(1-WI-R)

   abstract: In the theory of quadratic programming, the positive semidefiniteness of the underlying matrix induces well-known duality results. In this paper, the authors study the interesting converse problem of finding duality results that must hold for a pair of dual quadratic programs such that the underlying matrix Q is positive definite or semidefinite. They have shown many duality results which induce positive definiteness or semidefiniteness in Q. Thus they develop new characterizations for positive definite and semidefinite matrices. For example, it is shown that the underlying matrix is positive definite if a strict local minimum of a quadratic program exceeds or equals a strict global maximum of the dual.

4. In [413], Conjugate cone characterization of positive definite and semidefinite matrices, by Han, S.-P.(1-WI-R); Mangasarian, O. L.(1-WI-R),

   abstract: The authors consider real matrices A (not necessarily symmetric) which are positive definite [semidefinite] in the sense that $x^T Ax > 0$ [$x^T Ax \geq 0$] for all real vectors x in some cone. They prove a conjugate decomposition theorem for an arbitrary real vector with respect to a given matrix which is positive definite on a given closed cone (a generalization of J.-J. Moreau's polar decomposition [C. R. Acad. Sci. Paris 255 (1962), 238–240; MR 25 #3346]). They use the conjugate decomposition to characterize positive definite and semidefinite matrices (on all of R^n) in terms of positive definite and semidefinite conditions on closed cones in R^n. A typical result: A is positive definite on R^n if and only if for some closed convex cone K, A is positive definite on K and $(A + A^T)^{-1}$ exists and is positive semidefinite on the polar cone K^0.

5. In [411], A conjugate decomposition of the Euclidean space, by Han, S.-P.(1-WI-R); Mangasarian, O. L.(1-WI-R),

   abstract: Author's summary: "Given a closed convex cone K in the n-dimensional real Euclidean space R^n and an $n \times n$ real matrix A that is positive definite on K, we show that each vector in R^n can be decomposed into a component that lies in K and another that lies in the conjugate cone induced by A and such that the two vectors are conjugate to each other with respect to $A + A^T$. As a consequence of this decomposition we establish the following characterization of positive definite matrices: An $n \times n$ real matrix A is positive definite if and only if it is positive definite on some closed convex cone K in R^n and $(A + A^T)^{-1}$ exists and is positive semidefinite on the polar cone K°. If K is a subspace of R^n, then K° is its orthogonal complement $K^\perp$. Other applications include local duality results for nonlinear programs and other characterizations of positive definite and semidefinite matrices."

6. In [633], Restricted quadratic forms, inertia theorems, and the Schur complement, by Maddocks,

   Let $\mathbf{R^m}$ be an m-dimensional real vector space, S a subspace of $\mathbf{R^m}$, $Q(x) = x^T Ax$ a quadratic form on $\mathbf{R^m}$ (A is a real symmetric $m \times m$ matrix), $Q|S$ the restriction of Q to S, In A the triple (P_+, P_-, P_0), where P_+, P_-, P_0 are the numbers of positive, negative and zero eigenvalues of A, $\mathrm{In}^*(S; A)$ the triple corresponding to $Q|S$. In the paper the author studies relations between $\mathrm{In}A$ and $\mathrm{In}^*(S; A)$. Examples of the theorems proved: (1) $\mathrm{In}^*(Y^A; A) = \mathrm{In}^*(S; A) + \mathrm{In}^*(S^A \cap Y^A; A) + (d^0(S), d^0(S), -d^0(S + Y) - \dim(S \cap S^A))$. Here Y is a subspace of $\mathbf{R^m}$ such that $S \subset Y^A$, the superscript A denotes the A-orthogonal subspace, $Y \cap Y^A \subset \mathrm{Ker}(A)$, $d^0(S)$ is defined to be $\dim[A(S \cap S^A)]$ (Theorem 2.6); (2) $\mathrm{In}A = \mathrm{In}^*(S; A) + \mathrm{In}^*(S^\perp \cap R(A); A^+) + (d, d, e - 2d)$, where $\perp$ is the symbol of usual orthogonal complement, $R(A)$ is the range of A, $d = \dim(AS \cap S^\perp)$, $e = \dim(\mathrm{Ker}(A)) - \dim[\mathrm{Ker}(A) \cap S]$, A^+ is the generalized inverse of A (Theorem 3.1). The author also studies the connections between $\mathrm{In}A$ and $\mathrm{In}B^T AB$ and the inertia of quadratic forms defined by partitioned matrices and forms involving the generalized Schur complement of A in $\left(\begin{pmatrix} A & B \\ B^T & C \end{pmatrix} \right)$ (defined as $M/A = C - B^T A^+ B$).

7. In [635], Clarke Generalized Jacobian of the Projection onto the Cone of Positive Semidefinite Matrices, by J. MALICK and H. SENDOV,

   This paper studies the differentiability properties of the projection onto the cone of positive semidefinite matrices. In particular, the expression of the Clarke generalized Jacobian of the projection at any symmetric matrix is given. (See related work on semismooth functions [791], and/or Item 9, below, and the references therein.)

8. In [456], The Algebraic Degree of Semidefinite Programming, Authors: Jiawang Nie, Kristian Ranestad, Bernd Sturmfels,

   Abstract: Given a semidefinite program, specified by matrices with rational entries, each coordinate of its optimal solution is an algebraic number. We study the degree of the minimal polynomials of these algebraic numbers. Geometrically, this degree counts the critical points attained by a linear functional on a fixed rank locus in a linear space of symmetric matrices. We determine this degree using methods from complex algebraic geometry, such as projective duality, determinantal varieties, and their Chern classes.

9. In [791], A Quadratically Convergent Newton Method for Computing the Nearest Correlation Matrix, by Houduo Qi, Defeng Sun

   Abstract: The nearest correlation matrix problem is to find a correlation matrix which is closest to a given symmetric matrix in the Frobenius norm. The well-studied dual approach is to reformulate this problem as an unconstrained continuously differentiable convex optimization problem. Gradient methods and quasi-Newton methods such as BFGS have been used directly to obtain globally convergent methods. Since the objective function in the dual approach is not twice continuously differentiable, these methods converge at best linearly. In this paper, we investigate a Newton-type method for the nearest correlation matrix problem. Based on recent developments on strongly semismooth matrix valued functions, we prove the quadratic convergence of the proposed Newton method. Numerical experiments confirm the fast convergence and the high efficiency of the method.

10. In [963], A recurring theorem about pairs of quadratic forms and extensions: a survey, by Uhlig,

    Author's summary: "This is a historical and mathematical survey of work on necessary and sufficient conditions for a pair of quadratic forms to admit a positive definite linear combination, and various extensions thereof."

    The "main theorem" whose history and extensions are presented was first proved by P. Finsler [Comm. Math. Helv. 9 (1937), 188–192; Zbl 16, 199]. For a real symmetric matrix S [complex Hermitian matrix H], let $Q_S = \{x \in R^n | x'Sx = 0\}[\{x \in C^n | x^*Hx = 0\}]$. The theorem states: (I) for $n \geq 3$ and S, T symmetric $n \times n$, there is a definite matrix of the form $aS + bT(a, b \in R)$ if and only if $Q_S \cap Q_T = \{0\}$; (II) the preceding is true for any n if and only if the quadratic form of S (or T) does not change sign on Q_T (or Q_S); (III) for H, K Hermitian, the result (I) holds for all n.

    The history—and rediscovery—of this theorem and its extensions are discussed with a thorough bibliography, followed by a sketch of the proof methods (which have been many). Discussion of recent extensions includes connections to Euclidean and real-closed fields and to the stability analysis of the definite generalized eigenvalue problem.

    An excellent survey, supported by a bibliography of 75 entries.

11. In [99], H.H. Bauschke, F. Deutsch, H. Hundal, Linear images and sums of closed cones in Euclidean spaces:

    Abstract: In 1997, Bauschke, Borwein, and Lewis have stated a trichotomy theorem that characterizes when the convergence of the method of alternating projections can be arbitrarily slow. However, there are two errors in their proof of this theorem. In this note, we show that although one of the errors is critical, the theorem itself is correct. We give a different proof that uses the multiplicative form of the spectral theorem, and the theorem holds in any real or complex Hilbert space, not just in a real Hilbert space.

1.2.4 Cones and Closure

1. In [176], Bromek, T.; Kaniewski, J.
   Linear images and sums of closed cones in Euclidean spaces.

The authors investigate to what extent the closedness of cones can be spoiled by taking linear images and sums. The main result states that each convex F_σ-cone of R^n is a linear image (in fact a projection) of a closed convex cone of R^{n+1}, and moreover it is generated by a compact convex set. The converse implication also holds. This theorem is closely related to a theorem of V. Klee [Acta Math. 102 (1959), 79–107; MR 21 #4390]. Klee proved that each convex F_σ-set of R^n is a projection of some closed convex set of R^{n+1}. In fact, in this paper, it is shown how to obtain these theorems one from another. For a sum of cones, the situation is different; not every convex F_σ-cone can be obtained as the sum of two closed cones. The following equivalence is established: a convex cone is the sum of two closed cones if and only if it is a closed or a non-pointed F_σ-cone. At the end of the paper, an example of a linear program over cones in finite-dimensional spaces such that the values of the primal and dual programs are different is given. Similar examples with this property are well known [A. Ben-Israel, A. Charnes, and K. Kortanek, Bull. Amer. Math. Soc. 75 (1969), 318–324; MR 43 #2982].

2. In [523], Some characterizations of convex polyhedra, by Klee, Victor

   Call a (perhaps unbounded) subset K of euclidean n-space "polyhedral" if it is the intersection of finitely many closed affine halfspaces. Suppose $2 \leq j \leq n-1$. The author proves that K is polyhedral (1) if all its j-sections are polyhedral, or (2) if K is bounded and convex and all its j-projections are polyhedral, or (3) if $j \geq 3$ and K is closed and convex and all its j-projections have polyhedral closure, or (4) if K is a convex cone and all its j-projections are closed. These are the main results, but he establishes many refinements, and supplies counterexamples against other refinements. His last theorem deals with polyhedral approximation: For any subset X of n-space let $S(X,\varepsilon)$ be the union of open ε-balls centered at points $x \in X$, and define the "Hausdorff distance" $h(X,Y)$ between subsets X and Y to be $\inf\{d\colon X \subset S(Y,d) \text{ and } Y \subset S(X,d)\}$. Then for any closed convex Q, either $h(P,Q)=\infty$ for every polyhedron P, or else for every ε there is some polyhedron P with $h(P,Q)<\varepsilon$.

3. In [760], On the closedness of the linear image of a closed convex cone. Part I: necessary and sufficient conditions for nice cones, and exact characterizations of badly behaved semidefinite systems, by Gabor Pataki.

   The paper presents a collection of very simple, and intuitive conditions for a classic problem: the closedness of the linear image of a closed convex cone.

4. In [186], *Generators, extremals and bases of max cones*, Authors: Peter Butkovic, Hans Schneider, Sergei Sergeev.

   Abstract: We give simple algebraic proofs of results on generators and bases of max cones, some of which are known. We show that every generating set S for a cone in max algebra can be partitioned into two parts: the independent set of extremals E in the cone and a set F every member of which is redundant in S. We exploit the result that extremals are minimal elements under suitable scalings of vectors. We also give an algorithm for finding the (essentially unique) basis of a finitely generated cone.

