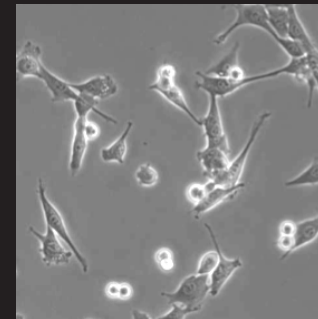# 3D Multilevel Aggregation for Segmentation and Tracking of Live Cells
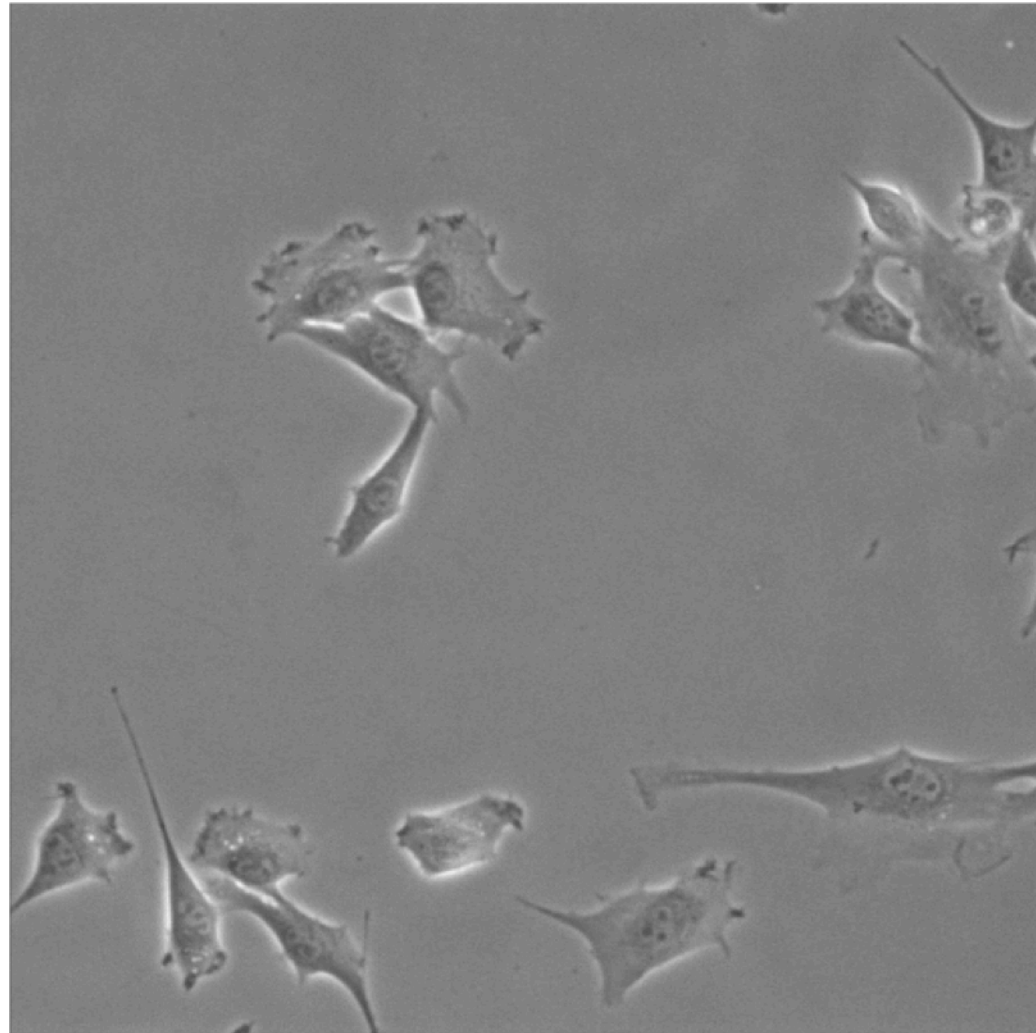
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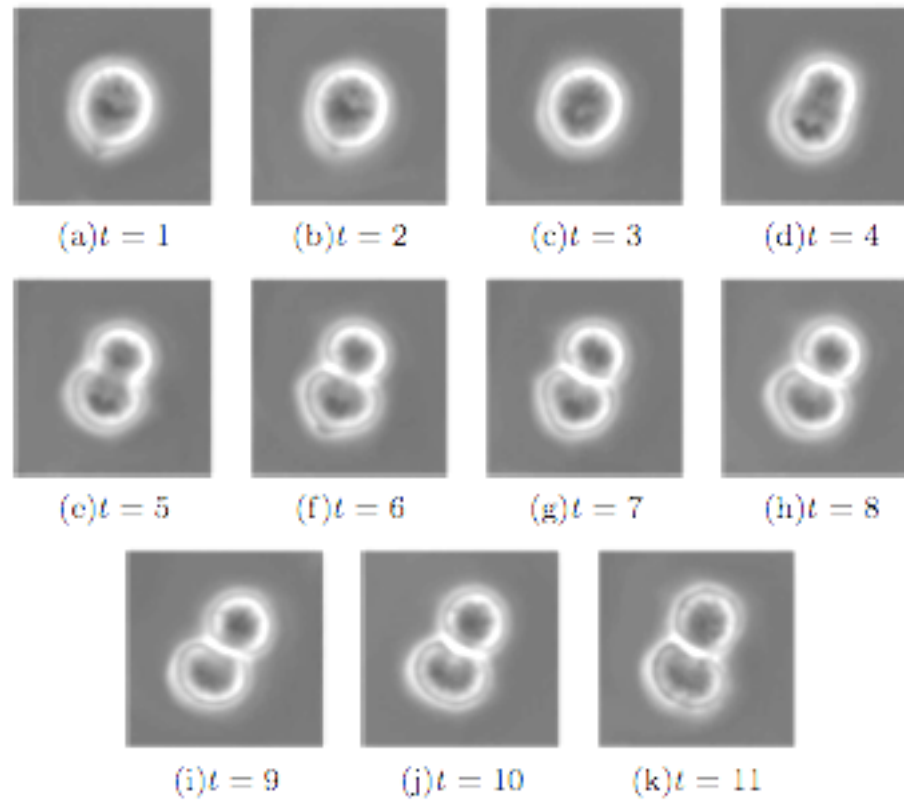**WATERLOO**

