Chapter 10

Conclusions and outlook

10.1 Conclusions

In this dissertation new results have been presented on the topology of stationary magnetohydrodynamic (MHD) bow shock flows. These findings result from interdisciplinary scientific research in several domains. Using techniques from Computational Fluid Dynamics (CFD), we have developed the PAR-MA (PARallel MAgnetohydrodynamics) code which solves the nonlinear time-dependent MHD equations numerically. Simulation results were validated and interpreted using theoretical concepts which derive from the field of applied mathematics. The simulation results were applied to space physics flows in the solar corona and in the earth's magnetic environment.

In this Section we briefly summarize the main findings described in this dissertation.

10.1.1 Contributions to the physics of MHD plasma flows with shocks

General theory of MHD bow shock flows

It is well-known that supersonic hydrodynamic flow of a neutral gas around a blunt obstacle generates a bow shock in front of the obstacle. For geometrically simple obstacles this bow shock has a single shock front and has a smooth paraboloid-like shape (Fig. 2.4). In an ionized gas or plasma magnetic forces influence the dynamics of the flow. Analogous to the case of a neutral gas, a bow shock is formed for superfast flow of an MHD plasma around an obstacle. When the upstream magnetic field strength is small and thermal or dynamic pressure effects dominate, a single-front bow shock flow is obtained which is morphologically similar to the bow shock in a neutral gas flow (Fig. 7.3).

The new result presented in this dissertation is that when the upstream magnetic field strength is large, intrinsically magnetic effects can alter the bow shock flow topology significantly. For upstream parameter values for which the intrinsically magnetic effect of MHD switch-on shocks occurs — we say that the upstream flow is magnetically dominated —, the leading bow shock front is followed by a secondary shock front of slow and intermediate MHD type which is attached to the leading front. This was shown in numerical simulation results of symmetrical 2D flows around cylinders (Fig. 6.2) and of 3D flows around spheres and paraboloids (Fig. 8.10). In this magnetically dominated parameter regime the bow shock flow thus assumes a complex topology (Fig. 7.5b) involving multiple shock fronts with segments of various MHD shock types.

We have concluded that there are two basic topologies for MHD bow shock flows. For pressure-dominated upstream flows the well-known single-front topology is obtained. For magnetically dominated upstream flows the bow shock flow assumes a complex multiple-front topology which was previously unknown. This result is an important addition to the general theory of MHD bow shock flows [114].

Existence of intermediate and compound shocks

We have clearly identified stationary MHD shock fronts of intermediate type in our 3D MHD simulation results (Fig. 8.13). This finding is relevant for the ongoing debate on the physical existence of intermediate shocks. For a long time it has been commonly believed that intermediate shocks cannot arise in MHD flows and are unphysical [87]. Recently, however, it has been shown theoretically that intermediate shocks can be stable in the dissipative MHD system for wide ranges of the dissipation coefficients [173, 43, 46, 110]. Our simulation results seem to be the first confirmation in 3D that intermediate shocks can indeed exist and persist for small dissipation MHD in a realistic flow configuration.

We have identified compound shocks, which are a manifestation of the non-convex nature of the MHD flux function, in our 2D and 3D simulation results (Figs. 6.25 and 7.20).

Characteristic analysis of 2D MHD flows

We have provided a new systematic derivation of the characteristic properties of stationary 2D MHD flows. We have shown how characteristic analysis can be a useful tool for the physical interpretation of 2D MHD flows and for grid convergence studies. We have applied this analysis to numerical simulation results which illustrate the most basic MHD wave phenomena, and to magnetically dominated bow shock flows with interacting shocks of different types and alternating elliptic and hyperbolic regions (Fig. 6.22).

Applications in space physics

Intrinsically magnetic effects are expected to play a role in many plasma flows with shocks in space physics and astrophysics. We have discussed two examples of bow shock flows in the solar system for which intrinsically magnetic effects can be important.

We predict that the new magnetically dominated complex bow shock flow topology which involves secondary slow shocks arises in shocks induced by fast solar coronal mass ejections (CMEs) propagating out of the solar corona (Fig. 8.3). It seems hard, however, to obtain conclusive observational evidence for this phenomenon given the limitations of present-day observational techniques.

