Introduction to numerical MHD modeling of the solar wind

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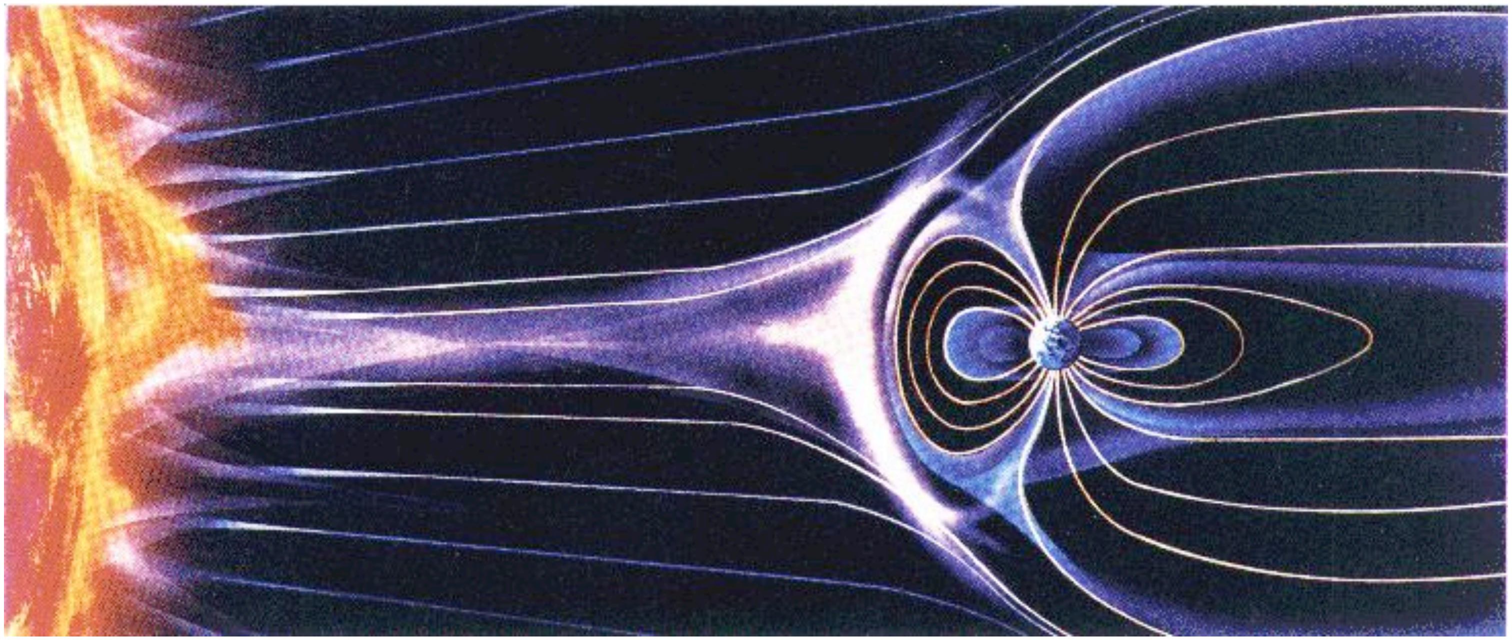
solar wind



- use continuum simulation model: magnetohydrodynamics (MHD)
- challenges: transition from subsonic to supersonic flow, shocks, scales, ...

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overview

1. the MHD model for plasma dynamics
2. numerical MHD methods
3. numerical MHD modeling of the solar wind

1. the MHD model for plasma dynamics

ideal MHD:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + (\nabla \times \vec{B}) \times \vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$$

$$\frac{\partial p}{\partial t} + (\vec{v} \cdot \nabla) p + \gamma p \nabla \cdot \vec{v} = 0$$

-PDE system

-8 equations in
8 unknowns

-functions
of space and time

ideal MHD

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + (\nabla \times \vec{B}) \times \vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$$

$$\frac{\partial p}{\partial t} + (\vec{v} \cdot \nabla) p + \gamma p \nabla \cdot \vec{v} = 0$$

MHD=gasdynamics +
electromagnetics

$$\vec{E} = -\vec{v} \times \vec{B}$$

$$\vec{J} = \nabla \times \vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

total energy: $$e = \frac{p}{\gamma - 1} + \rho \frac{\vec{v} \cdot \vec{v}}{2} + \frac{\vec{B} \cdot \vec{B}}{2}$$

ideal MHD plasma

- one-fluid plasma (ionized gas)
- quasi-neutral plasma
- isotropic pressure
- non-relativistic

- neglects some (important) kinetic effects
- leads to tractable models (full 3D simulations of solar-terrestrial system faster than real-time on parallel computers)

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ideal MHD in conservation law form

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho\,\vec{v}\\ \vec{B}\\ e\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\,\vec{v}\\ \rho\,\vec{v}\vec{v} + \mathbf{I}\left(p + \vec{B}\cdot\vec{B}\,/2\right) - \vec{B}\vec{B}\\ \vec{v}\vec{B} - \vec{B}\vec{v}\\ \left(e + p + \vec{B}\cdot\vec{B}\,/2\right)\vec{v} - \left(\vec{v}\cdot\vec{B}\right)\vec{B}\end{bmatrix}$$

$$= -\begin{bmatrix}0\\ \vec{B}\\ \vec{v}\\ \vec{v}\cdot\vec{B}\end{bmatrix}\nabla\cdot\vec{B}.$$

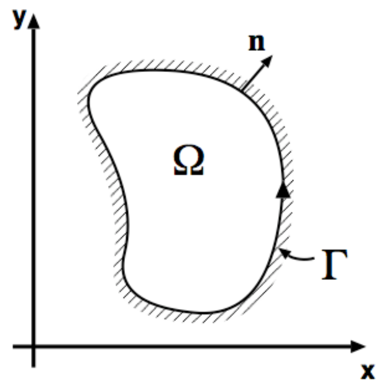
$$\frac{\partial\mathbf{U}}{\partial t} + \nabla\cdot\vec{\mathbf{F}}(\mathbf{U}) = \mathbf{S}^{Pow}$$

MHD=hyperbolic conservation law

conservation law

$$\frac{\partial u}{\partial t} + \nabla \cdot \mathbf{f}(u) = 0$$

$$\frac{d}{dt}\left(\int_\Omega u(\mathbf{x},t)d\Omega\right) + \int_\Omega \nabla \cdot \mathbf{f}(u)d\Omega = 0$$



$$\frac{d}{dt}Q_\Omega(t) + \oint_{\partial\Omega} (\mathbf{f}(u)\cdot\mathbf{n})d\ell = 0$$

conserved quantity: $Q_\Omega(t) = \int_\Omega u(\mathbf{x},t)d\Omega$

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ideal MHD in conservation law form

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\,\vec{v} \\ \vec{B} \\ e\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\,\vec{v} \\ \rho\,\vec{v}\vec{v} + \mathsf{I}\left(p + \vec{B}\cdot\vec{B}\,/2\right) - \vec{B}\vec{B} \\ \vec{v}\vec{B} - \vec{B}\vec{v} \\ \left(e + p + \vec{B}\cdot\vec{B}\,/2\right)\vec{v} - \left(\vec{v}\cdot\vec{B}\right)\vec{B}\end{bmatrix}$$

$$= -\begin{bmatrix}0 \\ \vec{B} \\ \vec{v} \\ \vec{v}\cdot\vec{B}\end{bmatrix}\nabla\cdot\vec{B}.$$

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla\cdot\vec{\mathbf{F}}(\mathbf{U}) = \mathbf{S}^{Pow}$$

conservation of mass, momentum, magnetic flux, energy

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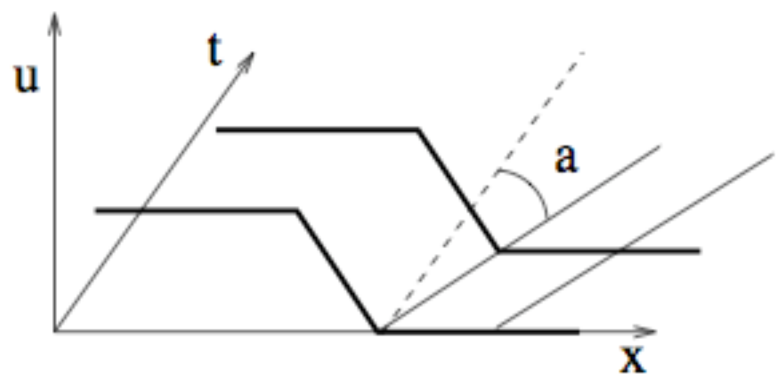
ideal MHD waves (hyperbolic PDEs)

linear advection equation in 1D:

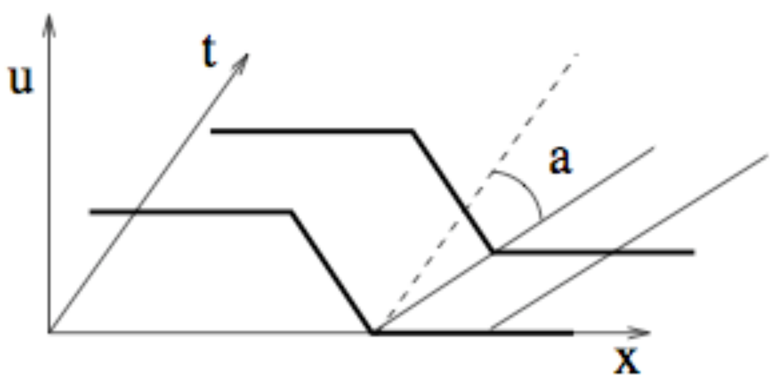
$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \qquad u(x,t) = f(x - at)$$



ideal MHD waves

1D linear advection:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$



1D MHD:

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \vec{\mathbf{F}}(\mathbf{U}) = \mathbf{S}^{Pow}$$

$$\frac{\partial \mathbf{U}}{\partial t} + \mathsf{A}(\mathbf{U}) \cdot \frac{\partial \mathbf{U}}{\partial x} = 0$$

$$\mathsf{A} = \mathsf{R} \cdot \Lambda \cdot \mathsf{L}$$

$$\mathbf{W} = \mathsf{L} \cdot \mathbf{U}$$

$$\frac{\partial \mathbf{W}}{\partial t} + \Lambda \cdot \frac{\partial \mathbf{W}}{\partial x} = 0$$

$$\frac{\partial w_i}{\partial t} + \lambda_i \frac{\partial w_i}{\partial x} = 0$$

ideal MHD in conservation law form

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\,\vec{v} \\ \vec{B} \\ e\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\,\vec{v} \\ \rho\,\vec{v}\vec{v} + \mathsf{I}\left(p + \vec{B}\cdot\vec{B}\,/2\right) - \vec{B}\vec{B} \\ \vec{v}\vec{B} - \vec{B}\vec{v} \\ \left(e + p + \vec{B}\cdot\vec{B}\,/2\right)\vec{v} - \left(\vec{v}\cdot\vec{B}\right)\vec{B}\end{bmatrix}$$

$$= -\begin{bmatrix}0 \\ \vec{B} \\ \vec{v} \\ \vec{v}\cdot\vec{B}\end{bmatrix}\nabla\cdot\vec{B}.$$