uwaterloo.ca

Hans De Sterck

Department of Applied Mathematics
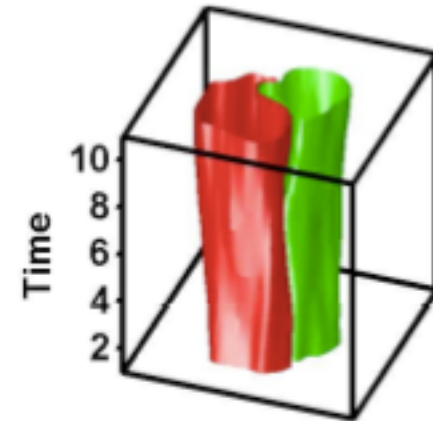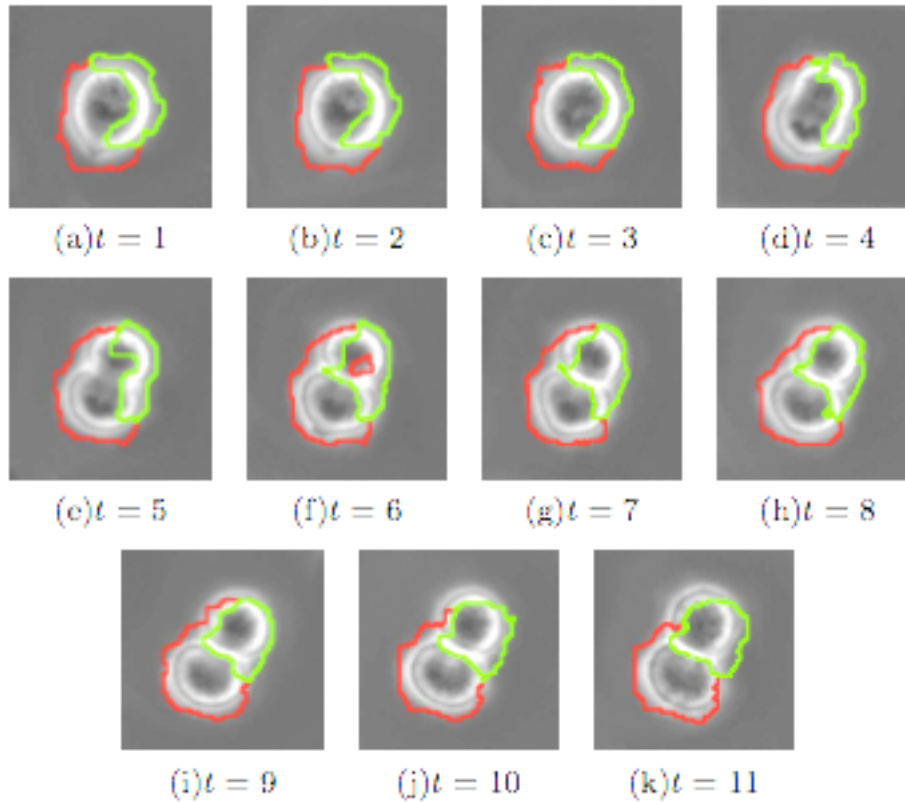University of Waterloo

1. cell segmentation

- segment and track individual cells

- bright circular cells

- touching and overlapping cells

- cell divisions

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cell segmentation

(a)$t = 1$ (b)$t = 2$ (c)$t = 3$ (d)$t = 4$

(e)$t = 5$ (f)$t = 6$ (g)$t = 7$ (h)$t = 8$

(i)$t = 9$ (j)$t = 10$ (k)$t = 11$
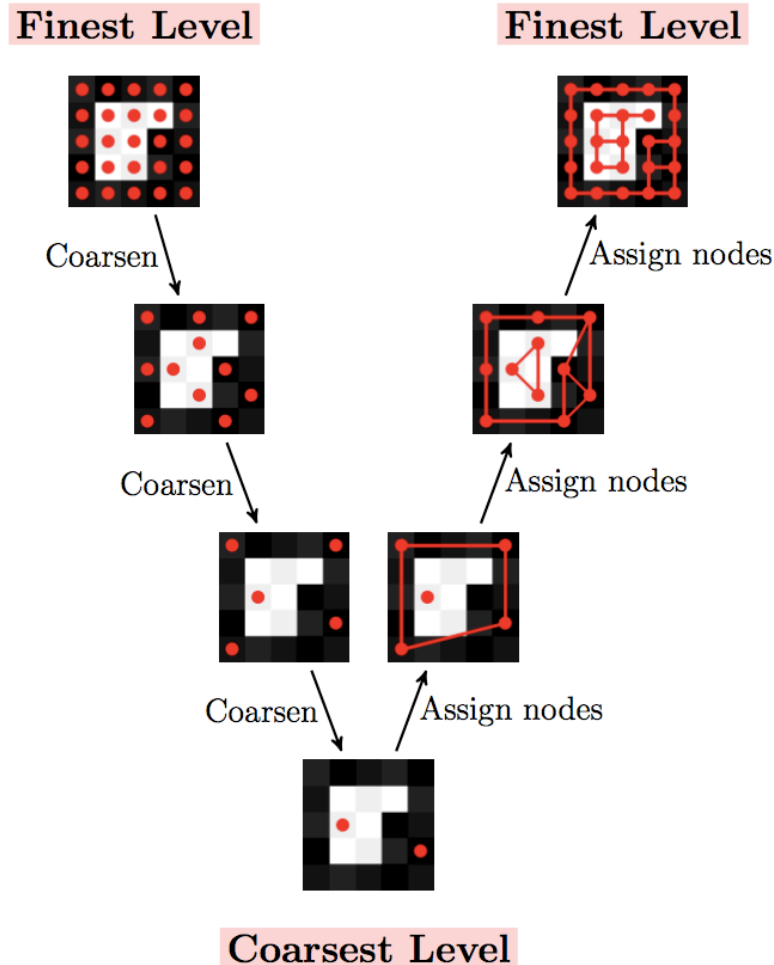
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cell segmentation

2. multilevel aggregation approach

[17] Eitan Sharon, Achi Brandt, and Ronen Basri. Fast multi-scale image segmentation. In *Proc. IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, pages 70–71, 2000.