5. In [825], *On common invariant cones for families of matrices*, Authors: Leiba Rodman, Hakan Seyalioglu, Ilya M. Spitkovsky

   Abstract: The existence and construction of common invariant cones for families of real matrices is considered. The complete results are obtained for 2x2 matrices (with no additional restrictions) and for families of simultaneously diagonalizable matrices of any size. Families of matrices with a shared dominant eigenvector are considered under some additional conditions.

1.2.5 Tensor Cones

1. In [43], "Cones and norms in the tensor product of matrix spaces", by T. Ando: This paper is of expository nature. In the tensor product of two matrix spaces $\mathbb{R}^m$ and $\mathbb{R}^n$ we consider three natural

cones; the first is the cone $\mathcal{P}_0$ of positive semi-definite matrices, the second is the cone $\mathcal{P}_+$ generated by tensor products of positive semi-definite matrices in the factor spaces, and the third cone $\mathcal{P}_-$ is defined as the dual cone of $\mathcal{P}_+$. Correspondingly, in the tensor space identified with $\mathbb{R}^{m\times n}$, besides the spectral and the trace norms, we consider several tensor norms generated by sepectral and/or trace norms in the factor spaces. We will study relations among those norms as well as relations among the three kinds of order intervals in the tensor product.

1.2.6 Harmonic Analysis and SDP

1. Invariant semidefinite programs Authors: Christine Bachoc, Dion C. Gijswijt, Alexander Schrijver, Frank Vallentin, [69].

   Abstract: In the last years many results in the area of semidefinite programming were obtained for invariant (finite dimensional, or infinite dimensional) semidefinite programs - SDPs which have symmetry. This was done for a variety of problems and applications. The purpose of this handbook chapter is to give the reader the necessary background for dealing with semidefinite programs which have symmetry. Here the basic theory is given and it is illustrated in applications from coding theory, combinatorics, geometry, and polynomial optimization.

2. Symmetry in semidefinite programs Authors: Frank Vallentin, [967].

   Abstract: This paper is a tutorial in a general and explicit procedure to simplify semidefinite programs which are invariant under the action of a symmetry group. The procedure is based on basic notions of representation theory of finite groups. As an example we derive the block diagonalization of the Terwilliger algebra of the binary Hamming scheme in this framework. Here its connection to the orthogonal Hahn and Krawtchouk polynomials becomes visible.

3. Lecture notes: Semidefinite programs and harmonic analysis, Authors: Frank Vallentin, (Submitted on 11 Sep 2008), [966].

   Abstract: Lecture notes for the tutorial at the workshop HPOPT 2008 - 10th International Workshop on High Performance Optimization Techniques (Algebraic Structure in Semidefinite Programming), June 11th to 13th, 2008, Tilburg University, The Netherlands.

1.3 Duality and Optimality Conditions

1.3.1 Central Path

1. In [601], On a special class of regularized central paths for semidefinite programs, by Anhua Lin. Abstract: We present a special class of regularized central paths for standard primal-dual semidefinite program (SDP) that can be used to design path-following algorithms for finding the projection of any symmetric matrix/vector onto the solution set of the SDP. We will study the existence, convergence, and analytical properties of these paths.

2. In [896], Characterization of the limit point of the central path in semidefinite programming, by G. Sporre and A. Forsgren:
   In linear programming, the central path is known to converge to the analytic center of the set of optimal solutions. Recently, it has been shown that this is not necessarily true for linear semidefinite programming in the absence of strict complementarity. The present paper deals with the formulation of a convex problem whose solution defines the limit point of the central path. This problem is closely related to the analytic center problem for the set of optimal solutions. In the strict complementarity case the problems are shown to coincide. See http://www.optimization-online.org/DB_HTML/2002/06/488.html

1.3.2 Characterizations/Duality Theory

1. In [836], A Class of Semidefinite Programs with a rank-one solution, by Author: Guillaume Sagnol,

   Abstract: We show that a class of semidefinite programs admits a solution which is a semidefinite positive matrix of rank 1, with the consequence that these problems actually reduce to Second Order Cone Programs (SOCP). The optimization problems of this class are semidefinite packing programs with the additional property that the objective function is defined by a matrix of rank 1. Such problems arise in statistics, in the optimal design of experiments.

   Cite as: arXiv:0909.5577v2 [math.OC]

2. In [101], Global optimality conditions for quadratic optimization problems with binary constraints, by Beck, Amir(IL-TLAV); Teboulle, Marc(IL-TLAV):
   Necessary and sufficient conditions for global minima are given for a quadratic—convex or not—problem with binary constraints. They involve the knowledge of the eigenvalues of the matrix of the quadratic form. An application to the max-cut problem, the original motivation for the research, is also presented.

3. In [313], Optimality conditions for nonconvex semidefinite programming, by Forsgren, Anders,
   Optimality conditions for nonlinear programs rely on stationary (first-order) conditions and on curvature (second-order) conditions on an appropriate matrix over a subspace. Forsgren examines a special case of nonconvex semidefinite programming: minimizing a (nonlinear) function $f(x)$ subject to the condition that $C(x) \succeq 0$, where $C(x)$ is a symmetric matrix whose elements are nonlinear functions of x. Using the concept of a local semidefinite Lagrangian, the author derives first- and second-order optimality conditions "from scratch". He also shows that conventional optimality conditions follow as a special case for diagonal matrices. In an appendix, he summarizes the relevant tools from matrix calculus. The paper is carefully written, guiding the reader step by step through the analysis, altogether a pleasure to read.

   (author's abstract: This paper concerns nonlinear semidefinite programming problems for which no convexity assumptions can be made. We derive first- and second-order optimality conditions analogous to those for nonlinear programming. Using techniques similar to those used in nonlinear programming, we extend existing theory to cover situations where the constraint matrix is structurally sparse. The discussion covers the case when strict complementarity does not hold. The regularity conditions used are consistent with those of nonlinear programming in the sense that the conventional optimality conditions for nonlinear programming are obtained when the constraint matrix is diagonal.)

4. In [861], On duality theory of conic linear problems, by Alexander Shapiro,
   abstract: In this paper we discuss duality theory of optimization problems with a linear objective function and subject to linear constraints with cone inclusions, referred to as conic linear problems. We formulate the Lagrangian dual of a conic linear problem and survey some results based on the conjugate duality approach where the questions of "no duality gap" and existence of optimal solutions are related to properties of the corresponding optimal value function. We discuss in detail applications of the abstract duality theory to the problem of moments, linear semi-infinite and continuous linear programming problems.

1.4 Factorizations

1.4.1 Cholesky Factorization

1. In [63], On Positive Semidefinite Matrices with Known Null Space
   by Peter Arbenz, Zlatko Drmac

   Abstract. We show how the zero structure of a basis of the null space of a positive semidefinite matrix can be exploited to determine a positive definite submatrix of maximal rank. We discuss consequences of this result for the solution of (constrained) linear systems and eigenvalue problems. The results are of particular interest if A and the null space basis are sparse. We furthermore execute a backward error analysis of the Cholesky factorization of positive semidefinite matrices and provide new elementwise bounds.

http://epubs.siam.org/sam-bin/dbq/article/38133

1.5 Sensitivity Analysis

1.5.1 Basic/General Results

1. In [1063], An Interior-Point Perspective on Sensitivity Analysis in Semidefinite Programming by Alper Yildirim

   Abstract:
   We study the asymptotic behavior of the interior-point bounds arising from the work of Yildirim and Todd on sensitivity analysis in semidefinite programming in comparison with the optimal partition bounds. For perturbations of the right-hand side vector and the cost matrix, we show that the interior-point bounds evaluated on the central path using the Monteiro-Zhang family of search directions converge to the symmetrized version of the optimal partition bounds under appropriate nondegeneracy assumptions, which can be weaker than the usual notion of nondegeneracy. Furthermore, the analysis does not assume strict complementarity as long as the central path converges to the analytic center in a relatively controlled manner. We also show that the same convergence results carry over to iterates lying in an appropriate (very narrow) central path neighborhood if the Nesterov-Todd direction is used to evaluate the interior-point bounds.

   http://www.optimization-online.org/DB_HTML/2001/06/343.html

2. In [1064], Unifying optimal partition approach to sensitivity analysis in conic optimization, by Alper Yildirim

   review:
   The author considers the following conic optimization problem:

   $$\min \langle c, x\rangle \quad \text{subject to } Ax = b, x \in K, \mathrm{P}$$

   where K is a closed, convex solid (with nonempty interior), a pointed (contains no nontrivial subspace) cone in a finite-dimensional real vector space X with inner product $\langle\,,\rangle$, $c \in X$, $A\colon X \to Y$ is a surjective linear operator, Y is another finite-dimensional real vector space, and $b \in Y$.

   The author considers in problem (P) that the right-hand side and the cost vectors vary linearly as functions of a scalar parameter, and extends the concept of the optimal partition in linear programming (LP) and semidefinite programming (SDP) to conic optimization.

   Similar to the optimal partition approach to sensitivity analysis in LP and SDP, the author shows that the range of perturbations for which the optimal partition remains constant can be computed by solving two conic optimization problems. It is shown that under a weaker notion of nondegeneracy, this range is simply given by a minimum ratio test.

   http://www.optimization-online.org/DB_HTML/2001/10/387.html

1.5.2 Analyticity of Central Path

1. In [240],

   "On the Curvature of the Central Path of Linear Programming Theory", by Jean-erre Dedieu, Gregorio Malajovich, Mike Shub

   Abstract : We prove a linear bound on the average total curvature of the central path of linear programming theory in terms on the number of independent variables of the primal problem, and independent on the number of constraints.

2. In [405], “Analyticity of the central path at the boundary point in semidefinite programming”, by Margareta Halicka.
Abstract : In this paper we study the limiting behavior of the central path for semidefinite programming. We show that the central path is an analytic function of the barrier parameter even at the limit point, provided that the semidefinite program has a strictly complementary solution. A consequence of this property is that the derivatives - of any order - of the central path have finite limits as the barrier parameter goes to zero.
http://www.optimization-online.org/DB_HTML/2001/04/318.html

3. In [406], "On the convergence of the central path in semidefinite optimization",
by M. Halicka, E. de Klerk and C. Roos
Technical report, June/01; Faculty ITS, TU Delft, Delft, The Netherlands.

   Abstract. The central path in linear optimization always converges to the analytic center of the optimal set. This result was extended to semidefinite programming by Goldfarb and Scheinberg (SIAM J. Optim. 8: 871-886, 1998). In this paper we show that this latter result is not correct in the absence of strict complementarity. We provide a counterexample, where the central path converges to a different optimal solution. This unexpected result raises many questions. We also give a rigorous proof that the central path always converges in semidefinite optimization, by using ideas from algebraic geometry.

   URL of the ps file: http://ssor.twi.tudelft.nl/~deklerk/publications/cpath7c1.ps

1.5.3 Ill-Conditioning in SDP

1. In [914], Avoiding numerical cancellation in the interior point method for solving semidefinite programs, by Sturm,J.F.

   abstract: The matrix variables in a primal-dual pair of semidefinite programs are getting increasingly ill-conditioned as they approach a complementary solution. Multiplying the primal matrix variable with a vector from the eigenspace of the non-basic part will therefore result in heavy numerical cancellation.This effect is amplified by the scaling operation in interior point methods.In order to avoid numerical problems in interior point methods, we therefore propose to maintain the matrix variables in a product form.We discuss how the factors of this product form can be updated after a main iteration of the interior point method with Nesterov-Todd scaling.