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla\cdot\vec{\mathbf{F}}(\mathbf{U}) = \mathbf{S}^{Pow}$$

conservation of mass, momentum, magnetic flux, energy

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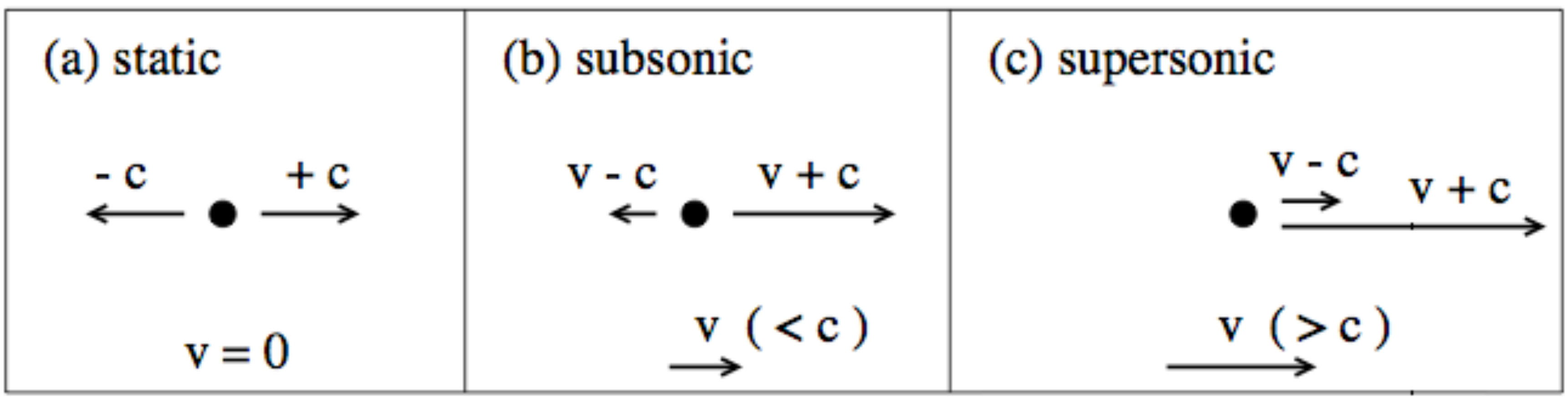
gasdynamics waves

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\,\vec{v} \\ e\end{bmatrix} + \nabla\cdot\begin{bmatrix}\rho\,\vec{v} \\ \rho\,\vec{v}\vec{v} + \mathsf{I}p \\ (e+p)\,\vec{v}\end{bmatrix} = \nabla\cdot(\mathsf{D}) \qquad \frac{\partial \mathbf{U}}{\partial t} + \mathsf{A}(\mathbf{U})\cdot\frac{\partial \mathbf{U}}{\partial x} = 0$$

wave speeds

λ= v, v, v, v+c, v-c

$$c = \sqrt{\frac{\gamma p}{\rho}} \qquad M = \frac{||\vec{v}||}{c}$$



MHD waves

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho\vec{v}\\ \vec{B}\\ e\end{bmatrix}+\nabla\cdot\begin{bmatrix}\rho\vec{v}\\ \rho\vec{v}\vec{v}+\mathsf{I}\left(p+\vec{B}\cdot\vec{B}/2\right)-\vec{B}\vec{B}\\ \vec{v}\vec{B}-\vec{B}\vec{v}\\ \left(e+p+\vec{B}\cdot\vec{B}/2\right)\vec{v}-\left(\vec{v}\cdot\vec{B}\right)\vec{B}\end{bmatrix}$$

$$=-\begin{bmatrix}0\\ \vec{B}\\ \vec{v}\\ \vec{v}\cdot\vec{B}\end{bmatrix}\nabla\cdot\vec{B}.$$

$$\frac{\partial\mathbf{U}}{\partial t}+\mathsf{A}(\mathbf{U})\cdot\frac{\partial\mathbf{U}}{\partial x}=0$$

$$\mathsf{A}=\begin{bmatrix}v_x&\rho&0&0&0&0&0&0\\ 0&v_x&0&0&0&B_y/\rho&B_z/\rho&1/\rho\\ 0&0&v_x&0&0&-B_x/\rho&0&0\\ 0&0&0&v_x&0&0&-B_x/\rho&0\\ 0&0&0&0&v_x&0&0&0\\ 0&B_y&-B_x&0&0&v_x&0&0\\ 0&B_z&0&-B_x&0&0&v_x&0\\ 0&c^2\rho&0&0&0&0&0&v_x\end{bmatrix}$$

MHD waves

$$\frac{\partial \mathbf{U}}{\partial t} + \mathsf{A}(\mathbf{U}) \cdot \frac{\partial \mathbf{U}}{\partial x} = 0$$

$$\lambda_{1,2} = v_x \pm c_{fx}, \quad \lambda_{3,4} = v_x \pm c_{Ax}, \quad \lambda_{5,6} = v_x \pm c_{sx}, \quad \lambda_{7,8} = v_x$$

$$c_{fx}^2 = \frac{1}{2}\left(\frac{\gamma p + B^2}{\rho} + \sqrt{\left(\frac{\gamma p + B^2}{\rho}\right)^2 - 4\frac{\gamma p B_x^2}{\rho^2}}\right)$$

fast, Alfven, slow:

$$c_{Ax}^2 = \frac{B_x^2}{\rho}$$

$$c_{fx} \geq c_{Ax} \geq c_{sx}$$

$$c_{sx}^2 = \frac{1}{2}\left(\frac{\gamma p + B^2}{\rho} - \sqrt{\left(\frac{\gamma p + B^2}{\rho}\right)^2 - 4\frac{\gamma p B_x^2}{\rho^2}}\right)$$

(anisotropic waves)

wave steepening (nonlinear hyperbolic PDE)

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0 \qquad \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

simple example: Burgers' equation

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wave steepening

$$\frac{d}{dt}u(x(t),t) = \frac{\partial u}{\partial x}\frac{dx(t)}{dt} + \frac{\partial u}{\partial t} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

"u is constant on characteristic curve"

characteristic curve: $$\frac{dx(t)}{dt} = u(x(t),t)$$

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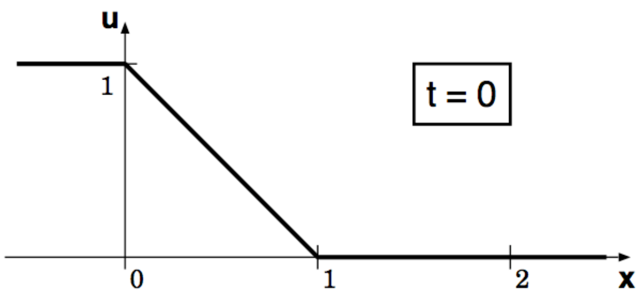
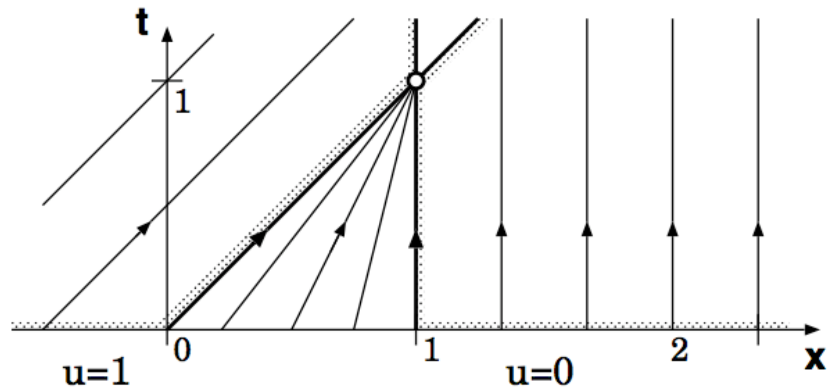
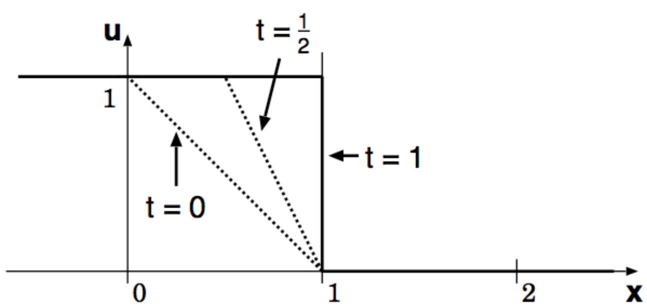
wave steepening: shock waves are formed

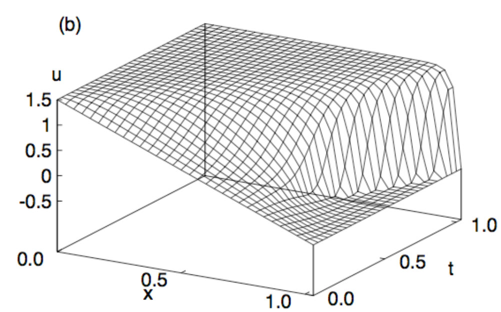
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

$$\frac{dx(t)}{dt} = u(x(t), t)$$









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shock speed - Rankine Hugoniot relations

1D conservation system: $$\frac{\partial \mathbf{U}(x,t)}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = 0$$

traveling wave solution: $$\mathbf{U}(x,t) = \mathbf{U}(y = x - st)$$

$$-s\frac{\partial \mathbf{U}(y)}{\partial y} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial y} = 0$$

$$-s\mathbf{U} + \mathbf{F}(\mathbf{U}) = \mathbf{F}_{const}$$

RH relation for shock wave: $$\mathbf{F}(\mathbf{U}_r) - \mathbf{F}(\mathbf{U}_l) = s\ (\mathbf{U}_r - \mathbf{U}_l)$$

MHD shocks

$$\mathbf{F}(\mathbf{U}_r) - \mathbf{F}(\mathbf{U}_l) = s\ (\mathbf{U}_r - \mathbf{U}_l)$$

s=0 in the rest frame of the shock wave

$$F(\begin{bmatrix} \rho \\ \rho\, v_x \\ \rho\, v_y \\ \rho\, v_z \\ B_x \\ B_y \\ B_z \\ e \end{bmatrix}) = \begin{bmatrix} \rho\, v_x \\ \rho\, v_x^2 + p + B^2/2 - B_x^2 \\ \rho\, v_x v_y - B_x B_y \\ \rho\, v_x v_z - B_x B_z \\ 0 \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \\ (e + p + B^2/2) v_x - B_x\, (\vec{v} \cdot \vec{B}) \end{bmatrix}$$