[19] Eitan Sharon, Meirav Galun, Dahlia Sharon, Ronen Basri, and Achi Brandt. Hierachy and adaptivity in segmenting visual scenes. *Nature*, 442:810–813, 2006.
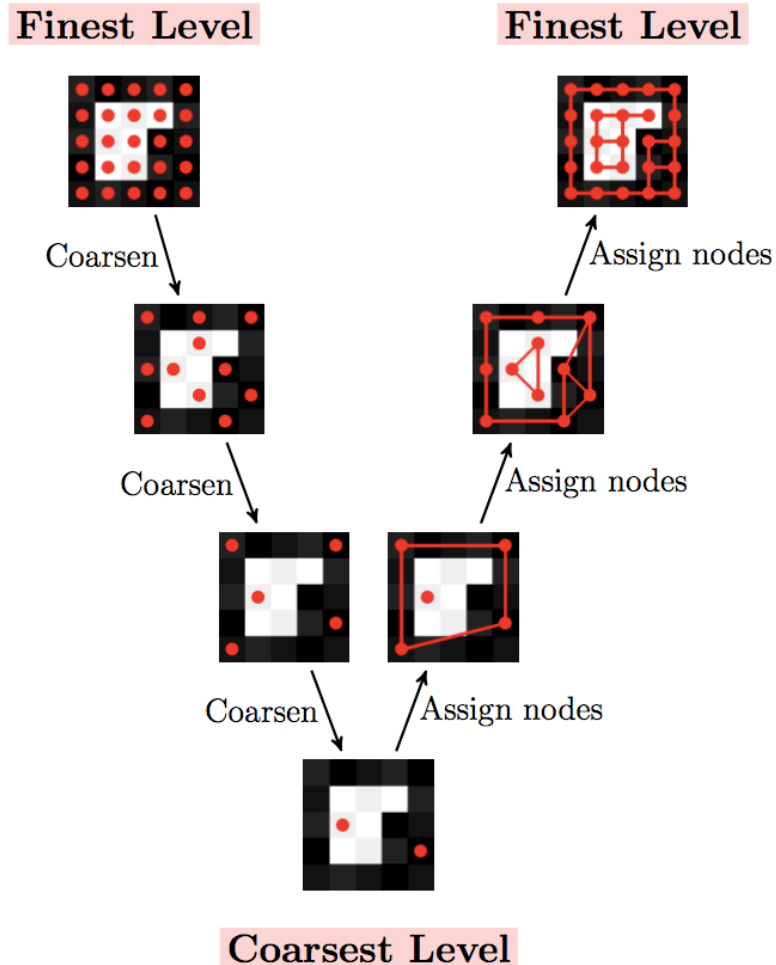
- we applied the 'segmentation by weighted aggregation' (SWA) algorithm to this problem
- we adapted the algorithm to our problem (in particular, new scale-invariant saliency measure)

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3. multilevel aggregation

Finest Level

Coarsen

Coarsen

Coarsen

Coarsest Level

Finest Level

Assign nodes

Assign nodes

Assign nodes

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- weights between neighbouring fine-level pixels based on intensity similarity (undirected graph)

- coarsen the graph (group fine-level pixels and calculate new weights)

- adjust weights based on coarse-level features (average intensity, intensity variance, shape, orientation, ...)

multilevel aggregation

Finest Level   Finest Level

Coarsen   Assign nodes

Coarsen   Assign nodes

Coarsen   Assign nodes

Coarsest Level

- coarse-level nodes are overlapping groups of fine-level nodes

- coarse-level node not connected to any other nodes ➔ 'salient' segment!

- stop coarsening when all nodes are salient

- upward phase: assign nodes on all levels to segments (remove overlap between groups)

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4. some algorithmic details

- fine-level coupling matrix

$$A_{ij}^{[1]} = \begin{cases} e^{-\alpha|I_i^{[1]} - I_j^{[1]}|} & \text{if } i, j \text{ are horizontal or} \\ & \qquad \text{vertical neighbours,} \\ 0 & \text{otherwise.} \end{cases}$$

- aggregate pixels using first pass of classical Ruge-Stueben AMG coarsening algorithm (strength of connection in coupling matrix) (first step in graph coarsening)

some algorithmic details

- interpolation from coarse to fine level

$$P_{ij}^{[1,2]} = \begin{cases} 1 & \text{if } i \in C^{[1]}, \ i = C_j^{[1]}, \\ 0 & \text{if } i \in C^{[1]}, \ i \neq C_j^{[1]}, \\ \dfrac{A_{iC_j^{[1]}}^{[1]}}{\displaystyle\sum_{k \in C^{[1]}} A_{ik}^{[1]}} & \text{if } i \notin C^{[1]}. \end{cases}$$

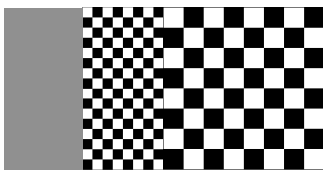- calculate coarse-level weights (second step in graph coarsening)

$$A^{[r+1]} = P^{[r,r+1]T} A^{[r]} P^{[r,r+1]}$$

some algorithmic details

- modify coarse-level weights:

  – average intensity (intensity similarity between coarse-level blocks)

$$A_{ij}^{[r+1]} \leftarrow A_{ij}^{[r+1]} e^{-\tilde{\alpha}|I_i^{[r+1]} - I_j^{[r+1]}|}$$

  – multilevel variance (as a measure of texture)

$$var(X) = E((X - E(X))^2) = E(X^2) - E(X)^2$$

$$A_{ij}^{[r+1]} \leftarrow A_{ij}^{[r+1]} e^{-\beta||\mathbf{s}_i^{[r+1]} - \mathbf{s}_j^{[r+1]}||_2}$$

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5. saliency measure

- when is a coarse-level node sufficiently decoupled from other nodes on the same level to make it (the representative of) a salient segment?

- SWA: energy functional

$$\Gamma^{[r]}(u^{[r]}) = \frac{\sum_{i>j} A_{ij}^{[r]} (u_i^{[r]} - u_j^{[r]})^2}{\sum_{i>j} A_{ij}^{[r]} u_i^{[r]} u_j^{[r]}}$$

saliency measure

- SWA: energy functional

$$\Gamma^{[r]}(u^{[r]}) = \frac{\sum_{i>j} A_{ij}^{[r]} (u_i^{[r]} - u_j^{[r]})^2}{\sum_{i>j} A_{ij}^{[r]} u_i^{[r]} u_j^{[r]}}$$

- seek segments (boolean vectors $u^{[r]}$) that yield low values of the energy functional (equivalent to normalized cut formulation, on finest level) (but: multilevel in SWA)

saliency measure

- low saliency Γ indicates segment

- numerator

$$\Gamma^{[r]}(u^{[r]}) = \frac{\sum\limits_{i>j} A^{[r]}_{ij} (u^{[r]}_i - u^{[r]}_j)^2}{\sum\limits_{i>j} A^{[r]}_{ij} u^{[r]}_i u^{[r]}_j}$$

  = sum of coupling coefficients along boundary

  = boundary length, weighted by similarity

- denominator

  = sum of coupling coefficients in interior

  = area, weighted by internal similarity

saliency measure

$$\Gamma^{[r]}(u^{[r]}) = \frac{\sum\limits_{i>j} A_{ij}^{[r]} (u_i^{[r]} - u_j^{[r]})^2}{\sum\limits_{i>j} A_{ij}^{[r]} u_i^{[r]} u_j^{[r]}} = \frac{u^{[r]T} L^{[r]} u^{[r]}}{\frac{1}{2} u^{[r]T} W^{[r]} u^{[r]}}$$

- graph Laplacian

$$L_{ij}^{[r]} = \begin{cases} -A_{ij}^{[r]} & \text{if } i \neq j, \\ \sum\limits_{k \neq i} A_{ik}^{[r]} & \text{if } i = j, \end{cases}$$