1.6 Related Inequalities and Graph Results

1.6.1 Löwner Order and Rank Inequalities

1. In [455], “Grothendieck inequalities for semidefinite programs with rank constraint”, by Authors: Jop Briet, Fernando Mario de Oliveira Filho, Frank Vallentin at arxiv.org/abs/1011.1754

   Grothendieck inequalities are fundamental inequalities which are frequently used in many areas of mathematics and computer science. They can be interpreted as upper bounds for the integrality gap between two optimization problems: A difficult semidefinite program with rank-1 constraint and its easy semidefinite relaxation where the rank constrained is dropped. For instance, the integrality gap of the Goemans-Williamson approximation algorithm for MAX CUT can be seen as a Grothendieck inequality. In this paper we consider Grothendieck inequalities for ranks greater than 1 and we give one application in statistical mechanics: Approximating ground states in the n-vector model.

2. In “The matricial relaxation of a linear matrix inequality”, by J. William Helton (helton***at***math.ucsd.edu) Igor Klep (igor.klep***at***fmf.uni-lj.si) Scott McCullough (sam***at***math.ufl.edu)

   www.optimization-online.orgDB_HTML2010032559.html

   Abstract: Given linear matrix inequalities (LMIs) L_1 and L_2, it is natural to ask: (Q1) when does one dominate the other, that is, does $L_1(X)$ PsD imply $L_2(X)$ PsD? (Q2) when do they have the same solution set? Such questions can be NP-hard. This paper describes a natural relaxation of an LMI, based on substituting matrices for the variables x_j. With this relaxation, the domination questions (Q1) and (Q2) have elegant answers, indeed reduce to constructible semidefinite programs. Assume there is an X such that $L_1(X)$ and $L_2(X)$ are both PD, and suppose the positivity domain of L_1 is bounded. For our "matrix variable" relaxation a positive answer to (Q1) is equivalent to the existence of matrices V_j such that $L_2(x) = V_1^* L_1(x) V_1 + ... + V_k^* L_1(x) V_k$. As for (Q2) we show that, up to redundancy, L_1 and L_2 are unitarily equivalent. Such algebraic certificates are typically called Positivstellens?tze and the above are examples of such for linear polynomials. The paper goes on to derive a cleaner and more powerful Putinar-type Positivstellensatz for polynomials positive on a bounded set of the form $X|L(X)PsD$. An observation at the core of the paper is that the relaxed LMI domination problem is equivalent to a classical problem. Namely, the problem of determining if a linear map from a subspace of matrices to a matrix algebra is "completely positive". Complete positivity (CP) is one of the main techniques of modern operator theory. Thus on one hand it provides tools for studying LMIs and in the other direction, since CP is well developed, it gives perspective on the difficulties in solving LMI domination problems.

3. In [291], TITLE: A rank minimization heuristic with application to minimum order system approximation, by AUTHORS: Maryam Fazel, Haitham Hindi, and Stephen P. Boyd,

   ABSTRACT: Several problems arising in control system analysis and design, such as reduced order controller synthesis, involve minimizing the rank of a matrix variable subject to linear matrix inequality (LMI) constraints. Except in some special cases, solving this rank minimization problem (globally) is very difficult. One simple and surprisingly effective heuristic, applicable when the matrix variable is symmetric and positive semidefinite, is to minimize its trace in place of its rank. This results in a semidefinite program (SDP) which can be efficiently solved. In this paper we describe a generalization of the trace heuristic that applies to general non-symmetric, even non-square, matrices, and reduces to the trace heuristic when the matrix is positive semidefinite. The heuristic is to replace the (nonconvex) rank objective with the sum of the singular values of the matrix, which is the dual of the spectral norm. We show that this problem can be reduced to an SDP, hence efficiently solved. To motivate the heuristic, we show that the dual spectral norm is the convex envelope of the rank on the set of matrices with norm less than one. We demonstrate the method on the problem of minimum order system approximation. STATUS: Proceedings American Control Conference, June 2001, volume 6, pages 4734-4739.

   URL: http://www.stanford.edu/˜boyd/nucnorm.html

4. In [656] Mesbahi, M.(1-CAIT-J); Papavassilopoulos, G. P.(1-SCA-D) On the rank minimization problem over a positive semidefinite linear matrix inequality. (English. English summary) IEEE Trans. Automat. Control 42 (1997), no. 2, 239–243.

   Summary: "We consider the problem of minimizing the rank of a positive semidefinite matrix, subject to the constraint that an affine transformation of it is also positive semidefinite. Our method for solving this problem employs ideas from the ordered linear complementarity theory and the notion of the least element in a vector lattice. This problem is of importance in many contexts, for example in feedback synthesis problems; such an example is also provided."

1.6.2 Miscellaneous Inequalities

1. In [263], Refinements of the Hermite-Hadamard Integral Inequality for Log-Convex Functions, by S.S. Dragomir,
   Abstract: Two refinements of the classical Hermite-Hadamard integral inequality for log-convex functions and applications for special means are given.

1.6.3 Optimization With Respect to Partial Orders

1. In [228], Principal majorization ideals and optimization, by Geir Dahl,

   Abstract:
   For a given vector $b \in \Re^n$ let the principal majorization ideal $M(b)$ be the set of vectors with non-increasing coordinates that are majorized by b. $M(b)$ is a polytope and we study the 1-skeleton and lattice properties of this set. Certain optimization problems involving $M(b)$ are studied and a related class of matrices which contains the positive semidefinite matrices is investigated.
   URL: http://www.elsevier.nl/gej-ng/10/30/19/156/25/34/show/?

1.6.4 Representations Using LMIs/Semi-algebraic Sets and Graph Results

1. In [437], Semidefinite representation of convex hulls of rational varieties, Authors: Didier Henrion (LAAS, CTU/FEE),

   Abstract: Using elementary duality properties of positive semidefinite moment matrices and polynomial sum-of-squares decompositions, we prove that the convex hull of rationally parameterized algebraic varieties is semidefinite representable (that is, it can be represented as a projection of an affine section of the cone of positive semidefinite matrices) in the case of (a) curves; (b) hypersurfaces parameterized by quadratics; and (c) hypersurfaces parameterized by bivariate quartics; all in an ambient space of arbitrary dimension.

   Cite as: arXiv:0901.1821v3 [math.OC]

2. In [985], " Convex Graph Invariants", by Authors: Venkat Chandrasekaran, Pablo A. Parrilo, Alan S. Willsky

   Abstract: The structural properties of graphs are usually characterized in terms of invariants, which are functions of graphs that do not depend on the labeling of the nodes. In this paper we study convex graph invariants, which are graph invariants that are convex functions of the adjacency matrix of a graph. Some examples include functions of a graph such as the maximum degree, the MAXCUT value (and its semidefinite relaxation), and spectral invariants such as the sum of the k largest eigenvalues. Such functions can be used to construct convex sets that impose various structural constraints on graphs, and thus provide a unified framework for solving a number of interesting graph problems via convex optimization. We give a representation of all convex graph invariants in terms of certain elementary invariants, and describe methods to compute or approximate convex graph invariants tractably. We also compare convex and non-convex invariants, and discuss connections to robust optimization. Finally we use convex graph invariants to provide efficient convex programming solutions to graph problems such as the deconvolution of the composition of two graphs into the individual components, hypothesis testing between graph families, and the generation of graphs with certain desired structural properties.

3. ???????polish up???? The matricial relaxation of a linear matrix inequality Authors: J. William Helton, Igor Klep, Scott McCullough (Submitted on 3 Mar 2010)

   Abstract: Given linear matrix inequalities (LMIs) L_1 and L_2, it is natural to ask: (Q1) when does one dominate the other, that is, does $L_1(X)$ PsD imply $L_2(X)$ PsD? (Q2) when do they have the same solution set? Such questions can be NP-hard. This paper describes a natural relaxation of an LMI, based on substituting matrices for the variables x_j. With this relaxation, the domination questions (Q1) and (Q2) have elegant answers, indeed reduce to constructible semidefinite programs. Assume there is an X such that $L_1(X)$ and $L_2(X)$ are both PD, and suppose the positivity domain of L_1 is bounded. For our "matrix variable" relaxation a positive answer to (Q1) is equivalent to the existence of matrices V_j such that $L_2(x) = V_1^* L_1(x) V_1 + ... + V_k^* L_1(x) V_k$. As for (Q2) we show that, up to redundancy, L_1 and L_2 are unitarily equivalent. Such algebraic certificates are typically called Positivstellensaetze and the above are examples of such for linear polynomials. The paper goes on to derive a cleaner and more powerful Putinar-type Positivstellensatz for polynomials positive on a bounded set of the form $\{X|L(X)PsD\}$. An observation at the core of the paper is that the relaxed LMI domination problem is equivalent to a classical problem. Namely, the problem of determining if a linear map from a subspace of matrices to a matrix algebra is "completely positive".

Comments: 34 pages; supplementary material is available in the source file, or see this http URL Subjects: Operator Algebras (math.OA); Optimization and Control (math.OC) MSC classes: Primary 46L07, 14P10, 90C22; Secondary 11E25, 46L89, 13J30 Cite as: arXiv:1003.0908v1 [math.OA]

4. In [631], On a parametrization of positive semidefinite matrices with zeros, Authors: Mathias Drton, Josephine Yu

   Abstract: We study a class of parametrizations of convex cones of positive semidefinite matrices with prescribed zeros. Each such cone corresponds to a graph whose non-edges determine the prescribed zeros. Each parametrization in this class is a polynomial map associated with a simplicial complex supported on cliques of the graph. The images of the maps are convex cones, and the maps can only be surjective onto the cone of zero-constrained positive semidefinite matrices when the associated graph is chordal and the simplicial complex is the clique complex of the graph. Our main result gives a semi-algebraic description of the image of the parametrizations for chordless cycles. The work is motivated by the fact that the considered maps correspond to Gaussian statistical models with hidden variables.

5. In [491], Linear Matrix Inequality Representation of Sets, by J. William Helton, Victor Vinnikov:

   This article concerns the question: which subsets of $\Re^m$ can be represented with Linear Matrix Inequalities, LMIs? This gives some perspective on the scope and limitations of one of the most powerful techniques commonly used in control theory. Also before having much hope of representing engineering problems as LMIs by automatic methods one needs a good idea of which problems can and cannot be represented by LMIs. Little is currently known about such problems. In this article we give a necessary condition, we call "rigid convexity", which must hold for a set $C \in \Re^m$ in order for C to have an LMI representation. Rigid convexity is proved to be necessary and sufficient when $m = 2$. This settles a question formally stated by Pablo Parrilo and Berndt Sturmfels in [PSprep].

6. In [63], ELA, Volume 11, pp. 281-291, November 2004, "Interpolation by Matrices", by Allan Pinkus

   abstract: Assume that two sets of k vectors in R^n are given, namely $\{x^1, ..., x^k\}$ and $\{y^1, ..., y^k\}$, and a class of matrices, e.g., positive definite matrices, positive matrices, strictly totally positive matrices, or P-matrices. The question considered in this paper is that of determining necessary and sufficient conditions on these sets of vectors such that there exists an nxn matrix A in the given class satisfying $Ax^j = y^j (j = 1, ..., k)$.