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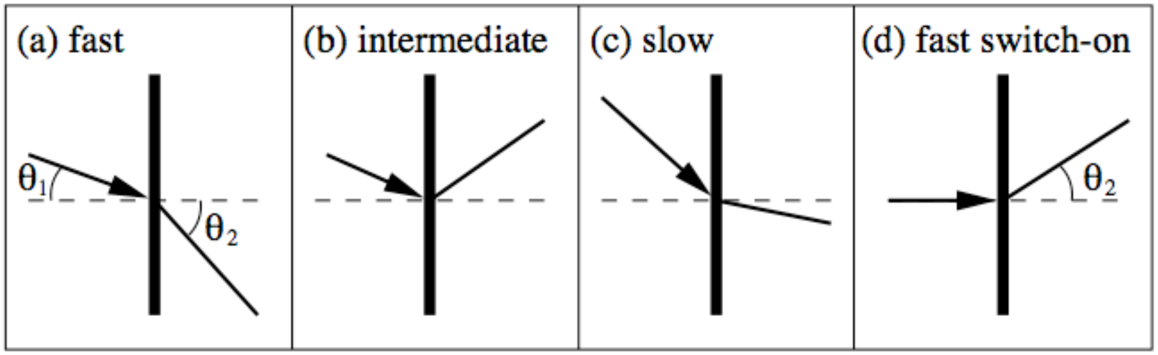
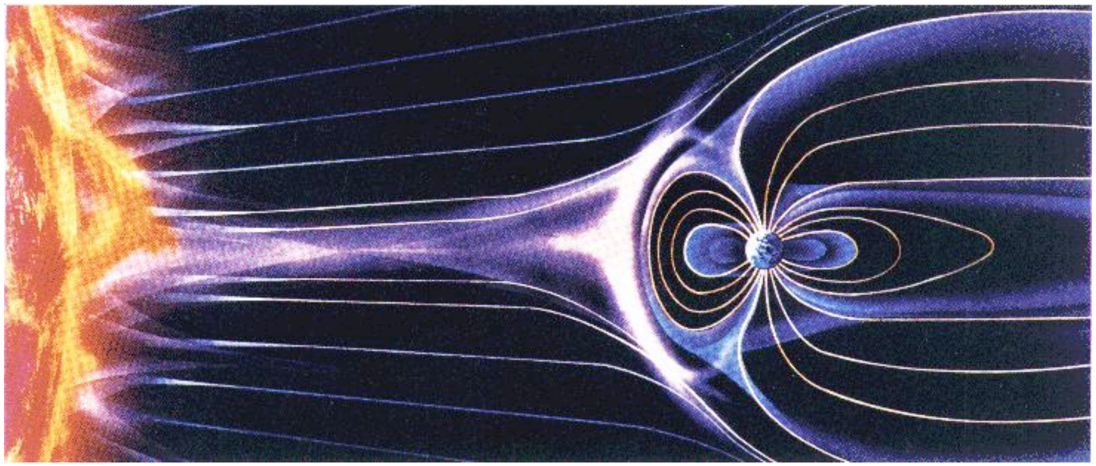
MHD shocks

3 types of MHD shocks



bow shocks in the solar wind



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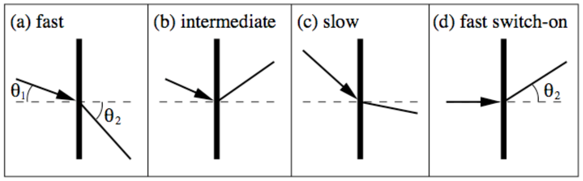
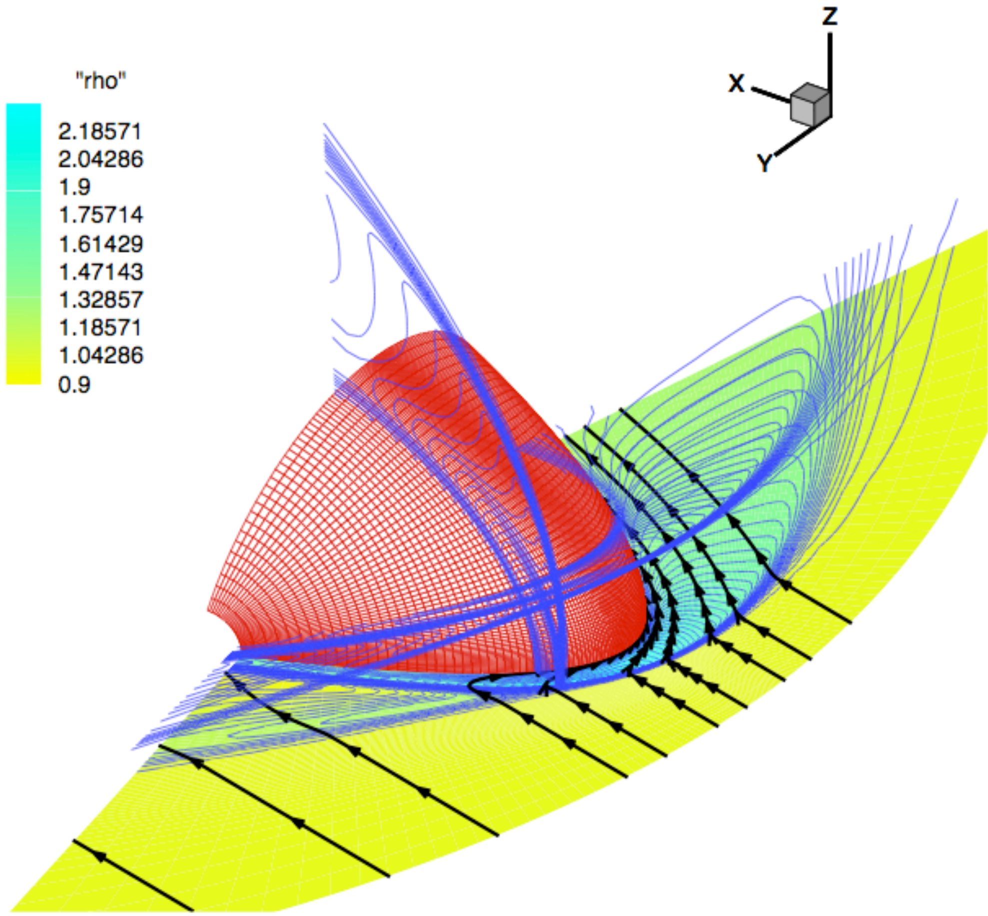
MHD shocks





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overview

1. the MHD model for plasma dynamics
2. numerical MHD methods
3. numerical MHD modeling of the solar wind

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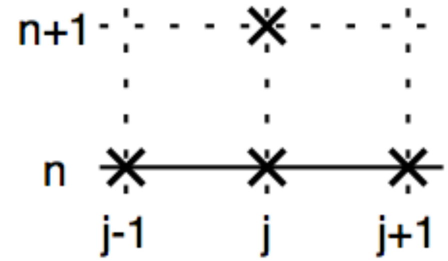
2. numerical MHD methods

linear advection equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

central differences

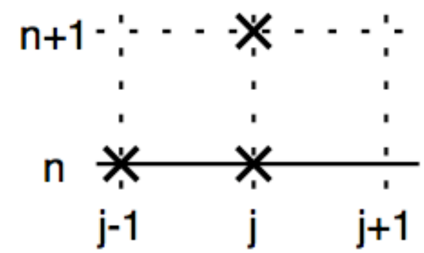
$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + a\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} = 0$$



unstable!

upwind differences

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + a\frac{u_i^n - u_{i-1}^n}{\Delta x} = 0$$



stable: $\Delta t < \frac{\Delta x}{a}$ (a > 0)

conservative form

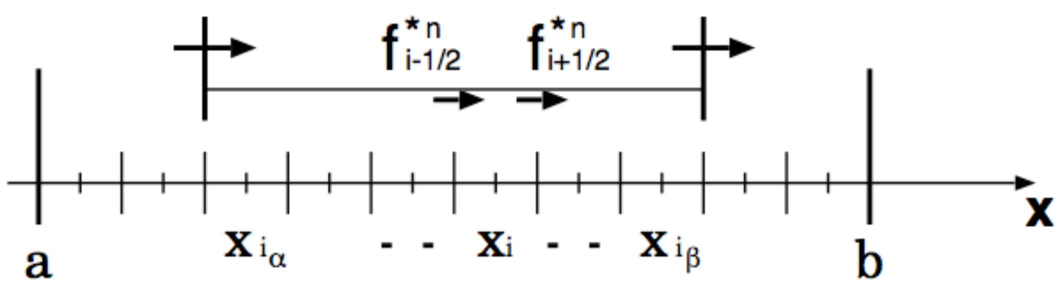
rewrite
$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+a\frac{u_i^n-u_{i-1}^n}{\Delta x}=0$$

as
$$\frac{u_i^{n+1}-u_i^n}{\Delta t}+\frac{f_{i+1/2}^{n*}-f_{i-1/2}^{n*}}{\Delta x}=0$$

with numerical flux function

$$f_{i+1/2}^{n*}=\frac{a\,u_{i+1}^n+a\,u_i^n}{2}-\frac{1}{2}|a|(u_{i+1}^n-u_i^n)$$

conservative form: exact discrete conservation



gives correct shock speeds

nonlinear conservation law

nonlinear flux function f(u):

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

conservative upwind method

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{f_{i+1/2}^{n*} - f_{i-1/2}^{n*}}{\Delta x} = 0$$

with numerical flux function

$$f_{i+1/2}^{n*} = \frac{f(u_{i+1}^n) + f(u_i^n)}{2} - \frac{1}{2}|f_{i+1/2}^{\prime n*}|(u_{i+1}^n - u_i^n)$$

nonlinear conservative system

nonlinear system: $$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{U})}{\partial x} = 0$$

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{f_{i+1/2}^{n*} - f_{i-1/2}^{n*}}{\Delta x} = 0$$

with

$$\mathbf{F}_{i+1/2}^{n*} = \frac{\mathbf{F}(\mathbf{U}_{i+1}^n) + \mathbf{F}(\mathbf{U}_i^n)}{2} - \frac{1}{2}\max_k(|\lambda_{i+1/2}^{(k)}|)(\mathbf{U}_{i+1}^n - \mathbf{U}_i^n)$$

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system in 2D: upwind finite volume method

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \vec{\mathbf{F}}(\mathbf{U}) = \mathbf{S}^{Pow}$$

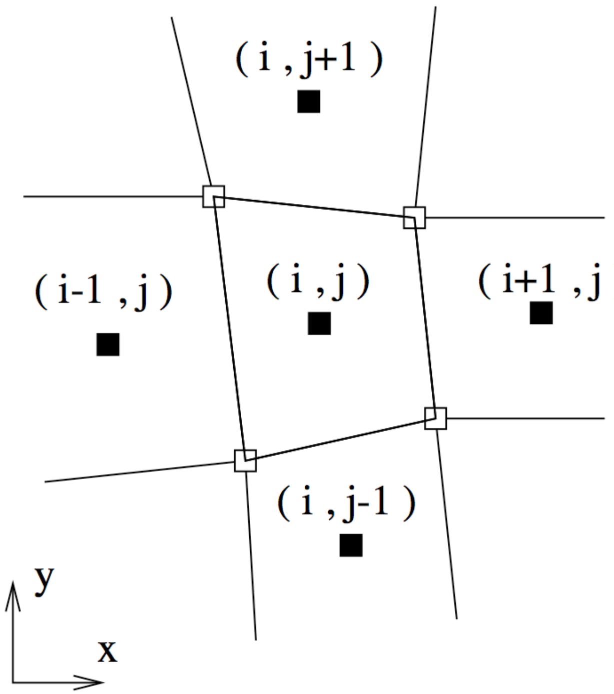
$$\frac{d}{dt}\left(\int_\Omega u(\mathbf{x},t)d\Omega\right) + \int_\Omega \nabla \cdot \mathbf{f}(u)d\Omega = 0.$$

use integrated form over finite volume cell:

$$\frac{\partial \overline{\mathbf{U}}_{i,j}}{\partial t} + 1/\Omega_{i,j} \sum_{k=1}^{4} \vec{\mathbf{F}}_k^* \cdot \vec{n}_k \, \Delta l_k = \overline{\mathbf{S}}_{i,j}^{Pow}$$

$$\overline{\mathbf{U}}_{i,j} = \left(\int\int \mathbf{U}(x,y,t)\, dx\, dy\right) / \Omega_{i,j}$$