- coupling matrix

$$W^{[r]} = A^{[r]}$$

- on coarse level

$$\Gamma_i^{[r]} = \Gamma(u^{[r],i}) = \frac{L_{ii}^{[r]}}{\frac{1}{2} W_{ii}^{[r]}}$$

$$\Gamma_i^{[r]} < \gamma.$$

saliency measure

$$\Gamma_i^{[r]} = \Gamma(u^{[r],i}) = \frac{L_{ii}^{[r]}}{\frac{1}{2}W_{ii}^{[r]}} \qquad \Gamma_i^{[r]} < \gamma.$$

problems with shape-invariance and scale-invariance

(due to ratio length / area)

(a)Small L_{ii}, large W_{ii}.   (b)Large L_{ii}, small W_{ii}.

saliency measure

$$L_{ij}^{[r]} = \begin{cases} -A_{ij}^{[r]} & \text{if } i \neq j, \\ \sum_{k \neq i} A_{ik}^{[r]} & \text{if } i = j, \end{cases}$$

$$\Gamma_i^{[r]} = \Gamma(u^{[r],i}) = \frac{L_{ii}^{[r]}}{\frac{1}{2} W_{ii}^{[r]}} \qquad \Gamma_i^{[r]} < \gamma$$

$$W^{[r]} = A^{[r]}$$

- unweighted coupling matrix (boundary length)

$$V_{ij}^{[1]} = \begin{cases} 0 & \text{if } A_{ij}^{[1]} = 0, \\ 1 & \text{if } A_{ij}^{[1]} \neq 0. \end{cases}$$

- unweighted Laplacian (area)

$$\Gamma_i^{[r]} = \frac{L_{ii}^{[r]}/G_{ii}^{[r]}}{W_{ii}^{[r]}/V_{ii}^{[r]}}$$

$$G_{ij}^{[1]} = \begin{cases} -V_{ij}^{[1]} & \text{if } i \neq j, \\ \sum_{k \neq i} V_{ik}^{[1]} & \text{if } i = j. \end{cases}$$

saliency measure

$$\Gamma_i^{[r]} < \gamma.$$

similarity-weighted boundary length /
similarity-weighted area

$$\Gamma_i^{[r]} = \frac{L_{ii}^{[r]}}{\frac{1}{2} W_{ii}^{[r]}}$$

➔ normalize boundary length and area

average similarity along boundary /
average internal similarity

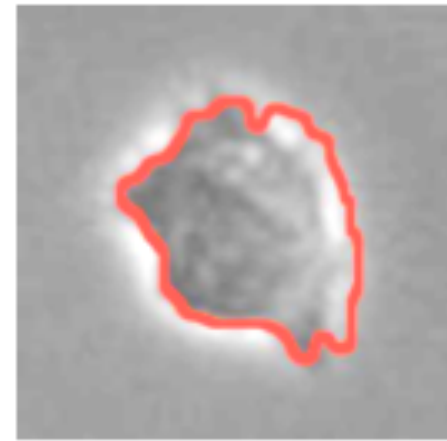$$\Gamma_i^{[r]} = \frac{L_{ii}^{[r]}/G_{ii}^{[r]}}{W_{ii}^{[r]}/V_{ii}^{[r]}}$$

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6. cell segmentation
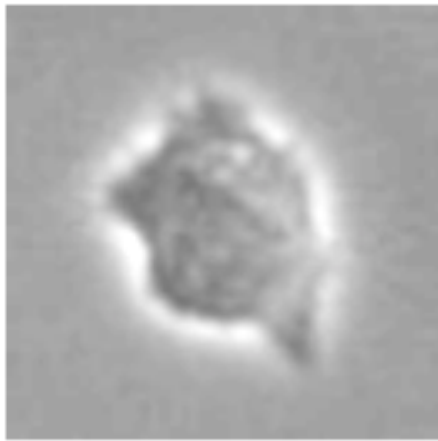
(a)Original image
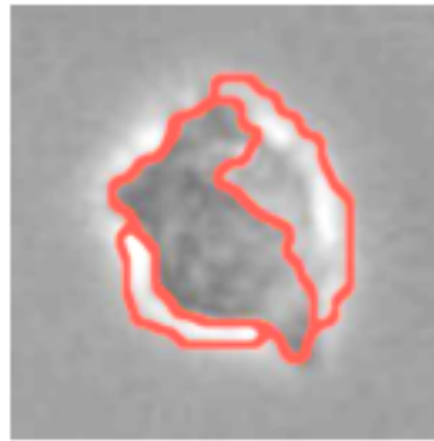
(b)4 segments found (including the background segment), not using variance