7. In [433], "Sufficient and Necessary Conditions for Semidefinite Representability of Convex Hulls and Sets", Authors: J. William Helton, Jiawang Nie (Submitted on 25 Sep 2007)

   Abstract: A set $S \subseteq \Re^n$ is called to be *Semidefinite (SDP)* representable if S equals the projection of a set in higher dimensional space which is describable by some Linear Matrix Inequality (LMI). The contributions of this paper are: (i) For bounded SDP representable sets $W_1, ..., W_m$, we give an explicit construction of an SDP representation for $?cv\cup_{k=1}^m W_k$. This provides a technique for building global SDP representations from the local ones. (ii) For the SDP representability of a compact convex semialgebraic set S, we prove sufficient condition: the boundary $?bdS$ is positively curved, and necessary condition: $?bdS$ has nonnegative curvature at smooth points and on nondegenerate corners. This amounts to the strict versus nonstrict quasi-concavity of defining polynomials on those points on $?bdS$ where they vanish. The gaps between them are $?bdS$ having positive versus nonnegative curvature and smooth versus nonsmooth points. A sufficient condition bypassing the gaps is when some defining polynomials of S are sos-concave. (iii) For the SDP representability of the convex hull of a compact nonconvex semialgebraic set T, we find that the critical object is $?pt_cT$, the maximum subset of $?ptT$ contained in $?pt?cvT$. We prove sufficient conditions for SDP representability: $?pt_cT$ is positively curved, and necessary conditions: $?pt_cT$ has nonnegative curvature at smooth points and on nondegenerate corners. The gaps between them are similar to case (ii). The positive definite Lagrange Hessian (PDLH) condition is also discussed.

8. Certificates of convexity for basic semi-algebraic sets, by Jean B. Lasserre (LAAS) (Submitted on 28 Jan 2009/ http://arxiv.org/abs/0901.4497v1)

   Abstract: We provide two certificates of convexity for arbitrary basic semi-algebraic sets of $\mathbb{R}^n$. The first one is based on a necessary and sufficient condition whereas the second one is based on a sufficient (but simpler) condition only. Both certificates are obtained from any feasible solution of a related semidefinite program and so can be obtained numerically (however, up to machine precision).

9. Exposed faces of semidefinite representable sets by Tim Netzer; Daniel Plaumann; Markus Schweighofer (Submitted on 19 Feb 2009)

   http://arxiv.org/abs/0902.3345v1

   Abstract: A linear matrix inequality (LMI) is a condition stating that a symmetric matrix whose entries are affine linear combinations of variables is positive semidefinite. Motivated by the fact that diagonal LMIs define polyhedra, the solution set of an LMI is called a spectrahedron. Linear images of spectrahedra are called semidefinite representable sets. Part of the interest in spectrahedra and semidefinite representable sets arises from the fact that one can efficiently optimize linear functions on them by semidefinite programming, like one can do on polyhedra by linear programming. It is known that every face of a spectrahedron is exposed. This is also true in the general context of rigidly convex sets. We study the same question for semidefinite representable sets. Lasserre proposed a moment matrix method to construct semidefinite representations for certain sets. Our main result is that this method can only work if all faces of the considered set are exposed. This necessary condition complements sufficient conditions recently proved by Lasserre, Helton and Nie.

2 ALGORITHMS

Many current references are given at the web site:

2.1 Barrier Problems

1. At URL hrefarxiv.org/abs/0803.1990http://arxiv.org/abs/0803.1990: Subsampling Algorithms for Semidefinite Programming by Alexandre d'Aspremont

   We derive a stochastic gradient algorithm for semidefinite optimization using randomization techniques. The algorithm uses subsampling techniques to reduce the computational cost of each iteration. The subsampling ratio explicitly controls the granularity of the algorithm, i.e. the tradeoff between cost per iteration and total number of iterations. Furthermore, the complexity of the algorithm is directly proportional to the complexity (i.e. rank) of the solution. We study numerical performance on some large-scale problems arising in statistical learning.

2. In [682], Penalty/Barrier Multiplier Algorithm for Semidefinite Programming by Leonid Mosheyev and Michael Zibulevsky

   We present a generalization of the Penalty/Barrier Multiplier algorithm for the semidefinite programming, based on a matrix form of Lagrange multipliers. Our approach allows to use among others logarithmic, shifted logarithmic, exponential and a very effective quadratic-logarithmic penalty/barrier functions. We present dual analysis of the method, based on its correspondence to a proximal point algorithm with nonquadratic distance-like function. We give computationally tractable dual bounds, which are produced by the Legendre transformation of the penalty function. Numerical results for large-scale problems from robust control, stable truss topology design and optimal material design demonstrate high efficiency of the algorithm.

   Preprint, September, 1999, Contact: michael@cs.unm.edu URL: http://www-unix.mcs.anl.gov/otc/InteriorPoint/abstra

2.2 Empirical Results

2.2.1 Benchmarking

1. In [658], "An Independent Benchmarking of SDP and SOCP Solvers", by Hans Mittlemann, abstract: Results from the DIMACS challenge.

   URL at Hans Mittlemann's web page: http://plato.la.asu.edu/papers.html

2.3 General Convex Programming

In [530], "A generalized augmented Lagrangian method for semidefinite programming, by M. Kocvara and M. Stingl. This article describes a generalization of the PBM method by Ben-Tal and Zibulevsky to convex semidefinite programming problems. The algorithm used is a generalized version of the Augmented Lagrangian method. We present details of this algorithm as implemented in a new code PENNON. The code can also solve second-order conic program- ming (SOCP) problems, as well as problems with a mixture of SDP, SOCP and NLP constraints. Results of extensive numerical tests and comparison with other SDP codes are presented."

2.3.1 Second Order Cone Programming

1. In [191], Solving second order cone programming via the augmented systems, by Zhi Cai and Kim-Chuan Toh

   Abstract : The standard normal equation based implementation of interior-point methods for second order cone programming encounters stability problems in the computation of search directions. Based on the eigenvalue decomposition of the $(1, 1)$ block of the augmented equation, a reduced augmented equation approach is proposed to overcome the stability problems. Numerical experiments show that the new approach is much more stable than the normal equation based approach.

   URL at optimization online: http://www.optimization-online.org/DB_HTML/2002/08/517.html

2.3.2 The Convex Programming Problem

1. In [65], A BFGS-IP algorithm for solving strongly convex optimization problems with feasibility enforced by an exact penalty approach, by Authors: Paul Armand , Jean Charles Gilbert , Sophie Jan,

   This paper introduces and analyses a new algorithm for minimizing a convex function subject to a finite number of convex inequality constraints. It is assumed that the Lagrangian of the problem is strongly convex. The algorithm combines interior point methods for dealing with the inequality constraints and quasi-Newton techniques for accelerating the convergence. Feasibility of the iterates is progressively enforced thanks to shift variables and an exact penalty approach. Global and q-superlinear convergence is obtained for a fixed penalty parameter; global convergence to the analytic center of the optimal set is ensured when the barrier parameter tends to zero, provided strict complementarity holds.

   URL at optimization online: http://www.optimization-online.org/DB_HTML/2001/02/267.html

2.4 Large Sparse Case

2.4.1 Symmetrization Techniques

2.4.2 Gauss-Newton Direction

1. In [317], Software Performance on Nonlinear Least-Squares Problems, by Christina Fraley.

2.4.3 Augmented System

1. In [945], Solving large scale semidefinite programsvia an iterative solver onthe augmented systems, by Kim-Chuan Toh

   Abstract : The search directions in an interior-point method for large scale semidefinite programming (SDP) can be computed by applying a Krylov iterative method to either the Schur complement equation (SCE) or the augmented equation. Both methods suffer from slow convergence as interior-point iterates approach optimality. Numerical experiments have shown that diagonally preconditioned conjugate residual method on the SCE typically takes a huge number of steps to converge. However, it is difficult to incorporate cheap and effective preconditioners into the SCE. This paper proposes to apply the preconditioned symmetric quasi-minimal residual (PSQMR) method to a reduced augmented equation that is derived from the augmented equation by utilizing the eigenvalue structure of the interior-point iterates. Numerical experiments on SDP problems arising from maximum clique and selected SDPLIB problems show that moderately accurate solutions can be obtained with a modest number of PSQMR steps using the proposed preconditioned reduced augmented equation. An SDP problem with 127600 constraints is solved in about 6.5 hours to an accuracy of 10^{-6} in relative duality gap.

   URL of the ps file at optimization online:
   http://www.optimization-online.org/DB_HTML/2003/01/596.html

2.4.4 Conjugate Gradient Methods

1. In [943], *Solving some large scale semidefinite programs via the conjugate residual method*, by Kim-Chuan Toh, and Masakazu Kojima,
   Research Report, Department of Mathematics, National University of Singapore, August 2000
   URL of the ps file:
   http://www.math.nus.edu.sg/~mattohkc/papers/largesdp.ps

   Most current implementations of interior-point methods for semidefinite programming use a direct method to solve the Schur complement equation (SCE) $M\Delta y = h$ in computing the search direction. When the number of constraints is large, the problem of having insufficient memory to store M can be avoided if an iterative method is used instead. Numerical experiments have shown that the conjugate residual (CR) method typically takes a huge number of steps to generate a high accuracy solution. On the other hand, it is difficult to incorporate traditional preconditioners into the SCE, except for block diagonal preconditioners. We decompose the SCE into a 2×2 block system by decomposing Δy (similarly for h) into two orthogonal components with one lying in a certain subspace that is determined from the structure of M. Numerical experiments on semidefinite programming problems arising from Lovász θ-function of graphs and MAXCUT problems show that high accuracy solutions can be obtained with moderate number of CR steps using the proposed equation.

2. In [229], *Smooth Optimization for Sparse Semidefinite Programs*, by Alexandre d'Aspremont, at http://arxiv.org/abs/math/0512344

   We show that the optimal complexity of Nesterov's smooth first-order optimization technique is preserved when the function value and gradient are only computed up to a small, uniformly bounded error. This means that only a partial eigenvalue decomposition is necessary when applying this technique to semidefinite programs, thus significantly reducing the method's computational and memory requirements. This also allows sparse problems to be solved efficiently.

2.4.5 Exploiting Low Rank Constraints

Many Combinatorial applications result in problems with constraints trace A_iX with the matrices A_i of low rank. This can be exploited in many different ways. URL: http://www.zib.de/helmberg/semidef.html in the *Interior Point Algorithms* section.

1. In [123], Benson, Steven J.; Ye, Yinyu; Zhang, Xiong,
   Solving large-scale sparse semidefinite programs for combinatorial optimization

   Summary: "We present a dual-scaling interior-point algorithm and show how it exploits the structure and sparsity of some large-scale problems. We solve the positive semidefinite relaxation of combinatorial and quadratic optimization problems subject to Boolean constraints. We report the first computational results of interior-point algorithms for approximating maximum cut semidefinite programs with dimension up to 3,000."