A slow shock is also expected to form in the earth's magnetosheath following the bow shock (Fig. 2.9) when the solar wind is in the magnetically dominated regime for which switch-on shocks can exist. Slow shocks have indeed been observed in the magnetosheath [141]. Our simulation results offer a physically attractive explanation for at least some of these observed slow shocks. Slow shocks are especially likely to form in the magnetosheath during magnetic cloud events. Magnetic clouds are believed to cause magnetic storms of concern to space weather. The characteristic time of the formation of the slow shock and the influence of the complex topology on reconnection processes at the magnetopause are potentially important for the mechanism and timing of magnetic storms. The temporary global reconfiguration of the magnetosheath flow during magnetic cloud events may thus be an important new element in scenarios for magnetic storms.

10.1.2 Numerical simulation of MHD flows with shocks

The use of advanced numerical techniques and fast parallel computers was essential for obtaining the new results on intrinsically magnetic effects in 3D MHD bow shock flows. We have shown that well-developed techniques from Computational Fluid Dynamics which are directly based on the wave properties of the hyperbolic system of equations, can be extended to MHD for the successful simulation of complex flows with multiple shocks of different types. This is not a trivial result, because the MHD system allows for three strongly anisotropic wave modes, whereas only one isotropic wave mode arises in hydrodynamic flows.

Numerical techniques similar to the one implemented in PAR-MA are currently being used in several application codes [118, 162, 124, 117]. When we started our thesis work, however, these techniques were not available, and we have been actively involved in the research on extending CFD techniques to MHD. We have invested a lot of effort in validating the simulation code through grid convergence studies (Fig. 5.8). We believe that such validation should be standard practice, but unfortunately this step is all too often omitted in the field of space physics simulations. These grid convergence studies also show that the efficient technique to treat the $\nabla \cdot \vec{B}$ constraint by adding a source term [118] produces valid solutions. Efficient implementation of the numerical algorithm on massively parallel computers made the simulation of large 3D MHD flow problems possible.

10.2 Outlook

10.2.1 Physics of MHD plasma flows with shocks

The results described in this dissertation regarding MHD bow shock flows can be extended in many ways. We now give a brief indication of some directions for future research.

General theory of MHD bow shock flows

Our study of 3D MHD bow shock topology for pressure-dominated and magnetically dominated upstream flows is not complete because only a limited region of the parameter space has been explored. This study of the general problem of 3D MHD flow around an obstacle deserves further investigation.

More detailed investigation of the turbulent wake behind the cylinder and sphere flows is another interesting topic. MHD turbulence is not a completely unexplored domain of research, but it seems that much remains to be discovered. Models which parametrize MHD turbulence could be developed and could be used in simulations similar to the way they are used in simulations of hydrodynamic flows [167, 101]. It would be interesting to study the stabilizing or destabilizing effect of magnetic fields on turbulent wakes [63].

Existence of intermediate shocks

We have argued that the 3D bow shock topology obtained for magnetically dominated upstream parameters is valid for small dissipation MHD. In our simulations the numerical dissipation plays a role analogous to a physical dissipation. The simulations should be repeated with a well-controlled modeled physical dissipation if we are to interpret the dissipation in a more meaningful way. It would then be interesting to see if it is indeed true that the solution we obtained is reproduced for large ranges of the dissipative parameters, and that the same solution is found in the limit of vanishing dissipation [48]. If a 3D MHD code can be developed which does not introduce any dissipation [48, 35], then it could be investigated what the ideal MHD solution to the 3D bow shock problem could be, which may not contain intermediate shocks as some recent work seems to imply [48, 46, 35]. The small dissipation case would seem the most appropriate to describe physical systems, so the discussion on vanishing dissipation limits and ideal MHD solutions is probably not so relevant for space physics applications [173], but these topics certainly remain interesting mathematical problems.

Applications in space physics

Time-dependent CME flows should be considered in a realistic solar wind background. Short of better observations than currently available, it is not trivial to find a realistic way to initiate a CME in a numerical simulation. Theoretical constraints and considerations are an important guideline in studying the CME initiation problem. Simulations of fast CMEs, preferentially based on theoretically and observationally credible initiation scenarios, could reproduce the formation of the complex bow shock topology with a slow shock following a fast shock. Line of sight integration could be performed on simulation results to mimic coronagraph images, which could be compared to real coronagraph images. Such scientific visualization techniques may contribute to the identification of secondary slow shocks on coronagraph images. It is interesting to know where exactly in the solar corona shocks would be induced by CMEs, and if reconnection flares can also generate shocks. Possible changes in shock types and topology at transitions in the solar wind from low-β to high-β and from subfast to superfast flow are other points of interest.