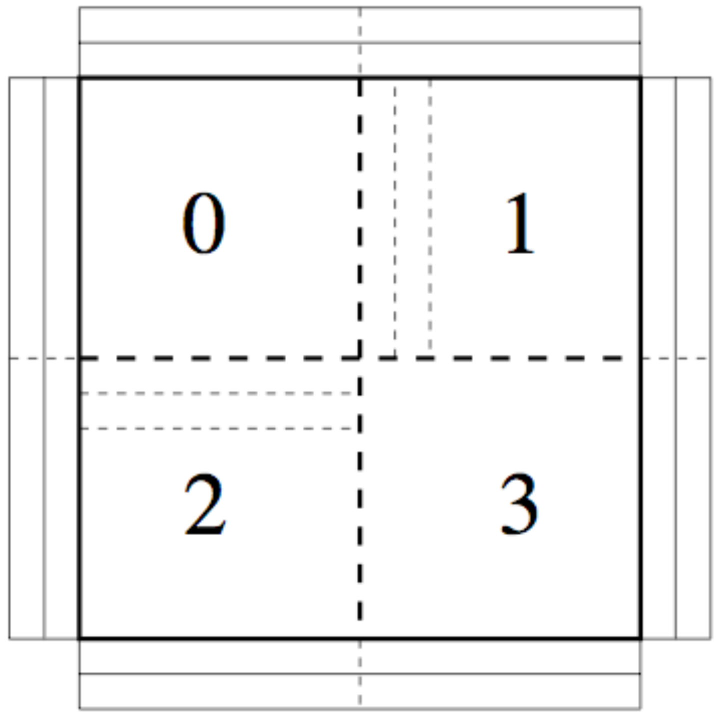
2D grid with discrete unknowns:



(use upwind numerical fluxes F*)

parallel implementation

-divide work over n processors
-communicate ghost cells between neighbour processors



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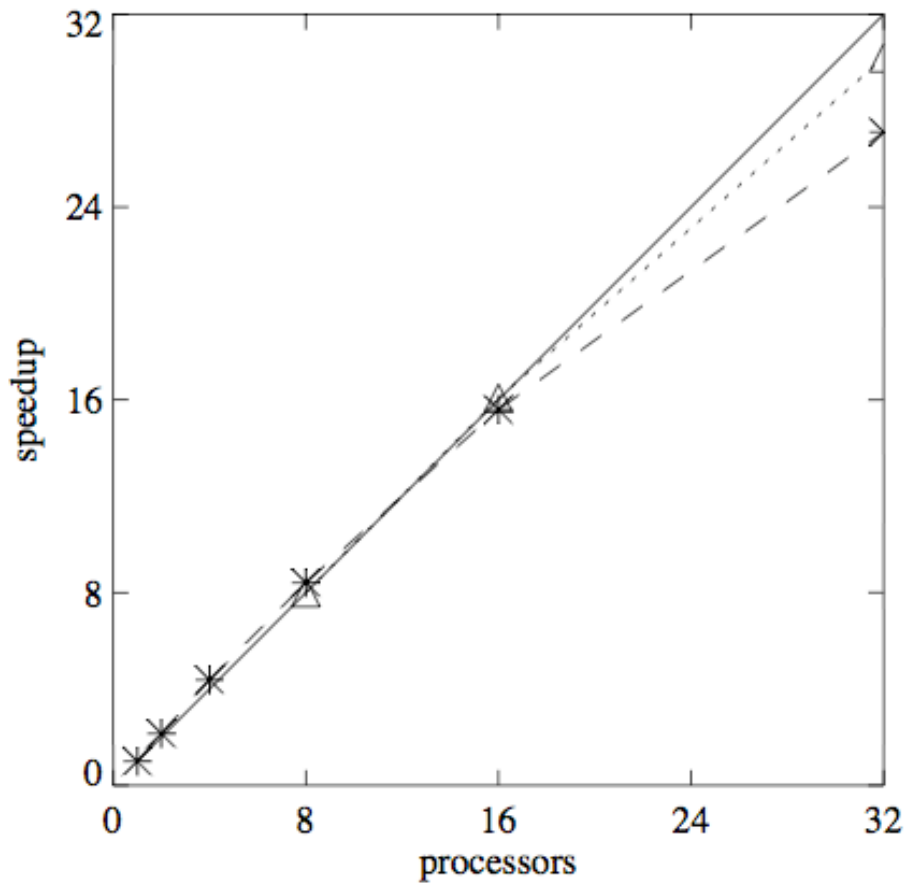
parallel implementation: scaling



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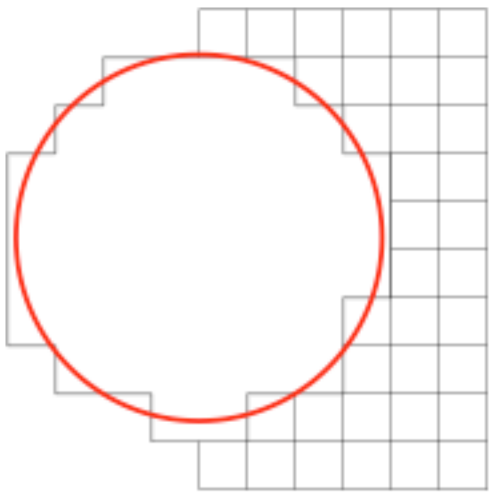
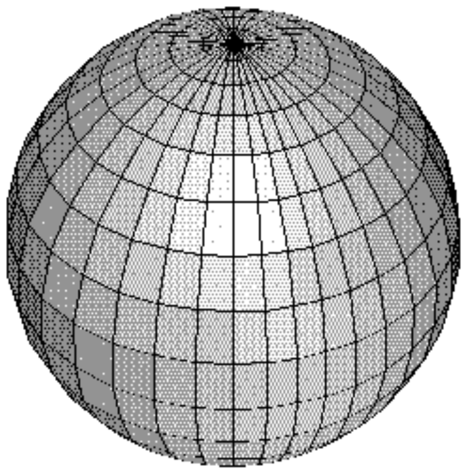
grids for outflow from spherical objects...





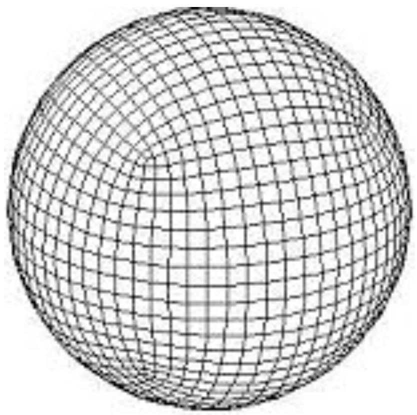
structured, Cartesian

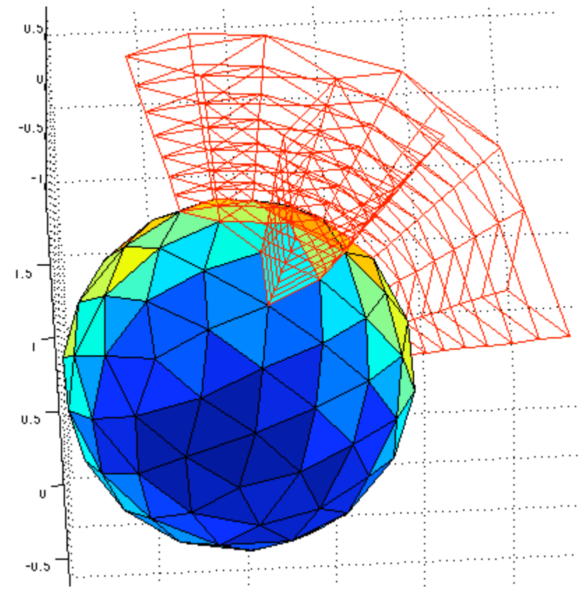
staircase effect

structured, spherical

singularities at 2 poles

grids for outflow from spherical objects...





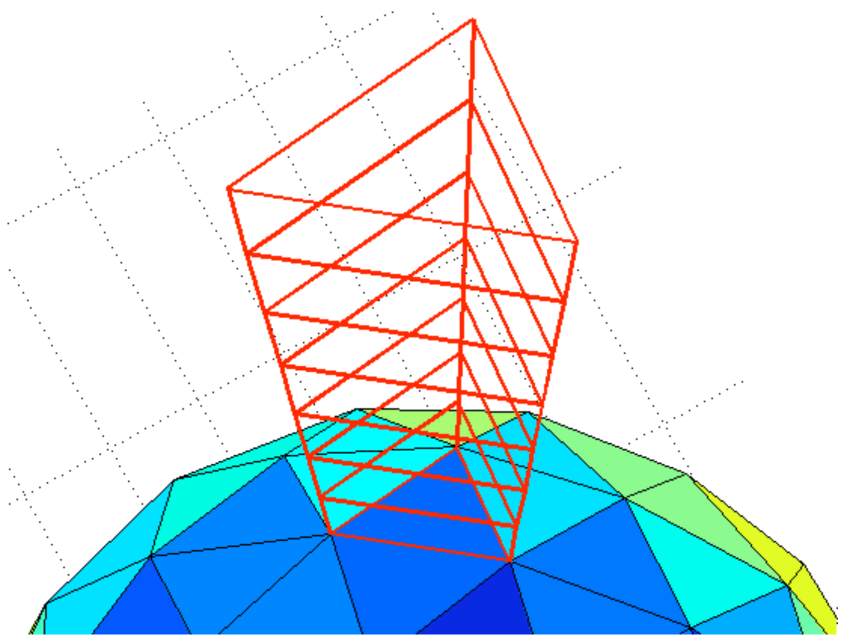
structured, 'cubed sphere'

singularities at 8 'poles'

unstructured!

triangles, uniform

grids for outflow from spherical objects...