2.4.6 Parallel Implementations

1. In [889], *A PARALLEL conic interior point decomposition approach for BLOCK-ANGULAR semidefinite programs*, by Kartik K. Sivaramakrishnan:
   URL: www4.ncsu.edu˜kksivara/publications/parallel-conic-blockangular.pdf Abstract: One can exploit the underlying sparsity and symmetry in a semidefinite program to preprocess it into an equivalent block-angular semidefinite program.

2. In [122], *Parallel Computing on Semidefinite Programs*, by Steven Benson:
   Abstract : This paper demonstrates how interior-point methods can use multiple processors efficiently to solve large semidefinite programs that arise in VLSI design, control theory, and graph coloring. Previous implementations of these methods have been restricted to a single processor. By computing and solving the Schur complement matrix in parallel, multiple processors enable the faster solution of large problems to moderate precision. The dual-scaling algorithm for semidefinite programming was adapted to a distributed-memory environment and used to solve larger problems than could previously be solved by interior-point algorithms. The results also show that interior-point algorithms possess good scalability on parallel architectures.

2.4.7 Exploiting Matrix Completions

1. In [181], *Semidefinite programming in the space of partial positive semidefinite matrices*, by Sam Burer. http://www.optimization-online.org/DB_HTML/2002/05/472.html

   Abstract : We build upon the work of Fukuda et al. [336] and Nakata et al. [689], in which the theory of partial positive semidefinite matrices has been applied to the semidefinite programming (SDP) problem as a technique for exploiting sparsity in the data. In contrast to their work, which improves an existing algorithm that is based on the HRVW/KSH/M search direction, we present a primal-dual path-following algorithm that is based on a new search direction, which, roughly speaking, is defined completely within the space of partial symmetric matrices. We show that the proposed algorithm computes a primal-dual solution to the SDP problem having duality gap less than a fraction $\varepsilon > 0$ of the initial duality gap in $\mathcal{O}(n \log(\varepsilon^{-1}))$ iterations, where n is the size of the matrices involved. Moreover, we present computational results showing that the algorithm possesses several advantages over other existing implementations.

2. In [689], *Exploiting sparsity in semidefinite programming via matrix completion II: implementation and numerical results*, by Kazuhide Nakata, Katsuki Fujisawa, Mituhiro Fukuda, Masakazu Kojima, Kazuo Murota. Research Report, Dept. of Information Sciences, Tokyo Institute of Technology, Tokyo, Japan, number B-368. Feb. 2001.
   URL of the ps file at optimization online:
   http://www.optimization-online.org/DB_HTML/2001/03/290.html.

   In Part I of this series of articles, we introduced a general framework of exploiting the aggregate sparsity pattern over all data matrices of large scale and sparse semidefinite programs (SDPs) when solving them by primal-dual interior-point methods. This framework is based on some results about positive semidefinite matrix completion, and it can be embodied in two different ways. One is by a conversion of a given sparse SDP having a large scale positive semidefinite matrix variable into an SDP having multiple but smaller positive semidefinite matrix variables. The other is by incorporating a positive definite matrix completion itself in a primal-dual interior-point method. The current article presents the details of their implementations. We introduce new techniques to deal with the sparsity through a clique tree in the former method and through new computational formulae in the latter one. Numerical results over different classes of SDPs show that these methods can be very efficient for some problems.

3. In [492] *A Parallel Primal-Dual Interior-Point Method for Semidefinite Programs Using Positive Definite Matrix Completion*, by K.Nakata, M.Yamashita, K.Fujisawa and M.Kojima, Research Report, Dept. of Information Sciences, Tokyo Institute of Technology, Tokyo, Japan, number B-398. Nov. 2003.
   URL of the ps file at optimization online:
   http://www.is.titech.ac.jp/~kojima/sdp.html

   Abstract: A parallel computational method SDPARA-C is presented for SDPs (semidefinite programs). It combines two methods SDPARA and SDPA-C proposed by the authors who developed a software package SDPA. SDPARA is a parallel implementation of SDPA and it features parallel computation of the elements of the Schur complement equation system and a parallel Cholesky factorization of its coefficient matrix. SDPARA can effectively solve SDPs with a large number of equality constraints, however, it does not solve SDPs with a large scale matrix variable with similar effectiveness. SDPA-C is a primal-dual interior-point method using the positive definite matrix completion technique by Fukuda et al, and it performs effectively with SDPs with a large scale matrix variable, but not with a large number of equality constraints. SDPARA-C benefits from the strong performance of each of the two methods. Furthermore, SDPARA-C is designed to attain a high scalability by considering most of the expensive computations involved in the primal-dual interior-point method. Numerical experiments with the three parallel software packages SDPARA-C, SDPARA and PDSDP by Benson show that SDPARA-C efficiently solve SDPs with a large scale matrix variable as well as a large number of equality constraints with a small amount of memory.

4. In [898], *A Fully Sparse Implementation of a Primal-Dual Interior-Point Potential Reduction Method for Semidefinite Programming* , by Gun Srijuntongsiri and Stephen Vavasis, http:www.optimization-online.org/DB_HTM

   Abstract : In this paper, we show a way to exploit sparsity in the problem data in a primal-dual potential reduction method for solving a class of semidefinite programs. When the problem data is sparse, the dual variable is also sparse, but the primal one is not. To avoid working with the dense primal variable, we apply Fukuda et al.'s theory of partial matrix completion and work with partial matrices instead. The other place in the algorithm where sparsity should be exploited is in the computation of the search direction, where the gradient and the Hessian-matrix product of the primal and dual barrier functions must be computed in every iteration. By using an idea from automatic differentiation in backward mode, both the gradient and the Hessian-matrix product can be computed in time proportional to the time needed to compute the barrier functions of sparse variables itself. Moreover, the high space complexity that is normally associated with the use of automatic differentiation in backward mode can be avoided in this case. In addition, we suggest a technique to efficiently compute the determinant of the positive definite matrix completion that is required to compute primal search directions. The method of obtaining one of the primal search directions that minimizes the number of the evaluations of the determinant of the positive definite completion is also proposed. We then implement the algorithm and test it on the problem of finding the maximum cut of a graph.

2.4.8 Exploiting Matrix Structure

1. In http://www.optimization-online.org/DB_HTML/2008/10/2107.html, Exploiting special structure in semidefinite programming: a survey of theory and applications by Etienne De Klerk(e.deklerk***at***uvt.nl) ¡br¿ Abstract: Semidefinite Programming (SDP) may be seen as a generalization of Linear Programming (LP). In particular, one may extend interior point algorithms for LP to SDP, but it has proven

much more difficult to exploit structure in the SDP data during computation. We survey three types of special structure in SDP data: 1) a common 'chordal' sparsity pattern of all the data matrices. This structure arises in applications in graph theory, and may also be used to deal with more general sparsity patterns in a heuristic way. 2) low rank of all the data matrices. This structure is common in SDP relaxations of combinatorial optimization problems, and SDP approximations of polynomial optimization problems. 3) the situation where the data matrices are invariant under the action of a permutation group, or, more generally, where the data matrices belong to a low dimensional matrix algebra. Such problems arise in truss topology optimization, particle physics, coding theory, computational geometry, and graph theory. We will give an overview of existing techniques to exploit these structures in the data. Most of the paper will be devoted to the third situation, since it has received the least attention in the literature so far.

2.5 Newton Method

1. In [431], Title: Newton's method on Graßmann manifolds, Authors: Uwe Helmke, Knut Hüper and Jochen Trumpf,

   MSC-class: 49M15; 53B20; 65F15; 15A18

   A general class of Newton algorithms on Graßmann and Lagrange-Graßmann manifolds is introduced, that depends on an arbitrary pair of local coordinates. Local quadratic convergence of the algorithm is shown under a suitable condition on the choice of coordinate systems. Our result extends and unifies previous convergence results for Newton's method on a manifold. Using special choices of the coordinates, new numerical algorithms are derived for principal component analysis and invariant subspace computations with improved computational complexity properties.

2.6 Nonlinear Semidefinite Programming

1. In [468], An interior method for nonconvex semidefinite programs, by Florian Jarre.

   The paper addresses the solution of nonlinear programming problems in the presence of nonlinear matrix inequality (semidefinite) constraints and/or the ordinary nonlinear function inequality constraints. The objective is to find a local optimum of the problem. The key idea is to consider a combined barrier function, using the logarithmic determinant for the matrix inequality constraint and the ordinary logarithmic barrier function for the other inequality constraints. Then, a predictor-corrector type method based on the barrier function is proposed. Interesting features of the method include its particular implementations at the predictor and the corrector steps. The predictor search direction is actually based on a barrier function of the linearized constraint functions, thus it is a kind of Dikin affine-scaling search direction. In the corrector step, a trust region method for minimizing the barrier function (for the fixed barrier parameter) is used, in combination with a kind of line search. Tests of the method on some standard problems show the potential of the method for solving nonconvex problems.

2. In [322], A sensitivity analysis and a convergence result for a sequential semidefinite programming method, by Roland W. Freund and Florian Jarre.

   The authors consider the solution of nonlinear programs with nonlinear semidefiteness constraints. In particular, a suitable symmetrization procedure needs to be chosen.

3. In [439], SDLS: a Matlab package for solving conic least-squares problems Authors: Didier Henrion (LAAS, CVUT), Jerome Malick (LJK), (Submitted on 17 Sep 2007)

   Abstract: This document is an introduction to the Matlab package SDLS (Semi-Definite Least-Squares) for solving least-squares problems over convex symmetric cones. The package is shortly presented through the addressed problem, a sketch of the implemented algorithm, the syntax and calling sequences, a simple numerical example and some more advanced features. The implemented method consists in solving the dual problem with a quasi-Newton algorithm. We note that SDLS is not the most competitive implementation of this algorithm: efficient, robust, commercial implementations are available (contact the authors). Our main goal with this Matlab SDLS package is to provide a simple, user-friendly software for solving and experimenting with semidefinite least-squares problems. Up to our knowledge, no such freeware exists at this date. by Roland W. Freund and Florian Jarre.

2.7 Stability

1. In [192], Solving second order cone programming via a reduced augmented system approach, by Cai, Zhi; Toh, Kim-Chuan,

   Abstract: In interior point algorithms, a linear system of equations is solved at each iteration. The usual approach is to solve the so-called Schur complement equations (SCE) that are obtained by simplifying the larger system of augmented equations. The drawback is that the SCE may be badly conditioned if the barrier parameter is small.

   The authors suggest a trade-off between solving the SCE and the augmented equations, by formulating a system of so-called reduced augmented equations that is larger than the SCE system, but in general smaller than the full system of augmented equations. The main result in the paper is that the coefficient matrix of the reduced augmented equations has a condition number that may be bounded independently of the barrier parameter, if the (second order cone) optimization problem under consideration satisfies strict complementarity (Theorem 5.2).

   The authors present detailed numerical results that confirm that the new approach allows one to obtain higher numerical accuracy at an acceptable increase in computational effort.

