Recent progress in matrix rank minimization

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We report on our progress in matrix rank minimization and its applications to problems in sensor network localization and data mining. Matrix rank minimization refers to the problem of finding a matrix X satisfying convex constraints with minimum rank. This problem is NP-hard in general, but our group at Waterloo and others have shown recently that convex relaxation shows great promise for solving certain important classes of problems exactly.

The sensor network localization, SNL, problem consists of locating the positions of ad hoc wireless sensors, given only the distances between sensors that are within radio range and the positions of a subset of the sensors (called anchors). Wireless sensor networks have many applications, e.g. in monitoring physical or environmental conditions (temperature, sound, vibration, pressure, battlefield surveillance, home automation, hospital patients, traffic control, etc.). The SNL problem can be relaxed to a weighted, nearest, (positive) semidefinite programming, SDP, completion problem. This relaxation is ill-conditioned because the feasible set is restricted to a low dimensional face of the SDP cone. This means that the Slater constraint qualification fails. Also, nonuniqueness of the optimal solution results in large sensitivity to small perturbations in the data.

We turn what is ordinarily regarded as a difficulty to our advantage with a preprocessing technique that solves the SNL problem. Our algorithm is based on repeatedly identifying faces of the semidefinite cone corresponding to intersections of cliques (i.e., the source of the degeneracy) in the SNL problem to reduce its size. In most cases, the SDP relaxation as well as the original SNL problem is completely solved without ever requiring an SDP solver.

Another thrust of our work concerns exact solution of NP-hard problems including clique, biclique and clustering using convex relaxation. Our results for these three problems can be described as follows. If the input graph consists of a clique, biclique, or clusters of nodes, but this data is obscured by an large amount of random noise (i.e., extra nodes and edges not part of the clique, biclique or clustering), then the convex relaxation can find the clique, biclique or clustered nodes with probability exponentially close to 1. The clique, biclique and clustering problems are all examples of data mining problems in which one seeks hidden structure in a large dataset.

Parts of this talk represent work with B. Ames, N. Krislock and S. Vavasis.