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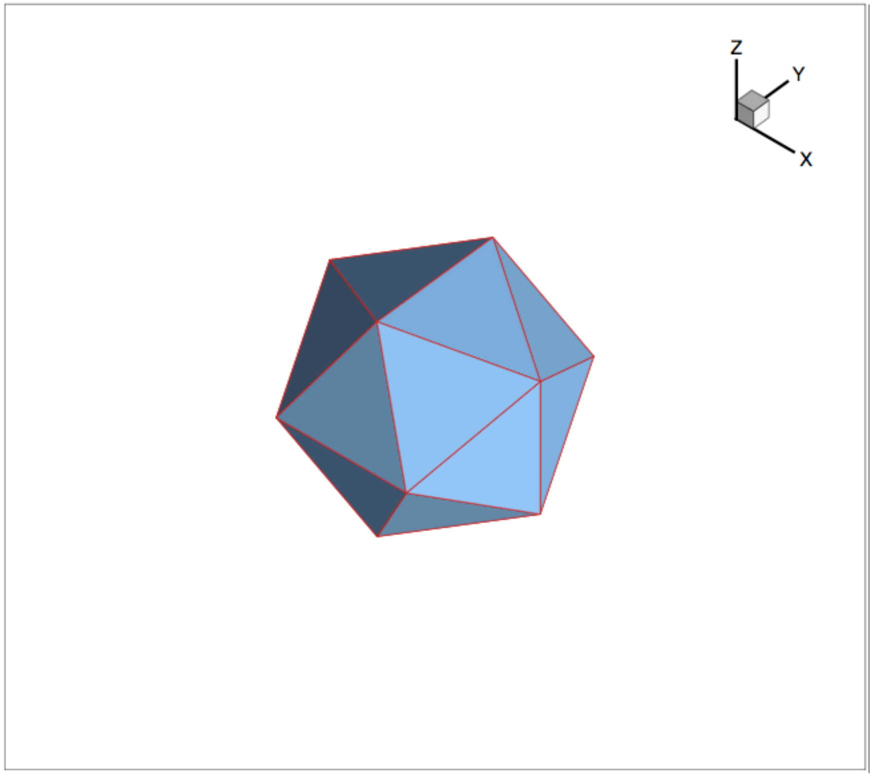
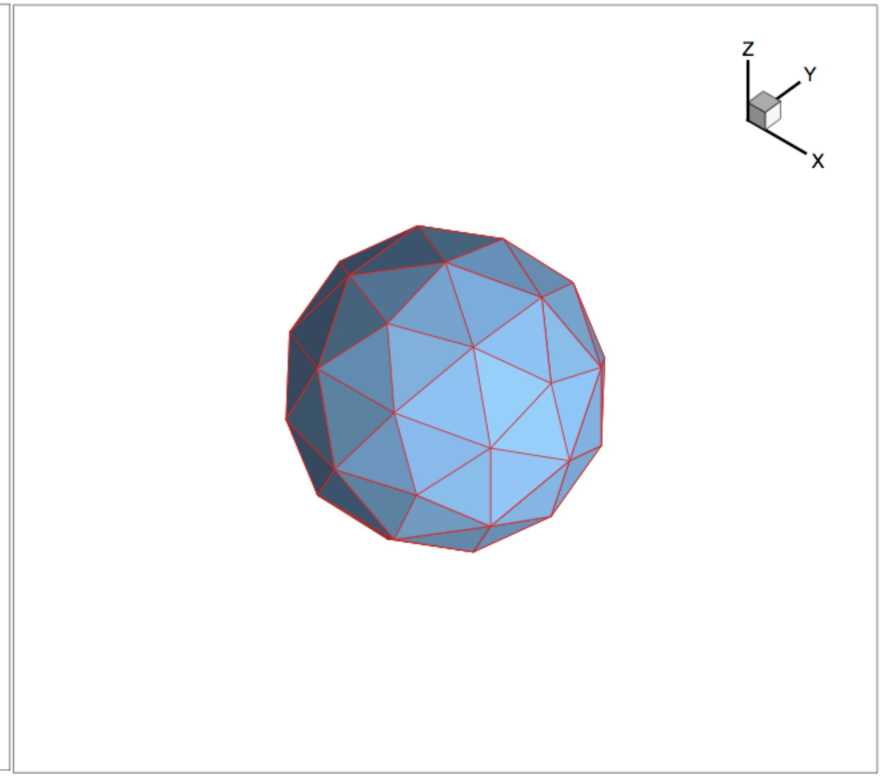
unstructured surface grids
(refine icosahedral grids)





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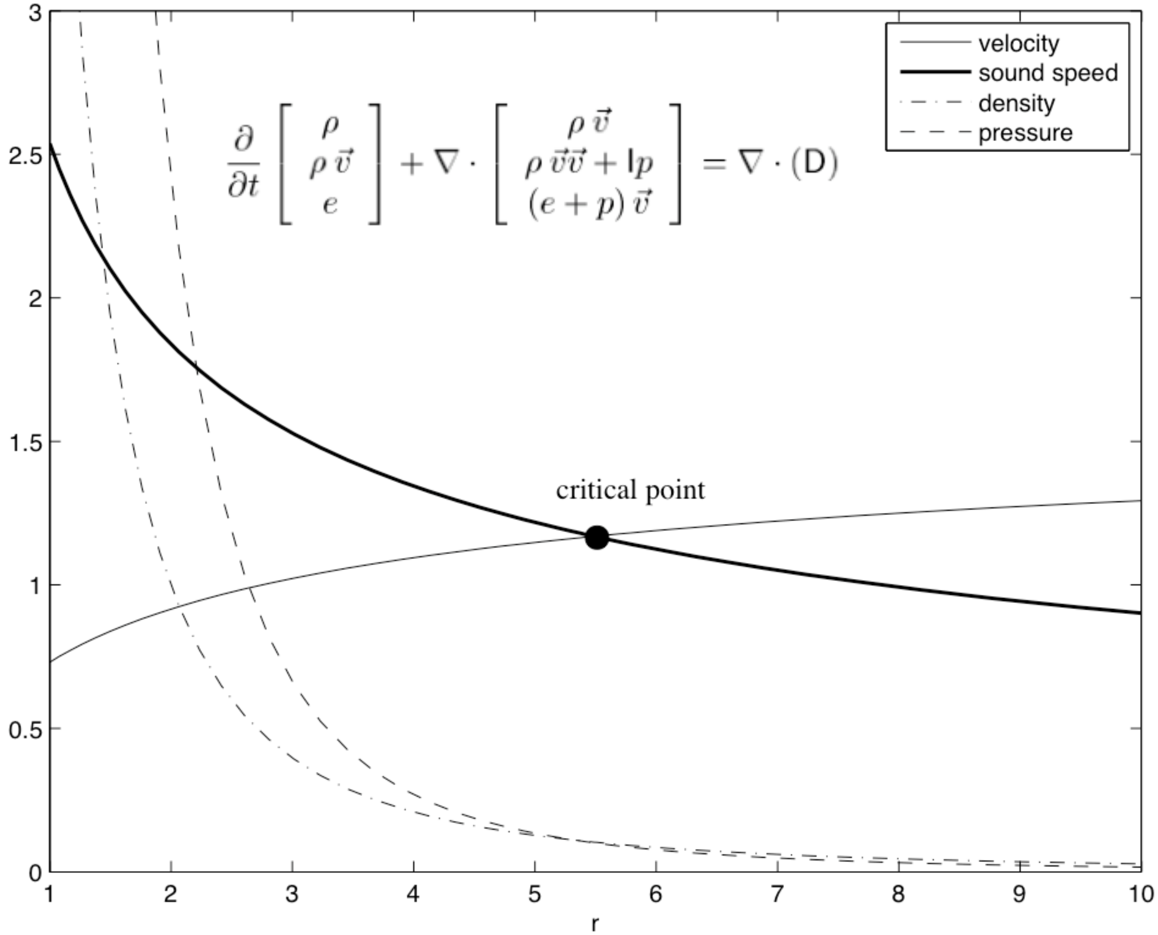
simple gasdynamic model of the solar wind:



subsonic ⇒|⇐ supersonic

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radial solar wind models (1D)

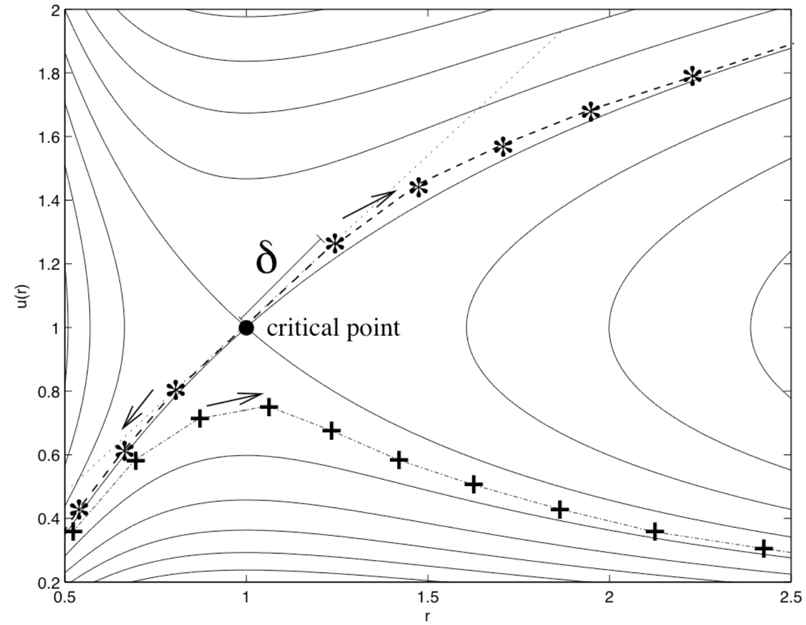
(includes Parker solar wind model)

- $$\frac{\partial}{\partial t}\begin{bmatrix} \rho\, r^2 \\ \rho\, u\, r^2 \end{bmatrix} + \frac{\partial}{\partial r}\begin{bmatrix} \rho\, u\, r^2 \\ \rho\, u^2\, r^2 + p\, r^2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\rho G M + 2\, p\, r \end{bmatrix}$$

- $$\frac{U^{n+1} - U^n}{\Delta t} + R(U^{n+1}) = S(U^{n+1})$$

Parker's solar wind

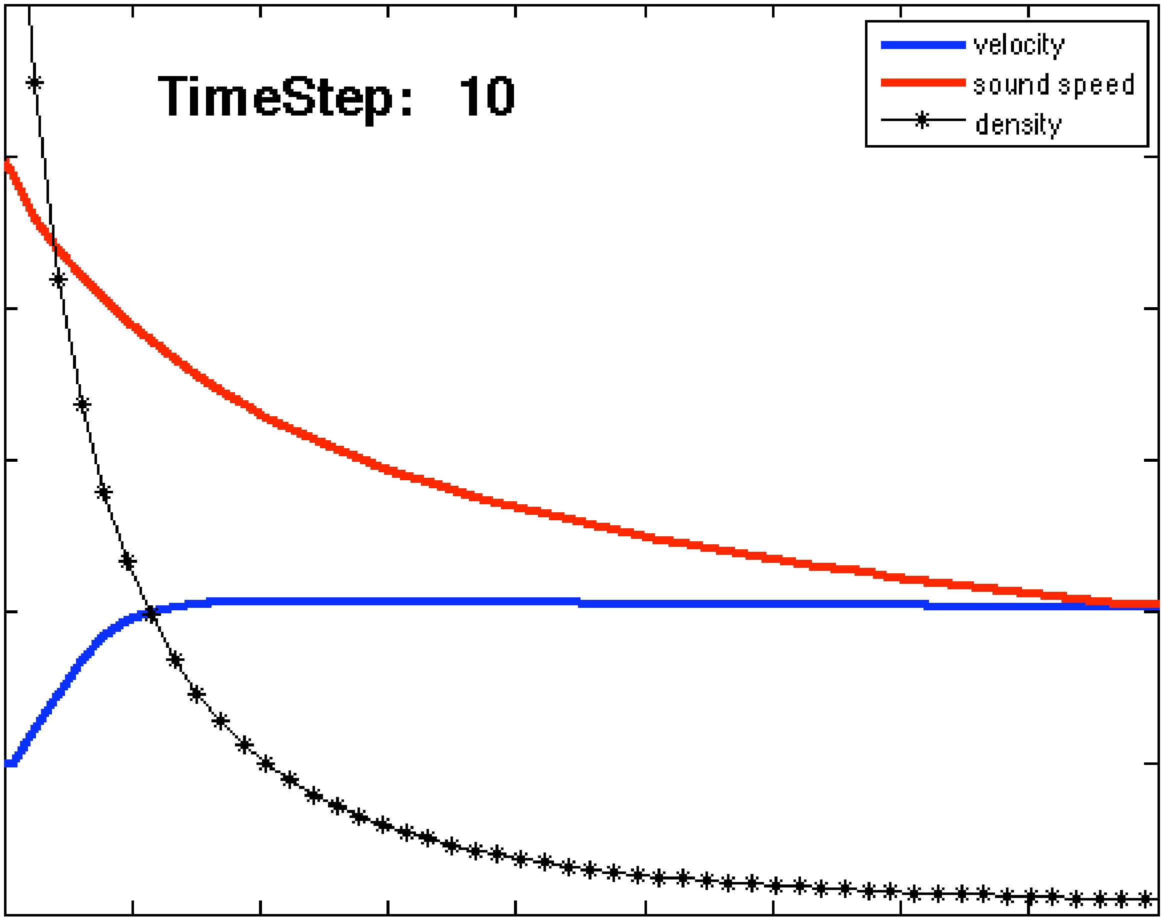
- critical point: u = c
  - wave speed u-c = 0
  - very slow convergence for time-marching methods
  - singular: 0/0 in ODE
  - 1 boundary condition for 2 ODEs specifies unique (transonic) solution



$$\frac{d}{dr}(\rho u r^2) = 0$$

$$\frac{du}{dr} = \frac{2\,u\,c^2\,(r - r_c)}{r^2\,(u^2 - c^2)}$$

use time marching method (explicit)