2. In [362], Product-form Cholesky factorization in interior point methods for second-order cone programming, by Goldfarb, D.(1-CLMB-I); Scheinberg, K.,

   abstract: Second-order cone programming (SOCP) problems include linear programming, convex quadratic programming, convex quadratically constrained quadratic programming and other applications. SOCP problems are typically solved by interior point methods. As in linear programming, interior point methods can, in theory, solve SOCP in polynomial time and can, in practice, exploit sparsity in the problem data. This paper is concerned with the efficient implementation of interior point methods for SOCP. The authors propose a product-form Cholesky factorization approach to solve a linear system of equations that are obtained by applying Newton's method to the system of perturbed optimality conditions in interior point methods, and they show that the proposed approach is numerically stable. They derive several product-form Cholesky factorization variants and compare their theoretical performance. They prove that the elements of L in the Cholesky factorizations LDL^T are uniformly bounded as the duality gap tends to zero as long as the iterates remain in some conic neighborhood of the central path. Theoretical results are analogous to similar results proved in D. Goldfarb and K. Scheinberg, Math. Program. 99 (2004), no. 1, Ser. A, 1–34; MR2031774 (2005b:90157), for linear programming problems. Finally some numerical results are given.

2.8 Path Following

2.9 Potential Reduction

3 APPLICATIONS

3.1 Best Approximation

3.1.1 Minimum Norm Problems

1. In [348], A conic formulation for l_p-norm optimization, by Frangois Glineur and Tamas Terlaky

   Abstract : In this paper, we formulate the l_p-norm optimization problem as a conic optimization problem, derive its standard duality properties and show it can be solved in polynomial time. We first define an ad hoc closed convex cone, study its properties and derive its dual. This allows us to express the standard l_p-norm optimization primal problem as a conic problem involving this cone. Using convex conic duality and our knowledge about this cone, we proceed to derive the dual of this problem and prove the well-known regularity properties of this primal-dual pair, i.e. zero duality gap and primal attainment. Finally, we prove that the class of l_p-norm optimization of problems can be solved up to a given accuracy in polynomial time, using the framework of interior-point algorithms and self-concordant barriers.

Keywords : convex optimization, conic optimization, l_p-norm optimization

Category 1 : Convex and Nonsmooth Optimization (Convex Optimization)

Category 2 : Linear, Cone and Semidefinite Programming (Other)

Citation : IMAGE0005, Service MATHRO, Faculti Polytechnique de Mons, Mons, Belgium, May/00

URL: http://www.optimization-online.org/DB_HTML/2001/03/292.html

3.1.2 Relaxations

1. In [532], A General Framework for Convex Relaxation of Polynomial Optimization Problems over Cones, by M. Kojima, Research Report B-380, Dept. of Mathematical and Computing Sciences, Tokyo Institute of Technology, Oh-Okayama, Meguro, Tokyo 152-8552, Japan, April 2002.

   Abstract:
   The class of POPs (polynomial optimization problems) over cones covers a wide range of optimization problems such as 0-1 integer linear and quadratic programs, nonconvex quadratic programs and bilinear matrix inequalities. This paper presents a new framework for convex relaxation of POPs over cones in terms of linear optimization problems over cones. It provides a unified treatment of many existing convex relaxationmethods based on the lift-and-project linear programming procedure, the reformulation-linearization technique and the semidefinite programming relaxation for a variety of problems. It also extends the theory of convex relaxation methods, and thereby brings flexibility and richness in practical use of the theory.

   URL of postscript file: www.is.titech.ac.jp/~kojima/sdp.html/

3.1.3 Differential Equations

1. In [438], "Moment and SDP relaxation techniques for smooth approximations of nonlinear differential equations", by Authors: Didier Henrion (LAAS, FEL-CVUT), Jean-Bernard Lasserre (LAAS), Martin Mevissen (LAAS) (Submitted on 24 Mar 2010)

   Abstract: Combining recent moment and sparse semidefinite programming (SDP) relaxation techniques, we propose an approach to find smooth approximations for solutions of nonlinear differential equations. Given a system of nonlinear differential equations, we apply a technique based on finite differences and sparse SDP relaxations for polynomial optimization problems (POP) to obtain a discrete approximation of its solution. In a second step we apply maximum entropy estimation (using moments of a Borel measure associated with the discrete solution) to obtain a smooth closed-form approximation. The approach is illustrated on a variety of linear and nonlinear ordinary differential equations (ODE) and partial differential equations (PDE) and preliminary numerical results are reported.

   Available at: arxiv.orgabs1003.4608

3.2 Discrete/Combinatorial Optimization

3.2.1 Abstract/General Problems

1. In "A Class of Semidefinite Programs with a rank-one solution" by Guillaume Sagnol, guillaume.sagnolinria.fr

Abstract: We show that a class of semidefinite programs admits a solution which is a semidefinite positive matrix of rank 1, with the consequence that these problems actually reduce to Second Order Cone Programs (SOCP). The optimization problems of this class are semidefinite packing programs with the additional property that the objective function is defined by a matrix of rank 1. Such problems arise in statistics, in the optimal design of experiments. Keywords SDP, Semidefinite Packing Program, rank 1-solution, SOCP, Optimal Experimental

2. In [435], Semidefinite geometry of the numerical range, by Didier Henrion (LAAS, Fel-Cvut) Abstract: Using elementary duality properties of positive semidefinite moment matrices and polynomial sum-of-squares decompositions, we prove that the convex hull of rationally parameterized algebraic varieties is semidefinite representable (that is, it can be represented as a projection of an affine of the cone of positive semidefinite matrices) in the case of (a) curves; (b) hypersurfaces parameterized by quadratics; and (c) hypersurfaces parameterized by bivariate quartics; all in an ambient space of arbitrary dimension.

3. In [434], Semidefinite geometry of the numerical range, by Didier Henrion (LAAS, Fel-Cvut)

   Abstract: The numerical range of a matrix is studied geometrically via the cone of positive semidefinite matrices (or semidefinite cone for short). In particular it is shown that the feasible set of a two-dimensional linear matrix inequality (LMI), an affine section of the semidefinite cone, is always dual to the numerical range of a matrix, which is therefore an affine projection of the semidefinite cone. Both primal and dual sets can also be viewed as convex hulls of explicit algebraic plane curve components. Several numerical examples illustrate this interplay between algebra, geometry and semidefinite programming duality. Finally, these techniques are used to revisit a theorem in statistics on the independence of quadratic forms in a normally distributed vector.

4. In [436], Semidefinite geometry of the numerical range, by Authors: Didier Henrion (LAAS, FEL-CVUT) (Submitted on 25 Mar 2010)

   Abstract: The numerical range of a matrix is studied geometrically via the cone of positive semidefinite matrices (or semidefinite cone for short). In particular it is shown that the feasible set of a two-dimensional linear matrix inequality (LMI), an affine section of the semidefinite cone, is always dual to the numerical range of a matrix, which is therefore an affine projection of the semidefinite cone. Both primal and dual sets can also be viewed as convex hulls of explicit algebraic plane curve components. Several numerical examples illustrate this interplay between algebra, geometry and semidefinite programming duality. Finally, these techniques are used to revisit a theorem in statistics on the independence of quadratic forms in a normally distributed vector.

   available at: arxiv.orgabs1003.4837

5. In [337], Idempotent interval analysis and optimization problems, by Authors: Grigori Litvinov, Andrei Sobolevskii,

   Many problems in optimization theory are strongly nonlinear in the traditional sense but possess a hidden linear structure over suitable idempotent semirings. After an overview of 'Idempotent Mathematics' with an emphasis on matrix theory, interval analysis over idempotent semirings is developed. The theory is applied to construction of exact interval solutions to the interval discrete stationary Bellman equation. Solution of an interval system is typically NP-hard in the traditional interval linear algebra; in the idempotent case it is polynomial. A generalization to the case of positive semirings is outlined.

   URL: http://arXiv.org/abs/math/0101080

3.2.2 Correlation Matrices

1. In [634], "A dual approach for conic least-squares problems", by J. Malick, abstract: We proose a method to solve "conic least-squares" probles: to project a point, in a finite-dimensional Euclidean

space, onto the interesection of a closed convex cone and an affine subspace.

2. In [1075], "Properties of a Covariance Matrix with an Application to D-optimal Design" ELA, Volume 10, pp. 65-76, March 2003; by Zewen Zhu, Daniel C. Coster and Leroy B. Beasley

   In this paper, a covariance matrix of circulant correlation, R, is studied. A pattern of entries in the inverse of R independent of the value r of the correlation coefficient is proved based on a recursive relation among the entries of the inverse of R. The D-optimal design for simple linear regression with circulantly correlated observations on [a, b] (a ¡ b) is obtained if even observations are taken and the correlation coefficient is between 0 and 0.5.

3.2.3 Euclidean Distance Matrices

1. In [83], Determinant of the distance matrix of a tree with matrix weights, by R.B. Bapat

   Abstract:
   The T be a tree with n vertices and let D be the distance matrix of T. According to a classical result due to Graham and Pollack, the determinant of D is a function of n, but does not depend on T. We allow the edges of T to carry weights, which are square matrices of a fixed order. The distance matrix D of T is then defined in a natural way. We obtain a formula for the determinant of D, which involves only the determinants of the sum and the product of the weight matrices.

3.2.4 SDP Relaxations of Graph Related Problems

1. In [659], High accuracy semidefinite programming bounds for kissing numbers
   Authors: Hans D. Mittelmann, Frank Vallentin

   Abstract: The kissing number in n-dimensional Euclidean space is the maximal number of non-overlapping unit spheres which simultaneously can touch a central unit sphere. Bachoc and Vallentin developed a method to find upper bounds for the kissing number based on semidefinite programming. This paper is a report on high accuracy calculations of these upper bounds for $n \leq 24$. The bound for $n = 16$ implies a conjecture of Conway and Sloane: There is no 16-dimensional periodic point set with average theta series $1 + 7680q^3 + 4320q^4 + 276480q^5 + 61440q^6 + \ldots$

   Cite as: arXiv:0902.1105v3

3.2.5 Max-Cut Problem

1. In [123], Solving large-scale sparse semidefinite programs for combinatorial optimization, by Steven J. Benson; Ye, Yinyu; Zhang, Xiong

   Summary: "We present a dual-scaling interior-point algorithm and show how it exploits the structure and sparsity of some large-scale problems. We solve the positive semidefinite relaxation of combinatorial and quadratic optimization problems subject to Boolean constraints. We report the first computational results of interior-point algorithms for approximating maximum cut semidefinite programs with dimension up to 3,000."

2. In [598], "Linear transformations which preserve Hermitian and positive semidefinite operators", by Li, C-K. and S. Pierce.

   Summary: "The linear operators that map the set of real or complex (rank one) correlation matrices onto itself are characterized".

3.2.6 Polynomial Optimization

1. In [458], Computation with Polynomial Equations and Inequalities arising in Combinatorial Optimization, Authors: Jesus A. De Loera, Peter N. Malkin, Pablo A. Parrilo (Submitted on 4 Sep 2009)

   Abstract: The purpose of this note is to survey a methodology to solve systems of polynomial equations and inequalities. The techniques we discuss use the algebra of multivariate polynomials with coefficients over a field to create large-scale linear algebra or semidefinite programming relaxations of many kinds of feasibility or optimization questions. We are particularly interested in problems arising in combinatorial optimization.

