Solving SDP moment problems for polynomial equations *

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Proposition 0.1. *Let* $x = \begin{pmatrix} x_1 & x_2 & \dots & x_{2n+1} \end{pmatrix}^T \in \mathbb{R}^{2n+1}$. *Define the index set*

$$\mathcal{E} := \{ij : i + j = k + 1, i > 1, j < n + 1, k = 3, 4, \dots, 2n - 1\} \tag{0.1}$$ {?}

and, for all $ij \in \mathcal{E}, k = i + j - 1$, *let*

$$A_{ij} := \begin{cases} E_{ij} - E_{1k}, & \textit{if } \ i \neq j, k \leq n + 1, \\ E_{ii} - \frac{1}{\sqrt{2}} E_{1k}, & \textit{if } \ i = j, k \leq n + 1, \\ E_{ij} - E_{(k-n)(n+1)}, & \textit{if } \ i \neq j, k > n + 1, \\ E_{ii} - \frac{1}{\sqrt{2}} E_{(k-n)(n+1)}, & \textit{if } \ i = j, k > n + 1. \end{cases}$$

Then $P = \mathcal{H}(x) = \mathcal{M}(x) \in \mathcal{S}^{n+1}$, *is a Hankel/moment matrix if, and only if,*

$$\begin{array}{c} P_{11} = x_1, P_{12} = x_2, P_{n,n+1} = x_{2n}, P_{n+1,n+1} = x_{2n+1}, \\ \operatorname{trace} A_{ij} P = 0, \forall ij \in E, \\ P \in \mathcal{S}^{n+1}. \end{array}$$

Let $n = 2$, so we have the following 3×3 matrices:

$$A_{1,2} = A_{2,1} = A_{3,2} = A_{1,3} = \dots = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Also,

$$A_{2,2} = \begin{pmatrix} 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 0 \end{pmatrix}$$

Therefore, trace $A_{ij} P = 0$ does not always generate the correct constraints (except $A_{2,2}$).
I think if

$$A_{1,2} = A_{2,1} = \begin{pmatrix} 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

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and

$$A_{3,2} = A_{2,3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 \end{pmatrix}$$

it will likely generate the correct constraints for the moment matrix.