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include heat source and heat conduction

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho\, r^2 \\ \rho\, u\, r^2 \\ \left(\frac{p}{\gamma-1}+\frac{\rho\, u^2}{2}\right)\, r^2 \end{bmatrix} + \frac{\partial}{\partial r}\begin{bmatrix} \rho\, u\, r^2 \\ \rho\, u^2\, r^2 + p\, r^2 \\ \left(\frac{\gamma\, p}{\gamma-1}+\frac{\rho\, u^2}{2}\right)\, u\, r^2 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ -\rho\, G M + 2\, p\, r \\ -\rho\, G M\, u + q_{heat}\, r^2 + \frac{\partial}{\partial r}\left(\kappa\, r^2\, \frac{\partial T}{\partial r}\right) \end{bmatrix}$$

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solar wind in 2D and 3D

- use time-marching for solar wind
- alternative: use implicit time integration for solar wind
- most existing codes (BATS-R-US, VAC, SCAMHD, ...) are block-structured (rectangular cells in a regular structure, using Cartesian or spherical coordinates)
- our proposed approach: use 'unstructured grids'

Mikic et al., Phys. Plasmas 1999

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}, \tag{2}$$

$$\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B} = \eta \mathbf{J}, \tag{3}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{4}$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = \frac{1}{c}\mathbf{J} \times \mathbf{B} - \nabla p - \nabla p_w + \rho \mathbf{g} + \nabla \cdot (\nu \rho \nabla \mathbf{v}), \tag{5}$$

$$\frac{\partial p}{\partial t} + \nabla \cdot (p \mathbf{v}) = (\gamma - 1)(-p \nabla \cdot \mathbf{v} + S), \tag{6}$$

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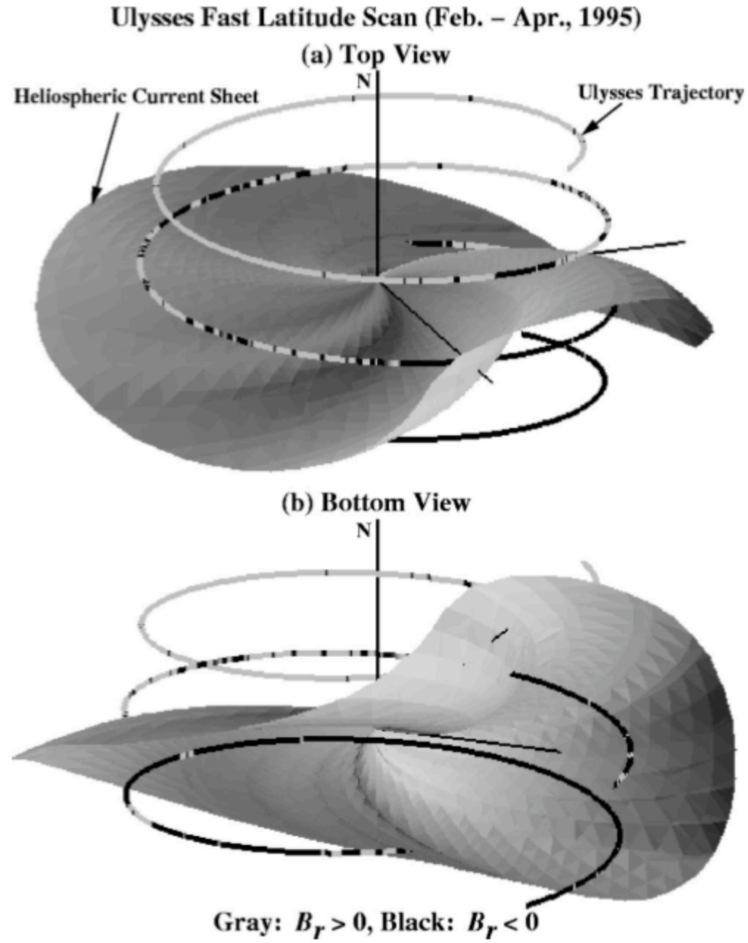
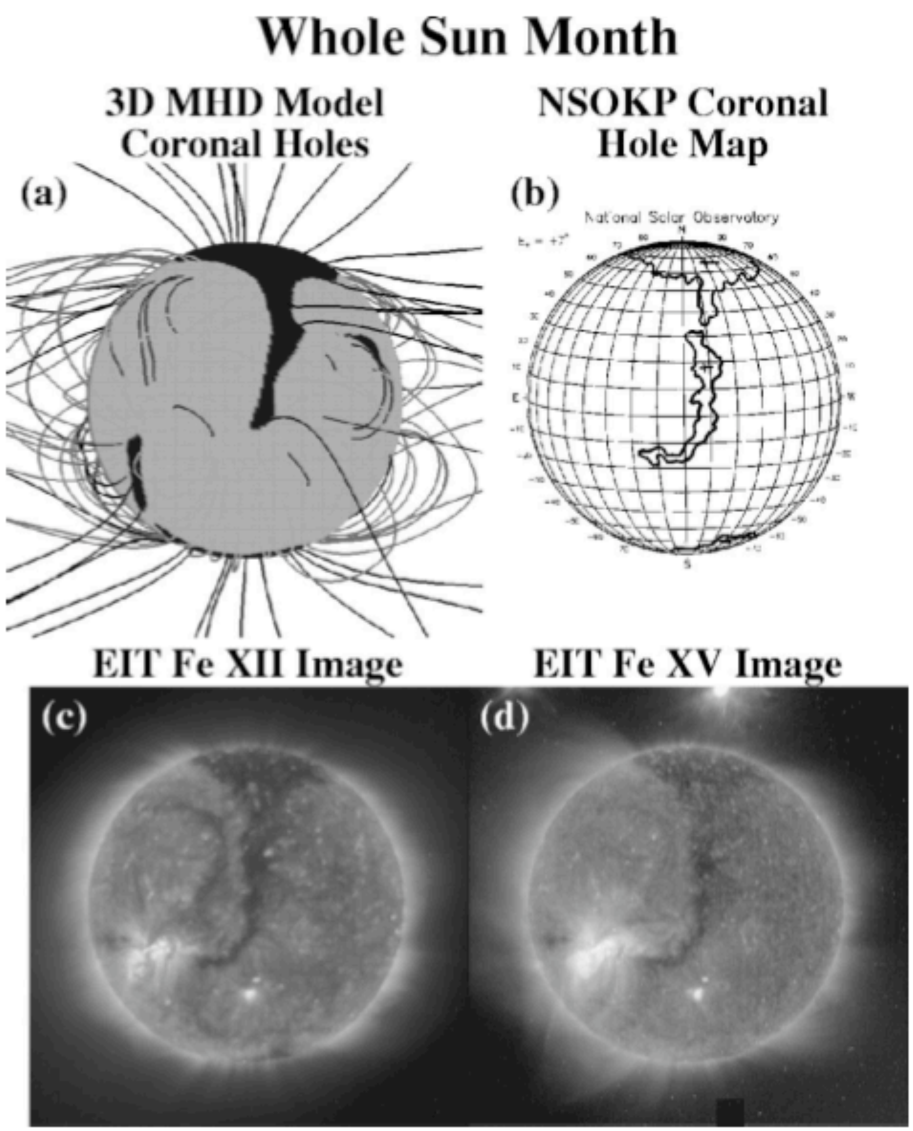
Mikic et al., Phys. Plasmas 1999



Mikic et al., Phys. Plasmas 1999



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Groth et al., JGR 2000

$$\frac{\partial\tilde{\rho}}{\partial\tilde{t}} + \left(\tilde{\mathbf{u}}\cdot\tilde{\nabla}\right)\tilde{\rho} + \tilde{\rho}\left(\tilde{\nabla}\cdot\tilde{\mathbf{u}}\right) = 0$$

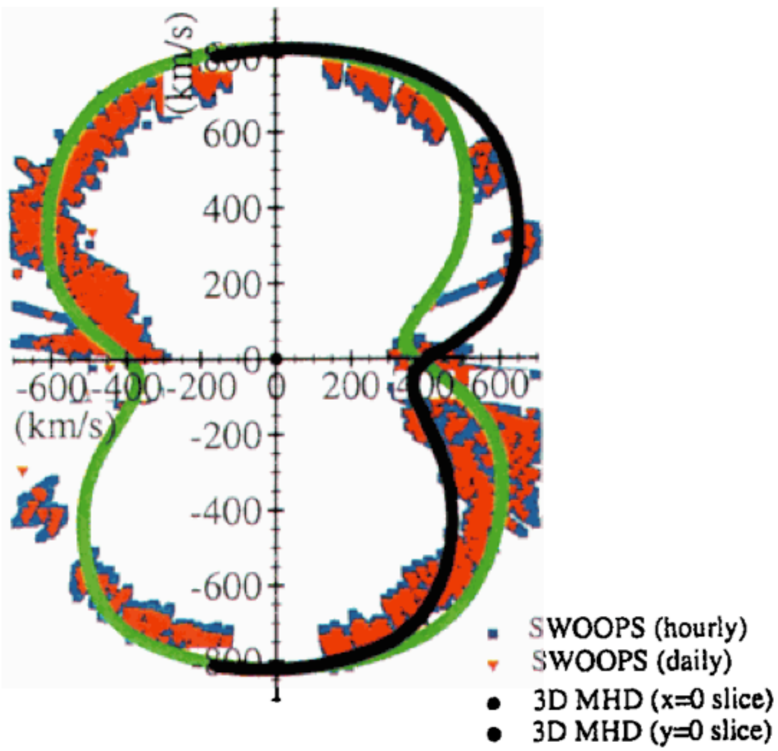
$$\frac{\partial\tilde{\mathbf{u}}}{\partial\tilde{t}} + \left(\tilde{\mathbf{u}}\cdot\tilde{\nabla}\right)\tilde{\mathbf{u}} + \frac{1}{\tilde{\rho}}\tilde{\nabla}\tilde{p} = \frac{1}{\tilde{\rho}}\left(\tilde{\mathbf{j}}\times\tilde{\mathbf{B}}\right) + \tilde{\mathbf{g}} - \tilde{\mathbf{\Omega}}\times\left(\tilde{\mathbf{\Omega}}\times\tilde{\mathbf{r}}\right) - 2\,\tilde{\mathbf{\Omega}}\times\tilde{\mathbf{u}}\,,$$

$$\frac{\partial\tilde{p}}{\partial\tilde{t}} + \left(\tilde{\mathbf{u}}\cdot\tilde{\nabla}\right)\tilde{p} + \gamma\tilde{p}\left(\tilde{\nabla}\cdot\tilde{\mathbf{u}}\right) = (\gamma-1)\,\tilde{Q}\,,$$

$$\frac{\partial\tilde{\mathbf{B}}}{\partial\tilde{t}} - \tilde{\nabla}\times\tilde{\mathbf{u}}\times\tilde{\mathbf{B}} = 0\,,$$