2. In [754], Minimizing Polynomial Functions by Authors: Pablo A. Parrilo, Bernd Sturmfels

   Comments: This paper was presented at the Workshop on Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, held at DIMACS, Rutgers University, March 12-16, 2001
   Subj-class: Optimization and Control; Algebraic Geometry
   MSC-class: 13J30, 90C22, 13P10, 65H10

   Abstract: We compare algorithms for global optimization of polynomial functions in many variables. It is demonstrated that existing algebraic methods (Gröbner bases, resultants, homotopy methods) are dramatically outperformed by a relaxation technique, due to N.Z. Shor and the first author, which involves sums of squares and semidefinite programming. This opens up the possibility of using semidefinite programming relaxations arising from the Positivstellensatz for a wide range of computational problems in real algebraic geometry.
   This paper was presented at the Workshop on Algorithmic and Quantitative Aspects of Real Algebraic Geometry in Mathematics and Computer Science, held at DIMACS, Rutgers University, March 12-16, 2001.

   URL: http://arXiv.org/abs/math/0103170

3. In [777], Convexity of quadratic transformations and its use in control and optimization
   Author: POLYAK, B.T.:

   This paper presents several generalizations of the theorems about convexity of a quadratic image of R^N. Results regarding the nonnegativity of a quadratic form subject to quadratic constraints are also obtained. Applications to ellipsoidal approximations and to optimization are discussed. Several examples and counterexamples are presented.

4. In [777], The extremal volume ellipsoids of convex bodies, their symmetry properties, and their determination in some special cases Authors: Osman Guler, Filiz G—utuna (Submitted on 5 Sep 2007)

   Abstract: A convex body K has associated with it a unique circumscribed ellipsoid CE(K) with minimum volume, and a unique inscribed ellipsoid IE(K) with maximum volume. We first give a unified, modern exposition of the basic theory of these extremal ellipsoids using the semi-infinite programming approach pioneered by Fritz John in his seminal 1948 paper. We then investigate the automorphism groups of convex bodies and their extremal ellipsoids. We show that if the automorphism group of a convex body K is large enough, then it is possible to determine the extremal ellipsoids CE(K) and IE(K) exactly, using either semi-infinite programming or nonlinear programming. As examples, we compute the extremal ellipsoids when the convex body K is the part of a given ellipsoid between two parallel hyperplanes, and when K is a truncated second order cone or an ellipsoidal cylinder.

3.2.7 Quadratic Objective, Quadratic Constraints Optimization

1. In [549], the author looks at the equivalence of several bounds presented in the papers: [776, 773]. He presents a *counterexample* to a result there. (However, this counterexample actually refers to an ommited assumption in one of the definitions, i.e. that a certain set is nonempty, i.e. the assumption

needed is that the trivial case does not occur. $\{u \in \Re^n : e^T u = 0, Q - \mathrm{Diag}\,(u) \preceq 0\} \neq \emptyset$, in which case the optimum is not finite.

2. In [180], Given two real vector spaces U and V, and a symmetric bilinear map $B : U \times U \to V$, let Q_B be its associated quadratic map Q_B. The problems we consider are as follows: (i) are there necessary and sufficient conditions, checkable in polynomial-time, for determining when Q_B is surjective?; (ii) if Q_B is surjective, given $v \in V$ is there a polynomial-time algorithm for finding a point $u \in Q_B^{-1}(v)$?; (iii) are there necessary and sufficient conditions, checkable in polynomial-time, for determining when B is indefinite? We present an alternative formulation of the problem of determining the image of a vector-valued quadratic form in terms of the unprojectivised Veronese surface. The relation of these questions with several interesting problems in Control Theory is illustrated.
URL:
http://arXiv.org/abs/math/0204068

3.3 Engineering

In [652], Band Gap Optimization of Two-Dimensional Photonic Crystals Using Semidefinite Programming and Subspace Methods by Han Men(men***at***nus.edu.sg), Ngoc-Cuong Nguyen(cuongng***at***mit.edu), Robert M. Freund(rfreund***at***mit.edu), Pablo A. Parrilo(parrilo***at***mit.edu), Jaume Peraire(peraire***at***mit.ed

Abstract: In this paper, we consider the optimal design of photonic crystal band structures for two-dimensional square lattices. The mathematical formulation of the band gap optimization problem leads to an infinite-dimensional Hermitian eigenvalue optimization problem parametrized by the dielectric material and the wave vector. To make the problem tractable, the original eigenvalue problem is discretized using the finite element method into a series of finite-dimensional eigenvalue problems for multiple values of the wave vector parameter. The resulting optimization problem is large-scale and non-convex, with low regularity and non-differentiable objective. By restricting to appropriate eigenspaces, we reduce the large-scale non-convex optimization problem via reparametrization to a sequence of small-scale convex semidefinite programs (SDPs) for which modern SDP solvers can be efficiently applied. Numerical results are presented for both transverse magnetic (TM) and transverse electric (TE) polarizations at several frequency bands. The optimized structures exhibit patterns which go far beyond typical physical intuition on periodic media design.

3.3.1 Copositivity

1. In [457], "A complete algorithm for determining copositive matrices" by Authors: Jia Xu, Yong Yao

   Abstract: In this paper, we present a complete algorithm called COPOMATRIX for determining the copositivity of an $n \times n$ matrix. The core of this algorithm is decomposition theorem, which is used to deal with simplicial subdivision of $\hat{T}^- = \{y \in \Delta_m | \beta^T y \leq 0\}$ on standard simplex Δ_m, where each component of the vector β^T is either -1 or 0 or 1.

2. In [917], On cones of nonnegative quadratic functions, by Sturm,J.F. and Zhang,S.

   abstract: We derive LMI-characterizations and dual decomposition algorithms for certain matrix cones which are generated by agiven set using generalized co-positivity.These matrix cones are in fact cones of non-convex quadratic functions that are nonnegative on a certain domain.As a domain, we consider for instance the intersection of a (upper) level-set of a quadratic function and a half-plane.We arrive at a generalization of Yakubovich's S-procedure result.As an application we show that optimizing a general quadratic function over the intersection of an ellipsoid and a half-plane can be formulated as SDP, thus proving the polynomiality of this class of optimization problems, which arise, e.g., from the application of the trust region method for nonlinear programming. Other applications are in control theory and robust optimization.

3. In [151], "Solving standard quadratic optimization problems via linear, semidefinite and copositive programming", by I.M. Bomze and E. de Klerk,

The problem of minimizing a (non-convex) quadratic function over the simplex has an exact convex reformulation as a copositive programming problem. In this paper we show how to approximate the optimal solution by approximating the cone of copositive matrices via systems of linear inequalities, and, more refined, linear matrix inequalities (LMI's). Examples from various applications, and simulations are provided showing the validity of this approach, which extends ideas of De Klerk and Pasechnik for the maximal stable set problem in a graph.

3.3.2 Machine Learning and EDM

1. TITLE: Learning a Kernel Matrix for Nonlinear Dimensionality Reduction [993]
   AUTHORS: Kilian Weinberger - University of Pennsylvania, Fei Sha - University of Pennsylvania, Lawrence Saul - University of Pennsylvania
   ABSTRACT: "We investigate how to learn a kernel matrix for high dimensional data thatlies on or near a low dimensional manifold. Noting that the kernel matriximplicitly maps the data into a non-linear feature space, we show how todiscover a mapping that "unfolds" the underlying manifold from which the datawas sampled. The kernel matrix is constructed by maximizing the variance infeature space subject to local constraints that preserve the angles anddistances between nearest neighbors. The main optimization involves aninstance of semidefinite programming— a fundamentally different computationthan previous algorithms for manifold learning, such as Isomap and locallylinear embedding. The optimized kernels perform better than polynomial and Gaussian kernels for problems in manifold learning, but worse for problems inlarge margin classification. We explain these results in terms of thegeometric properties of different kernels and comment on variousinterpretations of other manifold learning algorithms as kernel methods."

3.3.3 Sensor Network Localization

1. In Explicit Sensor Network Localization using Semidefinite Representations and Clique Reductions by Nathan Krislock, Henry Wolkowicz

   abstract: The sensor network localization, SNL, problem in embedding dimension r, consists of locating the positions of wireless sensors, given only the distances between sensors that are within radio range and the positions of a subset of the sensors (called anchors). Current solution techniques relax this problem to a weighted, nearest, (positive) semidefinite programming, SDPC completion problem, by using the linear mapping between Euclidean distance matrices, EDMC and semidefinite matrices. The resulting SDP is solved using primal-dual interior point solvers, yielding an expensive and inexact solution.

   This relaxation is highly degenerate in the sense that the feasble set is restricted to a low dimensional face of the SDP cone, implying that the Slater constraint qualification fails. The degeneracy in the SDP arises from cliques in the graph of the SNL problem. In this paper, we take advantage of the absence of the Slater constraint qualification and derive a technique for the SNL problem, with exact data, that explicitly solves the corresponding rank restricted SDP problem. No SDP solvers are used. We are able to efficiently solve this NP-HARD problem with high probability, by finding a representation of the minimal face of the SDP cone that contains the SDP matrix representation of the EDM. The main work of our algorithm consists in repeatedly finding the intersection of subspaces that represent the faces of the SDP cone that correspond to cliques of the SNL problem.

2. Robust Semidefinite Programming Approaches for Sensor Network Localization with Anchors by Nathan Krislock, Veronica Piccialli, Henry Wolkowicz

   abstract:
   We derive a robust primal-dual interior-point algorithm for a semidefinite programming, SDP, relaxation for sensor localization with anchors and with noisy distance information. The relaxation is based on finding a *Euclidean Distance Matrix, EDM,* that is nearest in the Frobenius norm for the known noisy distances and that satisfies given upper and lower bounds on the unknown distances. We show

that the SDP relaxation for this nearest EDM problem is usually underdetermined and is an ill-posed problem. Our interior-point algorithm exploits the structure and maintains exact feasibility at each iteration. High accuracy solutions can be obtained despite the ill-conditioning of the optimality conditions. Included are discussions on the strength and stability of the SDP relaxations, as well as results on invariant cones related to the operators that map between the cones of semidefinite and Euclidean distance matrices.

3. Sum of Squares Method for Sensor Network Localization Authors: Jiawang Nie Subj-class: Optimization and Control from: Mathematics, abstract math.OC/0605652

   We formulate the sensor network localization problem as finding the global minimizer of a quartic polynomial. Then sum of squares (SOS) relaxations can be applied to solve this problem. However, the general SOS relaxations are too expensive for practical problems. Exploiting special features of this polynomial, we propose a new structured SOS relaxation, and study its various properties. When distances are given exactly, our SOS relaxation often returns high quality sensor locations (they are exact if we ignore rounding errors), and can return more than one solution if the localization problem is not uniquely solvable, under flat extension condition. When distances have errors and localization is unique, we show that the sensor location returned by our SOS relaxation is accurate within a factor of the distance error under some technical assumptions. We also present some numerical simulations, which show that our structured SOS relaxation not only returns high quality solutions, but also takes much less CPU time than the SDP relaxation method.

3.3.4 Compressed Sensing

1. Testing the Nullspace Property using Semidefinite Programming by Authors: Alexandre d'Aspremont, Laurent El Ghaoui

   http://arxiv.org/abs/0807.3520

   Recent results in compressed sensing show that, under certain conditions, the sparsest solution to an underdetermined set of linear equations can be recovered by solving a linear program. These results rely on nullspace properties of the system matrix. So far, no tractable algorithm is known to test these conditions and most current results rely on asymptotic properties of sparse eigenvalues of random matrices. Given a matrix A, we use semidefinite relaxation techniques to test the nullspace property on A and show on some numerical examples that these relaxation bounds can prove perfect recovery of sparse solutions with relatively high cardinality.