(c)2 segments found (including the background segment), using variance

cell segmentation
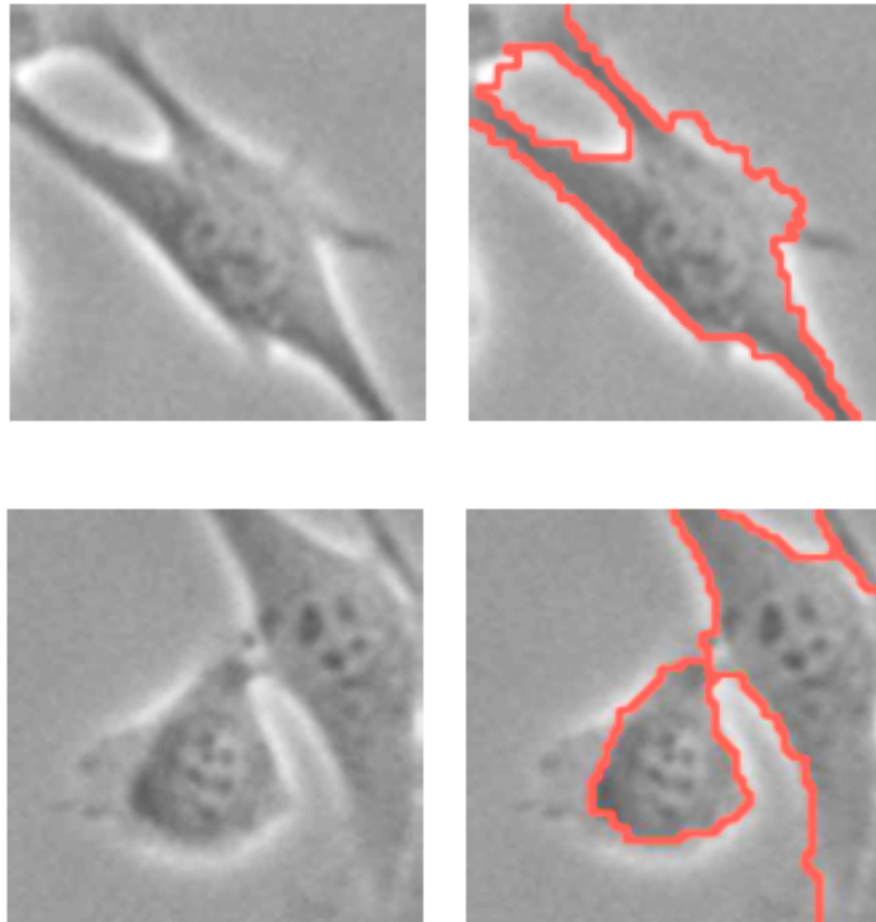
cell segmentation

7. cell tracking in space-time
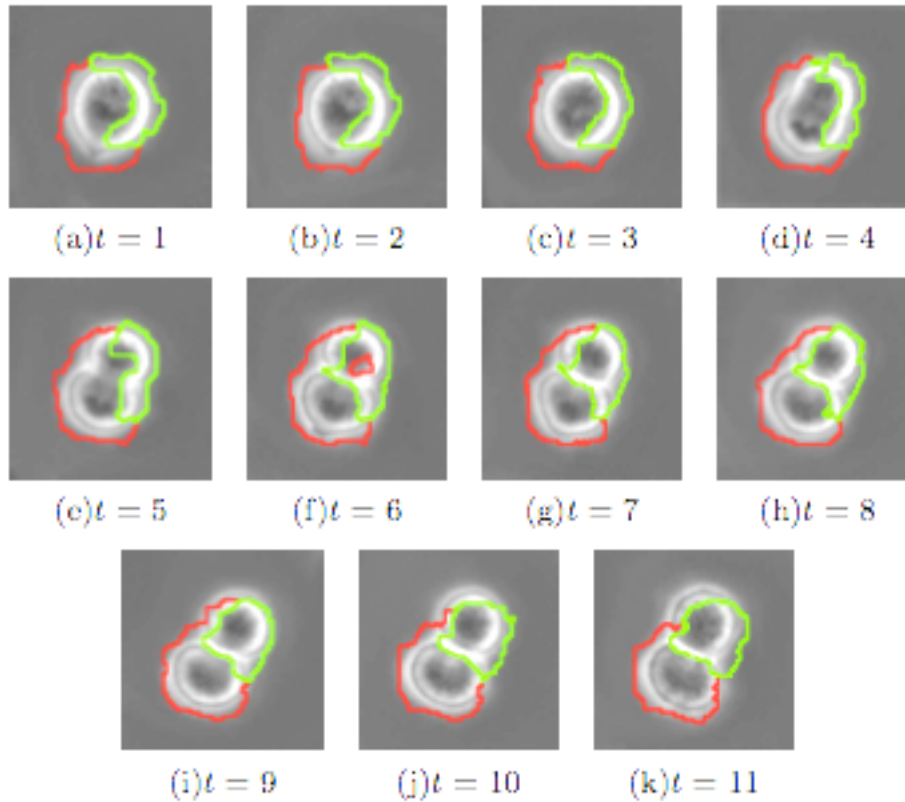

(a)$t = 1$ (b)$t = 2$ (c)$t = 3$

(d)$t = 4$ (e)$t = 5$ (f)$t = 6$

(g)$t = 7$ (h)$t = 8$ (i)$t = 9$

cell tracking in space-time

(a)$t = 1$ (b)$t = 2$ (c)$t = 3$ (d)$t = 4$

(e)$t = 5$ (f)$t = 6$ (g)$t = 7$ (h)$t = 8$