Time-dependent simulations of the earth's bow shock and magnetosheath flow under varying solar wind conditions should shed some light on how solar wind perturbations propagate through the magnetosheath. The characteristic time of the global reconfiguration of the magnetosheath flow through the formation of slow secondary shocks during magnetic cloud events may be important for magnetic storm scenarios. The influence of the resulting complex magnetic field topology on reconnection at the magnetopause should be investigated in simulations which include the inner magnetosphere and the ionosphere.

Ultimately CME flows in the solar wind and the influence of the CME perturbations on the magnetosphere could be simulated on one large simulation domain stretching from the sun to the earth. This could improve our understanding of CME propagation and of the processes affecting the whole solar-terrestrial environment. It may ultimately lead to simulation codes which can be useful for space weather nowcasting and possibly prediction.

CME-induced shocks and the earth's bow shock are just two examples of MHD shocks in space physics plasmas. The flow topology of cometary, lunar and planetary bow shock flows under magnetically dominated solar wind conditions should be investigated by simulations and observationally. Recent observations reveal that galactic and extragalactic jets may exhibit multiple bow shock fronts. It thus seems that our results on MHD bow shock flow topology may have many unexplored applications in space physics and astrophysics plasma flows.

10.2.2 New numerical simulation techniques

The numerical tools of solution-adaptive grid refinement and implicit time integration seem to be required to be able to tackle flow problems with varying spatial and temporal scales — like the space weather problem described above — efficiently. These tools have not been implemented in the PAR-MA code which was used for the simulations described in this dissertation. Rather than implementing these techniques in the finite volume PAR-MA code in a way which has become quite standard by now, it seems more promising to use new techniques. New generation shock-capturing numerical techniques using multi-dimensional approaches on unstructured grids have recently been developed for the hydrodynamic equations [167]. The unstructured grid approach naturally allows for adaptive refinement of grids. The multi-dimensional approach leads to good shock-capturing using compact numerical stencils. This compactness allows for efficient implicit time integration. A promising beginning has been made to apply this new approach to the MHD system [22]. Additional development is necessary during the coming years, and will probably lead to a numerical code well-suited to tackle space physics simulation problems.

10.2.3 Other directions

Many of the topics described in the two previous Sections are direct extensions of the thesis work and seem worthwhile of further investigation. However, it is sometimes interesting to look a little further and discover some less obvious related subjects which may deserve attention.

The MHD equations form a hyperbolic system. Our work involved numerical techniques and analytical concepts which are especially tailored to hyperbolic systems, and the simulation results give some insight into the physical properties of a hyperbolic system with multiple anisotropic wave modes. It may be interesting to apply this knowledge to other physical systems which are described by hyperbolic equations.

The equations of special relativistic MHD are an obvious candidate. It would be interesting to see if in relativistic MHD intrinsically magnetic effects influence bow shock flows in a way similar to the non-relativistic case. Applications would include the formation of shocks induced by relativistic jets in astrophysical plasmas. The hyperbolic nature of special relativistic MHD is well understood, and it seems that numerical schemes and analytical concepts would carry over smoothly from non-relativistic MHD.

On a next level, the equations of general relativity are also considered to form a hyperbolic system. However, a consistent hyperbolic formulation has not been found yet. The application of symmetrization theory, much like it has been used in the MHD case [7], may help to obtain such a formulation. It has been suggested by T. Barth that the general relativity constraints may be imposed weakly in a numerical scheme much in the same way as the $\nabla \cdot \vec{B}$ constraint is weakly imposed in our MHD scheme. Characteristic theory may help in clarifying the hyperbolic nature of the general relativistic equations, and in interpreting general relativistic flow solutions — also numerically obtained solutions. Recently analytical shock wave solutions of the general relativity equations have been found by matching regions with different metrics [137]. The possible existence of general relativistic shock waves suggests new scenarios for the cosmology of our universe. We have the impression that fascinating discoveries remain to be done in these fields.

The relevance of the results presented in this dissertation for the general class of hyperbolic systems with multiple characteristic waves becomes apparent when we summarize the physical lesson we have learned in this dissertation. We have discovered a new topology for magnetically dominated MHD bow shock flows involving multiple shock fronts with segments of various MHD shock types. The manner in which these various shock types arise on a front or the presence of multiple fronts all relate to the dynamics of information transfer by the MHD characteristic waves to guide the superfast flow around the obstacle. Put in these general terms, the numerical work presented in this dissertation has broadened our physical thinking on bow shock flows in MHD, in particular, and on hyperbolic systems with multiple characteristic waves, in general.