3.3.5 Signal Processing and Communications

1. TITLE: Handling Nonnegative Constraints in Spectral Estimation, [33]
   AUTHORS: B. Alkire and L. Vandenberghe
   ABSTRACT: We consider convex optimization problems with the constraint that the variables form a finite autocorrelation sequence, or equivalently, that the corresponding power spectral density is nonnegative. This constraint is often approximated by sampling the power spectral density, which results in a set of linear inequalities. It can also be cast as a linear matrix inequality via the positive-real lemma. The linear matrix inequality formulation is exact, and results in convex optimization problems that can be solved using interior-point methods for semidefinite programming. However, these methods require $O(n^6)$ floating point operations per iteration, if a general-purpose implementation is used. We introduce a much more efficient method with a complexity of $O(n^3)$ flops per iteration.
   STATUS: To appear in the Proceedings of the 34th IEEE Asilomar Conference on Signals, Systems and Computer, Pacific Grove, California, October 29 through November 1, 2000.
   DATE OF ENTRY: Feb. 5, 2001.
   Related paper: Interior-point methods for magnitude filter design

2. TITLE: Convex optimization problems involving finite autocorrelation sequences [34]
AUTHORS: B. Alkire and L. Vandenberghe
ABSTRACT: We discuss convex optimization problems where some of the variables are constrained to be finite autocorrelation sequences. Problems of this form arise in signal processing and communications, and we describe applications in filter design and system identification. Autocorrelation constraints in optimization problems are often approximated by sampling the corresponding power spectral density, which results in a set of linear inequalities. They can also be cast as linear matrix inequalities via the Kalman-Yakubovich-Popov lemma. The linear matrix inequality formulation is exact, and results in convex optimization problems that can be solved using interior-point methods for semidefinite programming. However, it has an important drawback: to represent an autocorrelation sequence of length n, it requires the introduction of a large number (n(n+1)/2) of auxiliary variables. This results in a high computational cost when general-purpose semidefinite programming solvers are used. We present a more efficient implementation based on duality and on interior-point methods for convex problems with generalized linear inequalities.
STATUS: Submitted to Mathematical Programming, Series B.
DATE OF ENTRY: Feb. 5, 2001.
RELATED PAPERS: Handling nonnegative constraints in spectral estimation, Interior-point methods for magnitude filter design

3. In [1044], "Fast Linear Iterations for Distributed Averaging", AUTHORS: L. Xiao and S. Boyd,

   ABSTRACT: We consider the problem of finding a linear iteration that yields distributed averaging consensus over a network, i.e., that asymptotically computes the average of some initial values given at the nodes. When the iteration is assumed symmetric, the problem of finding the fastest converging linear iteration can be cast as a semidefinite program, and therefore efficiently and globally solved. These optimal linear iterations are often substantially faster than several common heuristics that are based on the Laplacian of the associated graph.

   We show how problem structure can be exploited to speed up interior-point methods for solving the fastest distributed linear iteration problem, for networks with up to a thousand or so edges. We also describe a simple subgradient method that handles far larger problems, with up to one hundred thousand edges. We give several extensions and variations on the basic problem.

3.3.6 Systems and Control

1. In [753], Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization, PhD thesis of Pablo A. Parrilo, Caltech , 2000.

   Contains material on: approximating and solving a general class of semialgebraic problems, i.e. those that can be described by a finite number of polynomial equalities and inequalities. This leads to new (strengthened) relaxations.

3.4 Matrix Completions

3.4.1 SDP Completions

1. In [918], Multivariate Gaussians, Semidefinite Matrix Completion, and Convex Algebraic Geometry, Authors: Bernd Sturmfels, Caroline Uhler

   We study multivariate normal models that are described by linear constraints on the inverse of the covariance matrix. Maximum likelihood estimation for such models leads to the problem of maximizing the determinant function over a spectrahedron, and to the problem of characterizing the image of the positive definite cone under an arbitrary linear projection. These problems at the interface of statistics and optimization are here examined from the perspective of convex algebraic geometry.

   Cite as: arXiv:0906.3529v1 [math.ST]

3.4.2 Maximal/Minimal Rank Completions

1. In [212], Maximal rank Hermitian completions of partially specified Hermitian matrices, by Cohen, Nir(IL-TECH-E); Dancis, Jerome(1-MD) in Linear Algebra Appl. 244 (1996), 265–276.

   If M is an $n \times m$ partial matrix, K is a $p \times q$ submatrix of M, and T is a completion of M, define $\rho(K, M) = \operatorname{rank} K + (n-p) + (m-q)$ and $\rho(M) = \min\{\rho(K, M)\colon K$ is a submatrix of $M\}$, where the trivial row and the trivial column are included as submatrices of size $n \times 0$ and $0 \times m.\rho(M)$ is an upper bound for the maximal completion rank of M.

   The authors provide elementary proofs of the following two results: (i) There exists a completion T of M whose rank equals $\rho(M)$. (ii) If M is Hermitian, there exists a Hermitian completion T of M whose rank equals $\rho(M)$. Actually, in results (i) and (ii), one can choose the completion T in such a way that T is a strong rank maximizer for M, that is, any submatrix of T maximizes the completion rank of the corresponding partial submatrix of M. The authors also discuss these results for symmetric matrices over an arbitrary field. The same results are still true if the field is infinite. However, the situation for finite fields is not clear.

2. In [343], by Geelen, James F, Maximum rank matrix completion, in Linear Algebra Appl. 288 (1999), no. 1-3, 211–217.

   In this paper the author presents an algorithm that produces a maximal rank completion. In each step a free entry is perturbed when it either produces an increase in rank or an increase in the number of dependent lines. A dependent line is a row or column that is a linear combination of the other rows and columns, respectively. After at most $m(n+m)$ steps a maximal rank completion of an $m \times n$ partial matrix is obtained.

3. In [656], On the rank minimization problem over a positive semidefinite linear matrix inequality, by Mesbahi, M.(1-CAIT-J); Papavassilopoulos, G. P.(1-SCA-D) in IEEE Trans. Automat. Control 42 (1997), no. 2, 239–243.

   Summary: "We consider the problem of minimizing the rank of a positive semidefinite matrix, subject to the constraint that an affine transformation of it is also positive semidefinite. Our method for solving this problem employs ideas from the ordered linear complementarity theory and the notion of the least element in a vector lattice. This problem is of importance in many contexts, for example in feedback synthesis problems; such an example is also provided."

3.5 Nonlinear Programming

3.5.1 Trust Region Subproblems

1. In [199], by Chen, Xiongda, Yuan, Ya-Xiang, "On local solutions of the Celis-Dennis-Tapia subproblem",
   discuss the distribution of the local solutions of the CDT (two trust region subproblem).

2. In [763], J. PENG and Y. YUAN:
   Trust region methods require solving subproblems of the form $\min[q(x)\colon c_i(x) \leq 0, i = 1, \cdots, p]$, where q and c_i are quadratic functions. The case where $p = 1$ and the Hessian of c_1 is positive definite is well known. At the optimum, the Hessian of the Lagrangian is positive semi-definite. The paper deals with two constraints. It is shown that, if the gradients of c_1 and c_2 are linearly independent at the optimum, then the Hessian of the Lagrangian has at most one negative eigenvalue.

3. In [1062], "New Results on Quadratic Minimization" by Yinyu Ye and Shuzhong Zhang:

   Abstract : In this paper we present several new results on minimizing an indefinite quadratic function under quadratic/linear constraints. The emphasis is placed on the case where the constraints are two quadratic inequalities. This formulation is known as *the extended trust region subproblem* and the computational complexity of this problem is still unknown. We consider several interesting cases related to this problem and show that for those cases the corresponding SDP relaxation admits no gap with the true optimal value, and consequently we obtain polynomial time procedures for solving those special cases of quadratic optimization. For the extended trust region subproblem itself, we introduce a parameterized problem and prove the existence of a trajectory which will lead to an optimal solution. Combining with a result obtained in the first part of the paper, we propose a polynomial-time solution procedure for the extended trust region subproblem arising from solving nonlinear programs with a single equality constraint.

4 SURVEYS/MISCELLANEOUS PAPERS/BOOKS OF INTEREST

4.1 Survey Papers and Books

1. In [972, 1994] and [975, 1996] the authors L. VANDENBERGHE and S. BOYD provide surveys on Positive Definite Programming (semidefinite Programming).

4.2 Papers by Author: Henry Wolkowicz

[144] [145] [38] [37] [744] [745] [743] [742] [731] [789] [741] [1032] [557] [1029] [1028] [1021] [18] [718] [563] [1020] [1024] [1022] [1027] [258] [200] [253] [201] [555] [556] [15] [960] [254] [202] [203] [769] [46] [255] [554] [782] [256] [22] [374] [368] [13] [991] [959] [20] [1025] [801] [1026] [47] [49] [315] [316] [21] [50] [19] [564] [51] [1023] [692] [48] [52] [386] [55] [59] [17] [560] [562] [1019] [16] [561] [1039] [1074] [480] [798] [427] [805] [773] [429] [740] [278] [901] [902] [506] [804] [776] [1016] [1038] [1018] [802] [246] [399] [400] [472] [110] [398] [471] [906] [100] [803] [920] [1037] [381] [893] [166] [380] [654] [572] [482] [187] [892] [377] [378] [379] [653] [1014] [1012] [1013] [165] [1011] [162] [161] [164] [1010] [163] [1005] [1035] [1007] [1036] [1034] [1004] [1033] [1002] [1040] [905] [903] [904] [1017] [486] [160] [1009] [401] [1030] [1031] [1015] [847] [376] [1008] [646] [1006] [1003] [1001] [553] [990] [781] [559] [373] [252] [225] [314] [328] [558] [1073] [45] [1072] [840] [503] [397] [618] [899] [846] [568] see also: http://orion.math.uwaterloo.ca/~hwolkowi/henry/reports/ABSTRACTS.html

4.3 Papers by other authors

1. Kazuhide Nakata , Katsuki Fujisawa , Mituhiro Fukuda , Masakazu Kojima , Kazuo Murota

   Exploiting Sparsity in Semidefinite Programming via Matrix Completion II: Implementation and Numerical Results

   http://www.optimization-online.org/DB_HTML/2001/03/290.html

2. Samuel Burer , Renato D.C. Monteiro

   A Nonlinear Programming Algorithm for Solving Semidefinite Programs via Low-rank Factorization

   http://www.optimization-online.org/DB_HTML/2001/03/296.html

3. Raphael Hauser , Osman Guler

   Self-scaled barrier functions on symmetric cones and their classification

   http://www.optimization-online.org/DB_HTML/2001/03/307.html

   See also [420, 419, 418] for: The Nesterov-Todd direction and its relation to weighted analytic centers, Self-scaled barrier functions on symmetric cones and their classification, Self-scaled barriers for irreducible symmetric cones.

4. [432] AUTHOR = Helton, J. William and Lam, Daniel and Woerdeman, Hugo J., TITLE = Sparsity patterns with high rank extremal positive semidefinite matrices,

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