(i)$t = 9$ (j)$t = 10$ (k)$t = 11$

Time

10
8
6
4
2
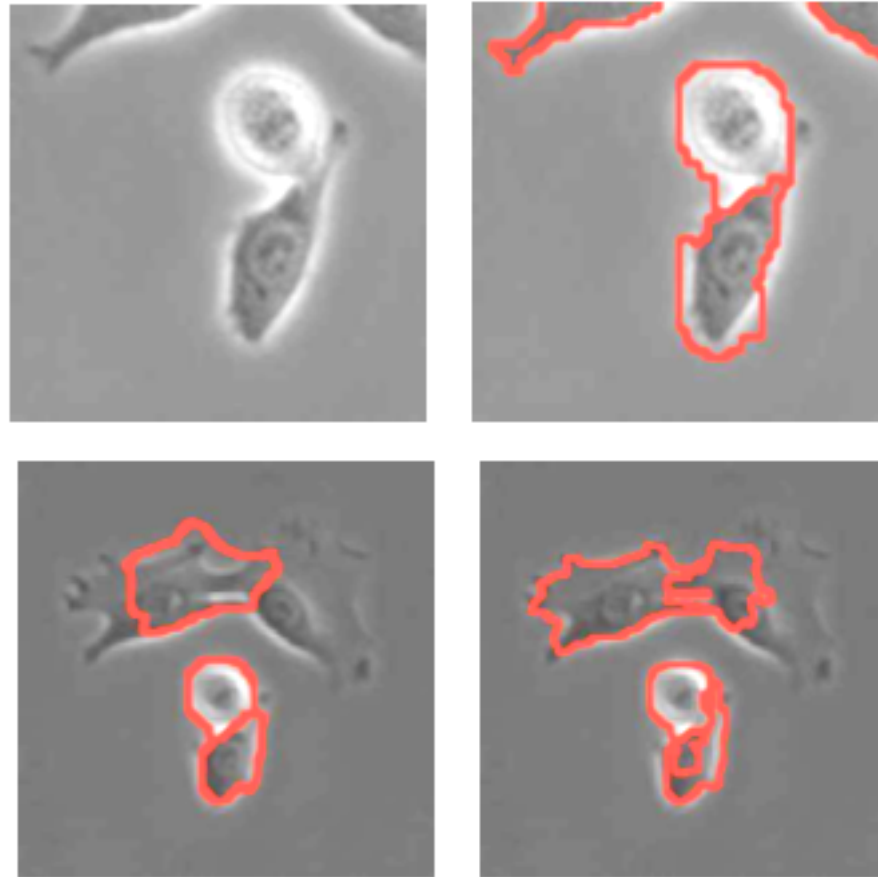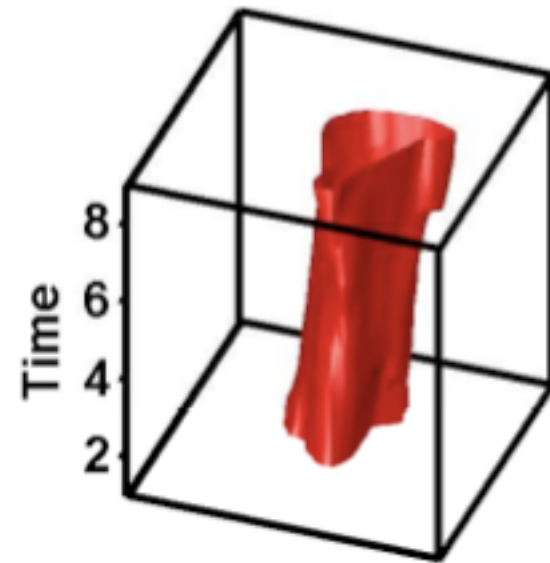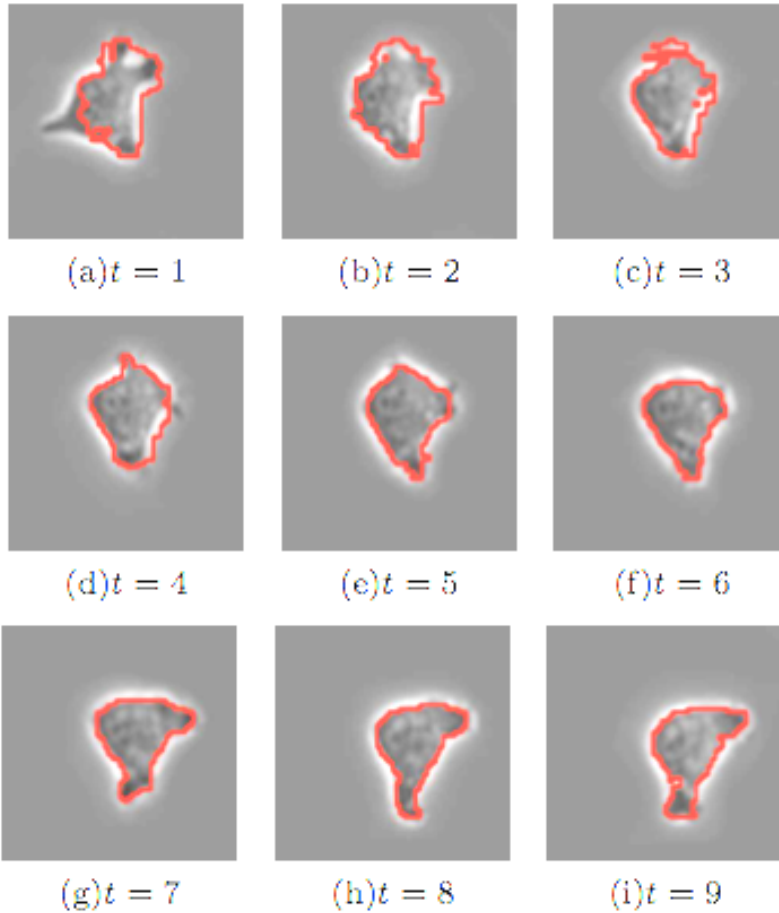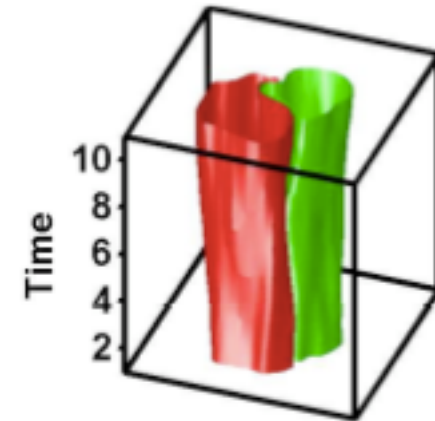
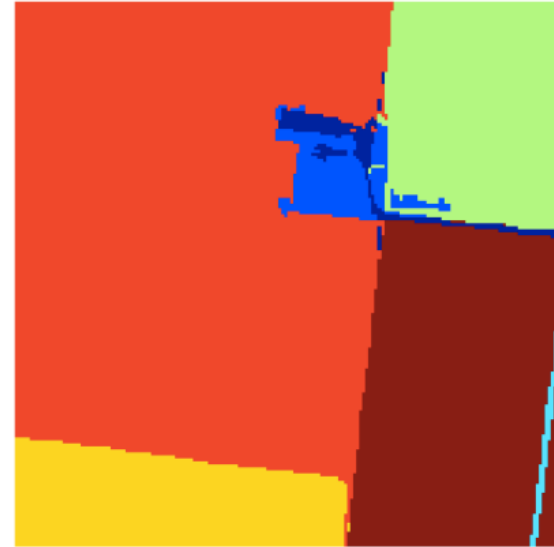# 8. performance
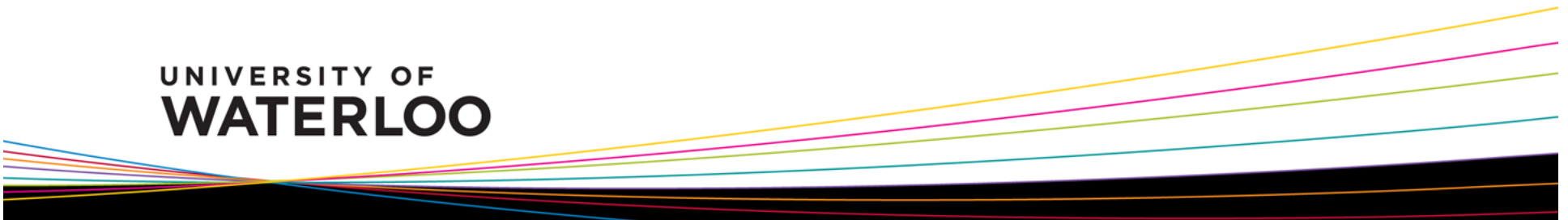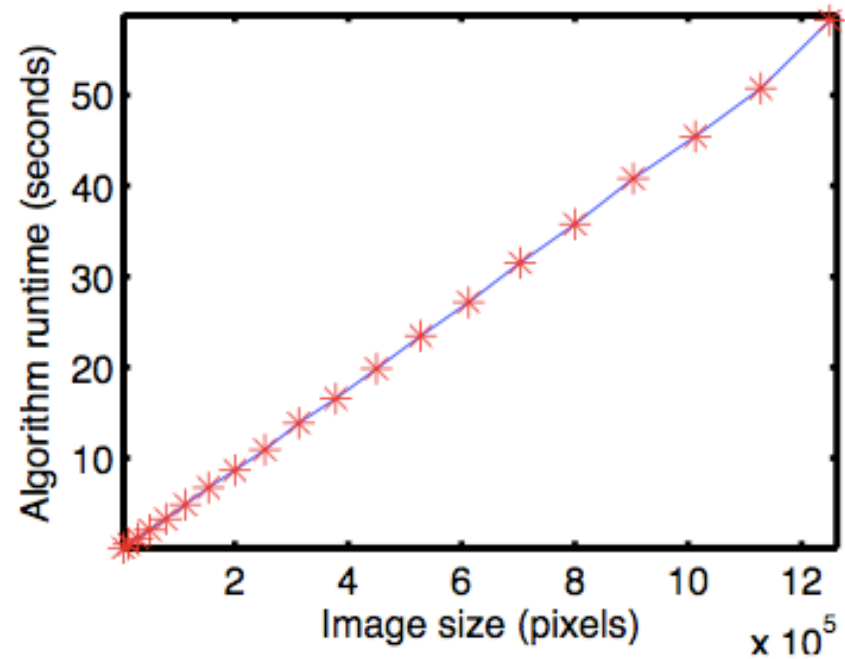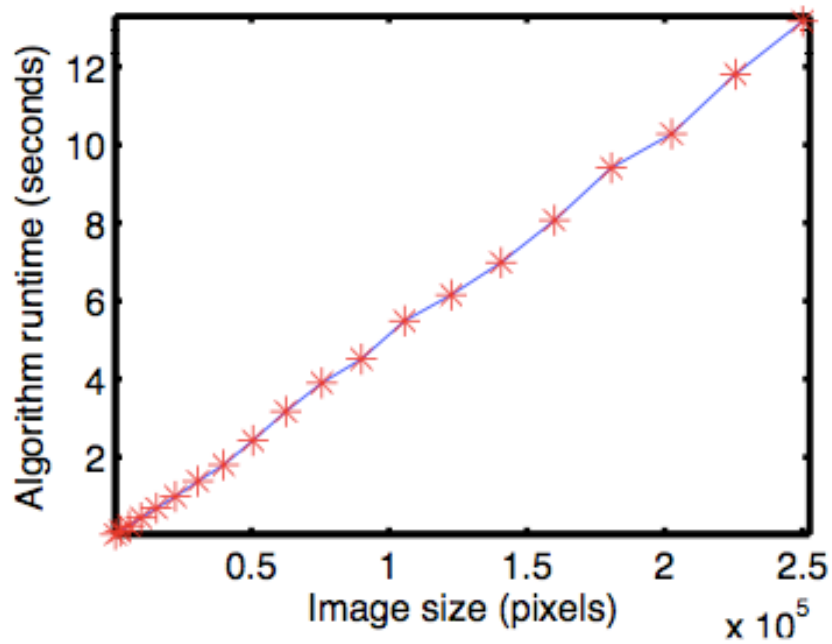
9. satellite segmentation

