

SENSOR NETWORK LOCALIZATION, EUCLIDEAN DISTANCE MATRIX COMPLETIONS, AND GRAPH REALIZATION

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Wireless sensor networks have many applications, e.g. in monitoring physical or environmental conditions (temperature, sound, vibration, pressure, battlefield surveillance, home automation, hospital patients, traffic control, etc.). The sensor network localization, *SNL*, problem consists of locating the positions of ad hoc wireless sensors, given only the distances between sensors that are within radio range and the positions of a subset of the sensors (called anchors). One main point is to view *SNL* as a (nearest) Euclidean Distance Matrix, *EDM*, completion problem that does not distinguish between the anchors and the sensors. We show that there are advantages for using the well-studied *EDM* model. This problem can be relaxed to a weighted, nearest, (positive) semidefinite programming, *SDP*, completion problem. This relaxation is ill-conditioned in two ways. First, it is, implicitly, highly degenerate in the sense that the feasible set is restricted to a low dimensional face of the *SDP* cone. This means that the Slater constraint qualification fails. Second, nonuniqueness of the optimal solution results in large sensitivity to small perturbations in the data.

The degeneracy in the *SDP* arises from cliques in the graph of the *SNL* problem. We take advantage of the absence of the Slater constraint qualification and derive a preprocessing technique that solves the *SNL* problem. With exact data, we explicitly solve the corresponding *SDP* problem without using any *SDP* solver. We do this by finding explicit representations of the faces of the *SDP* cone corresponding to intersections of cliques of the *SNL* problem. For problems with noise, we first solve nearest matrix problems to get best *EDM* approximations.