Groth et al., JGR 2000

$$\tilde{Q} = \tilde{\rho}\tilde{q}_\circ \exp\left[-\frac{\left(\tilde{r}-\tilde{R}_\circ\right)^2}{\sigma_\circ^2}\right]\left(\tilde{T}_\circ - \gamma\frac{\tilde{p}}{\tilde{\rho}}\right)$$



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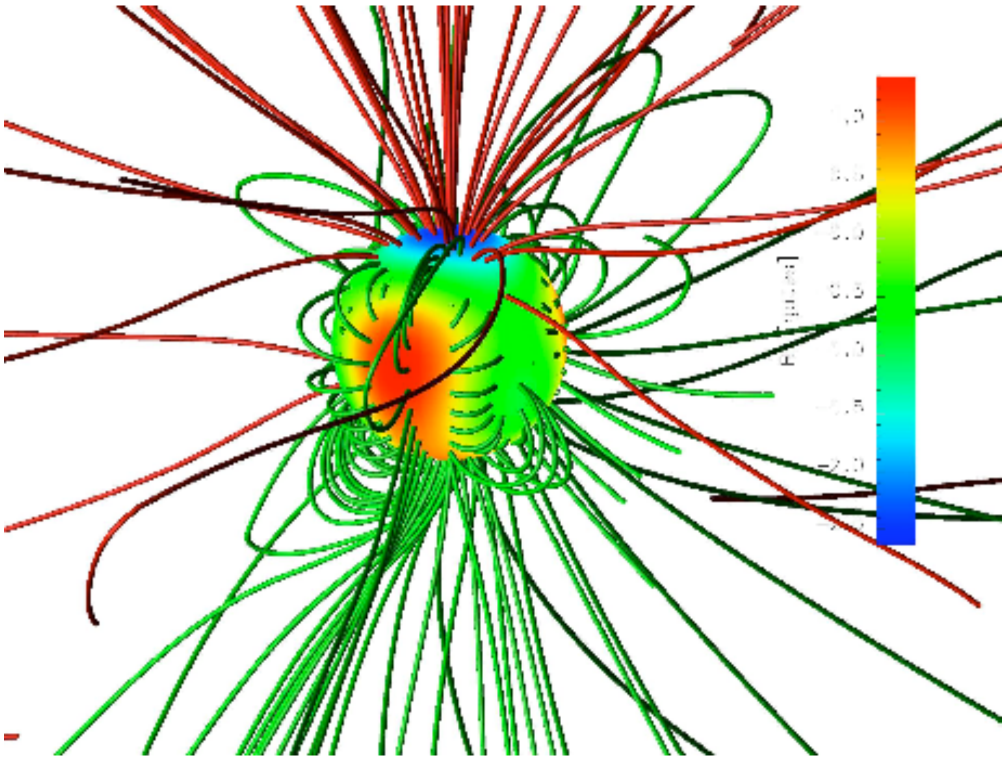
other examples of solar wind simulations

- 3D solar wind simulation based on MDI (SOHO) (Carla Jacobs, Stefaan Poedts, using AMRVAC)



current and planned activity

- use MDI (SOHO) magnetogram input for reconstructing photospheric radial magnetic field of the whole solar surface (Schrijver & DeRosa)
- use potential field with source surface to reconstruct 3D magnetic field (PFSS)
- relax potential field within MHD solar wind model (polytropic wind model)
- implicit time integration, adaptivity are essential
- unstructured grids are suitable for this problem
- future: use new satellite observations (Hinode, SDO)

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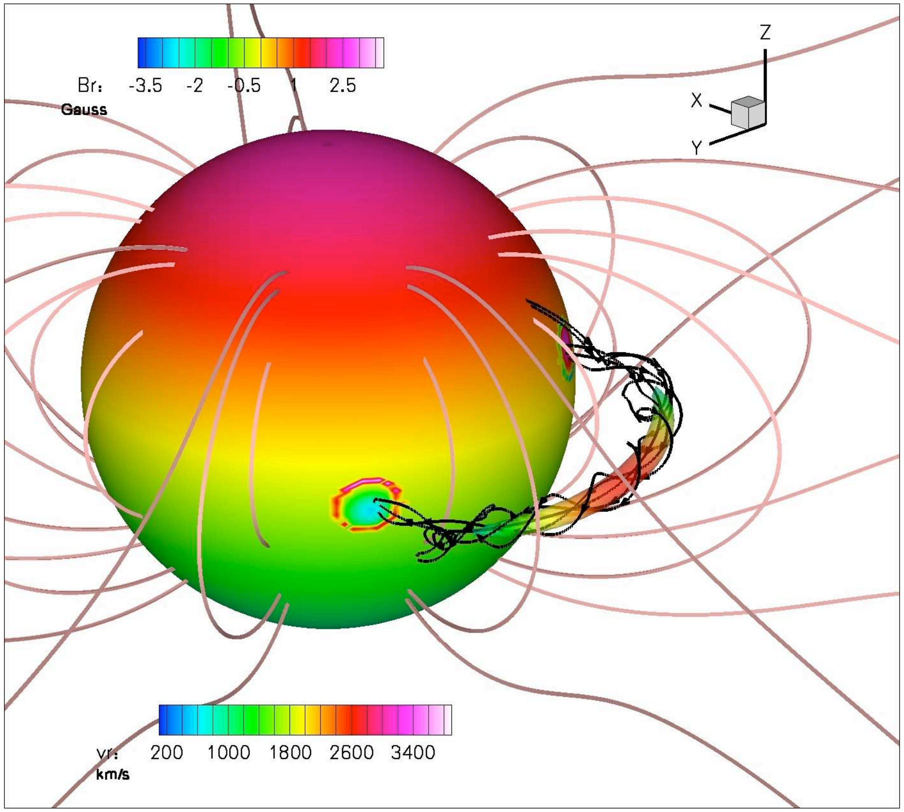
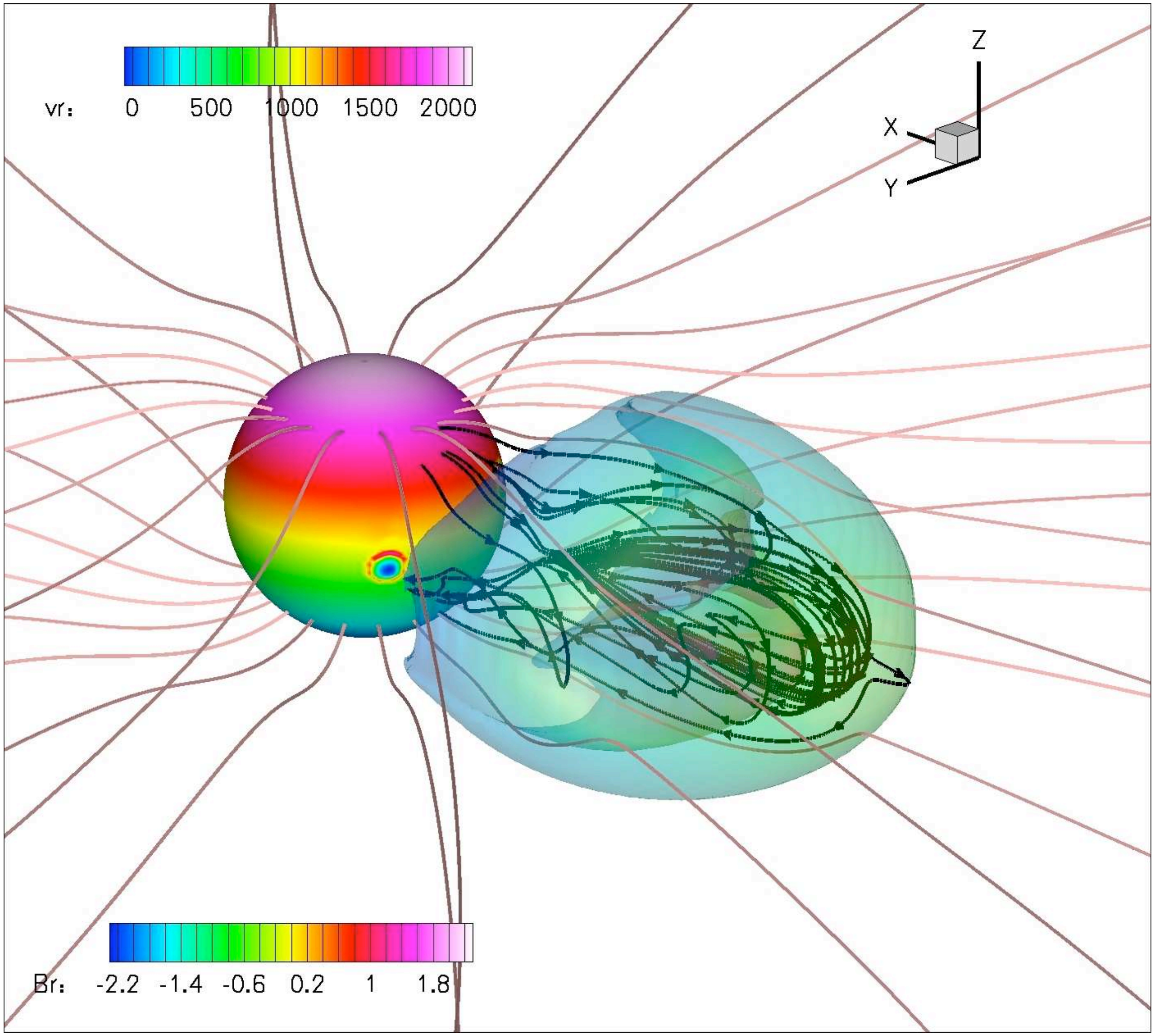
CME propagation in solar wind

- 3D simulation of flux-rope CME (Carla Jacobs, Stefaan Poedts, using AMRVAC)
- future: use STEREO data