Figure 5.1: RGB image of a farm (180x180).

satellite segmentation
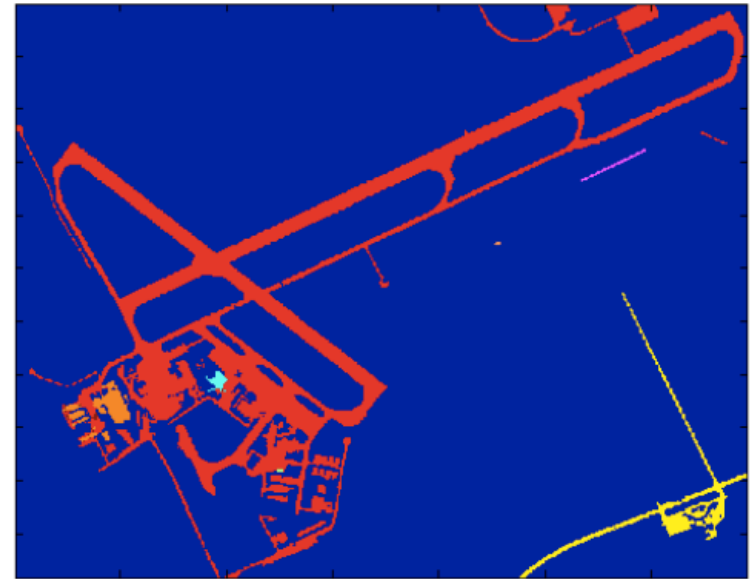
Figure 5.5: RGB image of the Region of Waterloo airport taken by Ikonos (2301x1801).

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satellite segmentation: multilevel elongation

$$\Lambda_k = \begin{bmatrix} (\overline{x^2})_k - (\overline{x})_k^2 & (\overline{xy})_k - (\overline{x})_k(\overline{y})_k \\ (\overline{xy})_k - (\overline{x})_k(\overline{y})_k & (\overline{y^2})_k - (\overline{y})_k^2 \end{bmatrix}$$

$$R_k = \frac{\min(\omega)}{\max(\omega)}$$
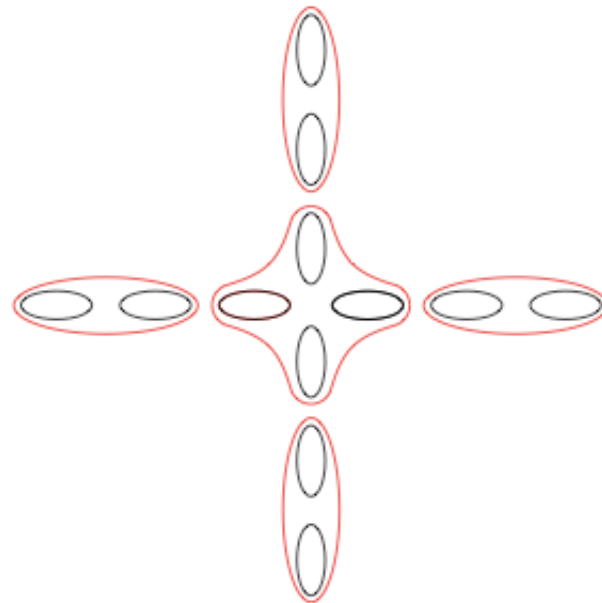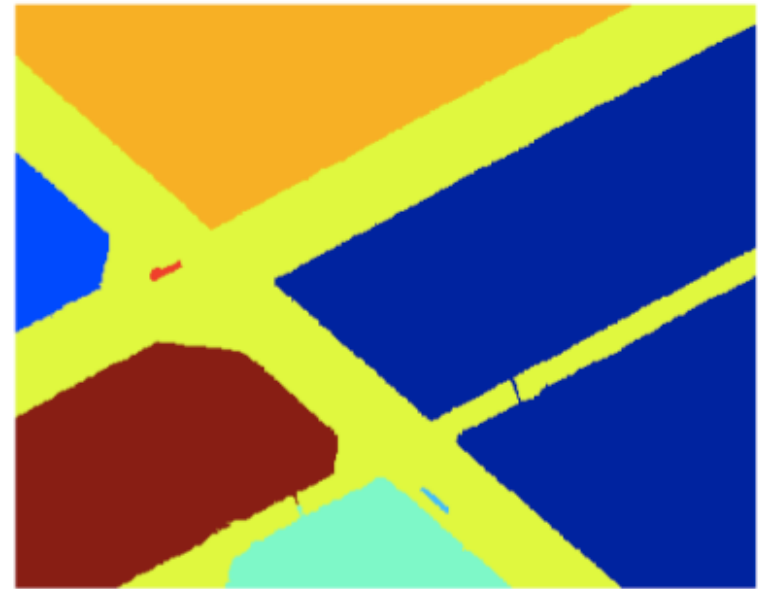
satellite segmentation