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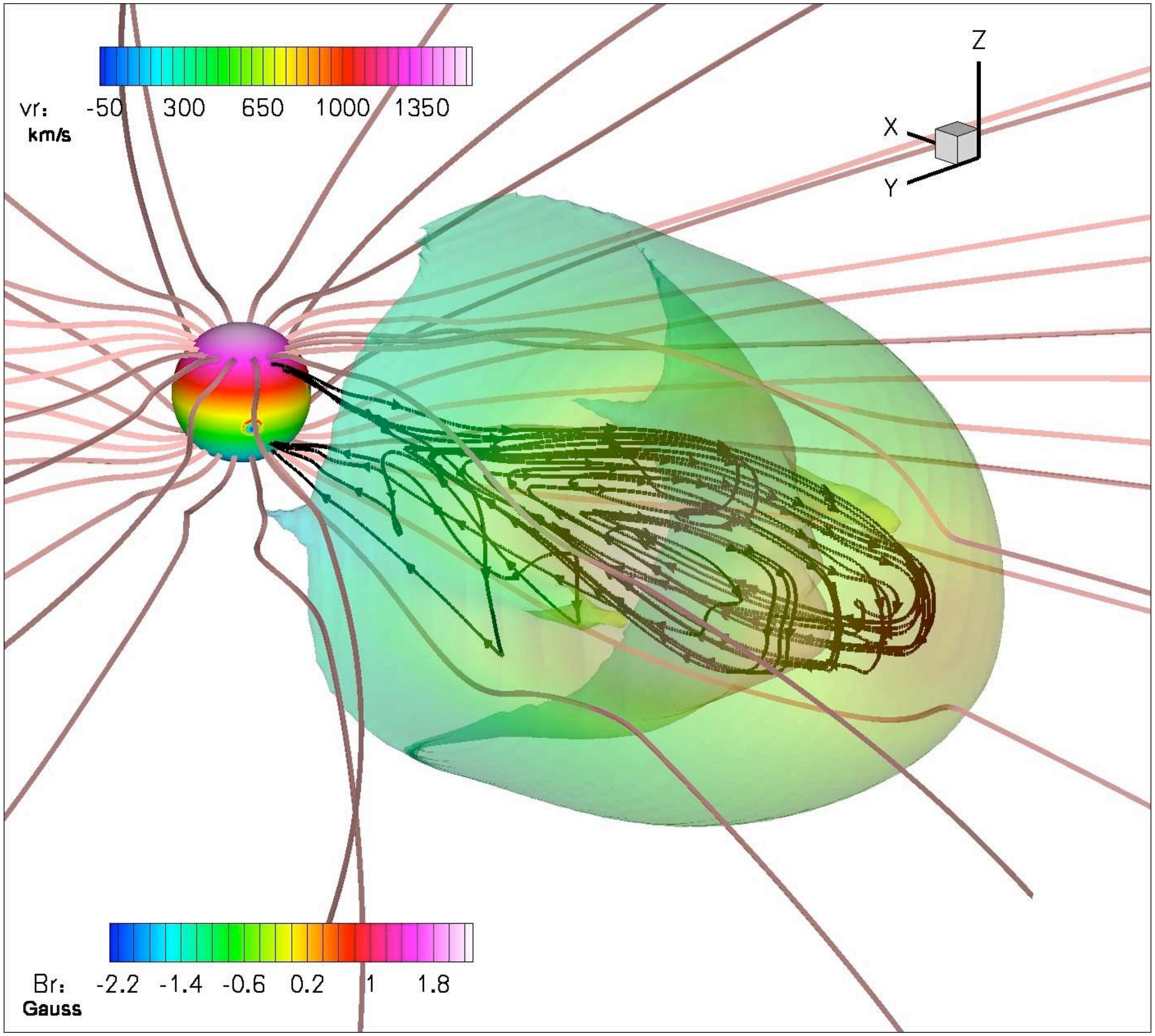

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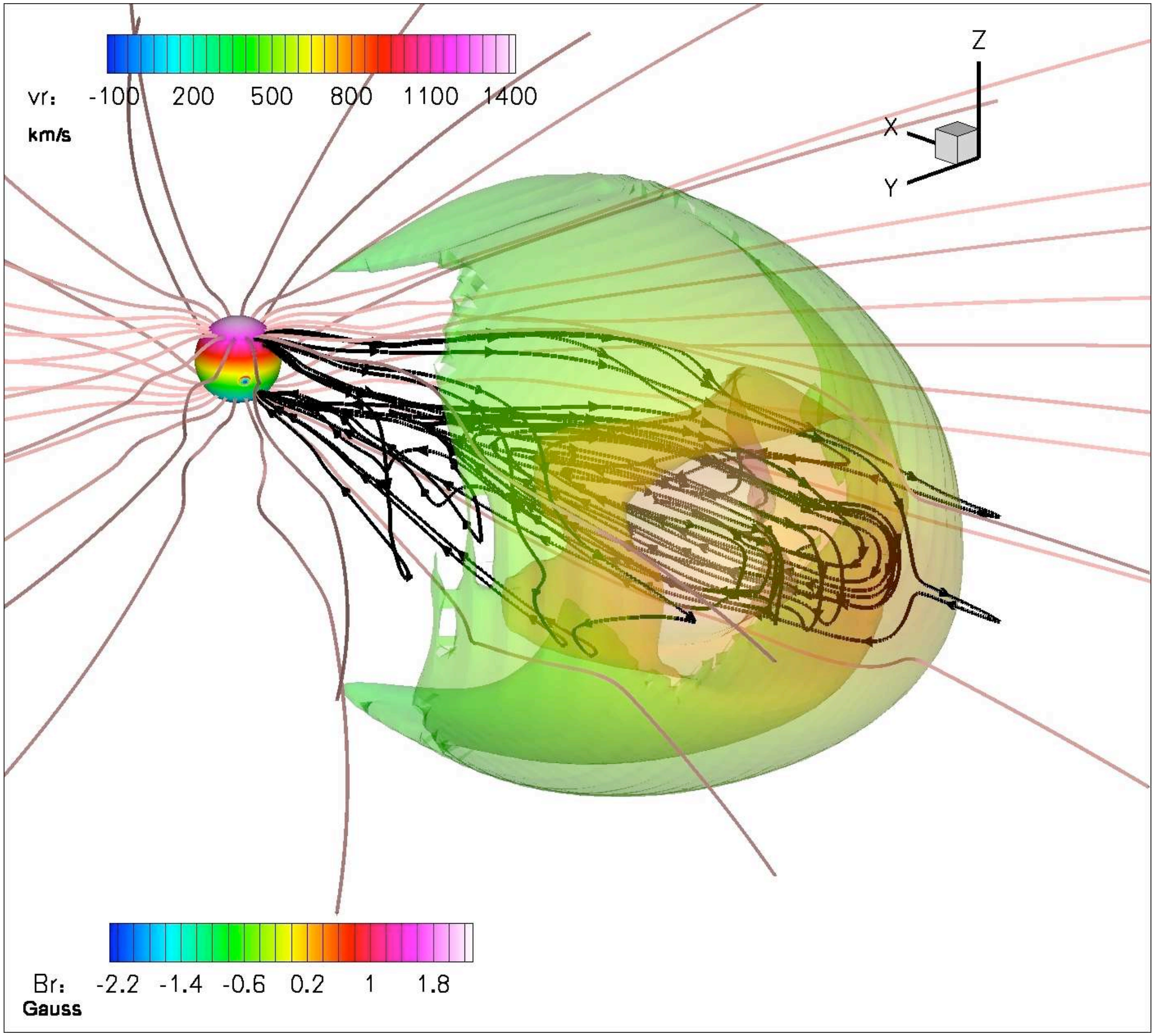
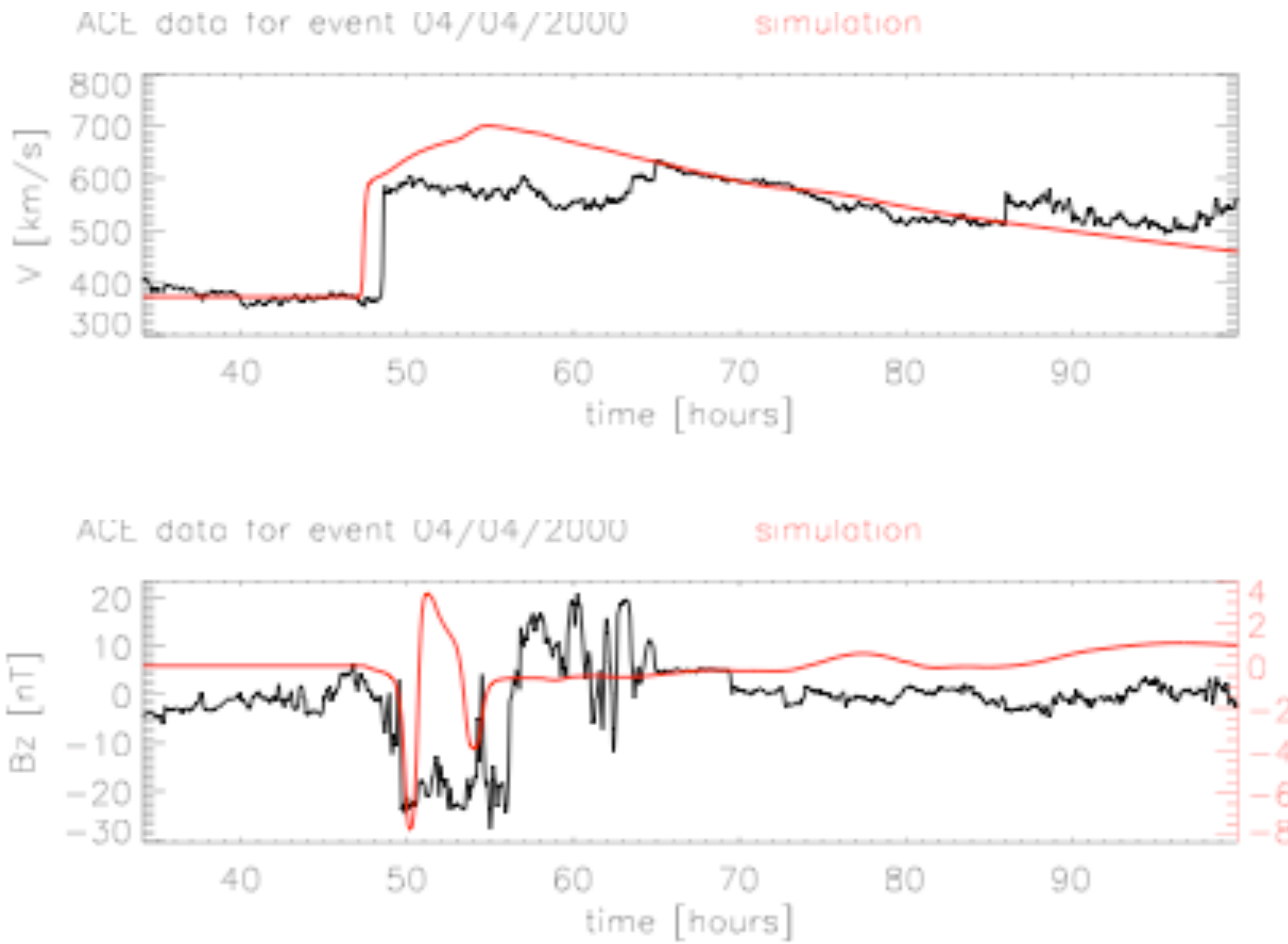

vr: -100 200 500 800 1100 1400
km/s
Z
X
Y
Br: -2.2 -1.4 -0.6 0.2 1 1.8
Gauss

comparison with ACE at 1AU



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overview

1. the MHD model for plasma dynamics

2. numerical MHD methods

3. numerical MHD modeling of the solar wind and CMEs

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questions?

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