Figure 5.7: RGB image of the Region of Waterloo airport runway (501x351).
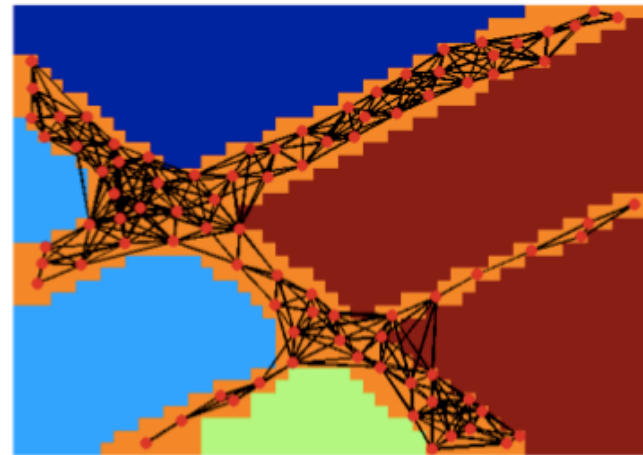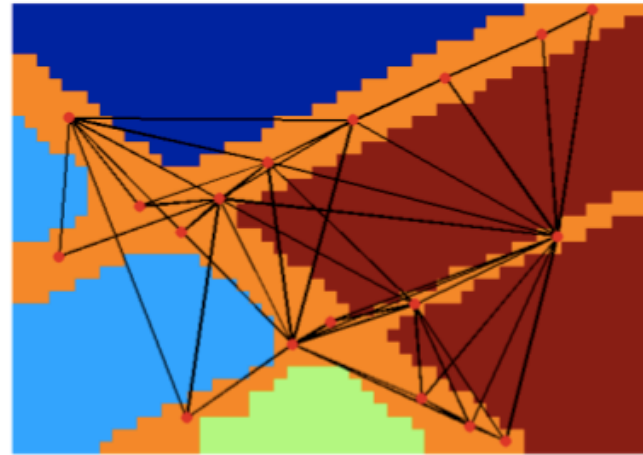
satellite segmentation

Figure 5.9: Scaled runway image (51x36).

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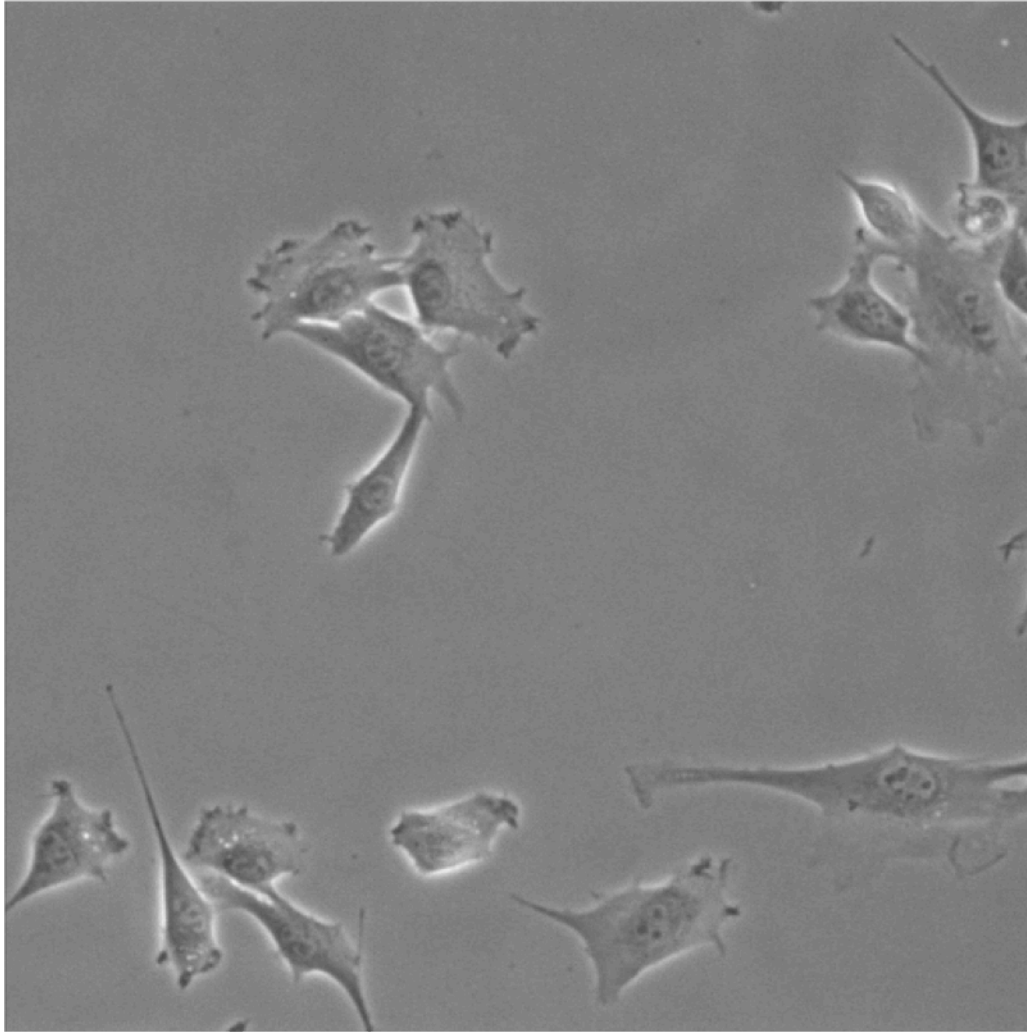**WATERLOO**

10. collaborators

- my Master's students at Waterloo:
  - Tiffany Inglis (cell application)
  - Adley Au (satellite application)
- Geoff Sanders (Boulder)
- Haig Djambazian, Robert Sladek, and Saravanan Sundarara (McGill University and Genome Quebec Innovation Centre, Montreal) and Thomas J. Hudson (Ontario Institute for Cancer Research, Toronto)

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11. conclusions

- multilevel aggregation (SWA) appears promising for cell segmentation and tracking, and satellite segmentation

- we propose a new scale-invariant and shape-invariant saliency measure for SWA

- problem: too many free parameters ('ideal' solution: include more coarse-level features up to the point that parameters can be fixed for a whole class of images)

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conclusions

- powerful (multilevel features), efficient (linear complexity)

- space-time aggregation automatically uses temporal information (touching, overlapping cells)

- advantage: multilevel hierarchy contains extensive feature information about segments (cell (part) classification, road extraction, ...)

- commercial application: search for famous people in online movies

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questions?

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