Solar Wind-like Outflows from Planetary Atmospheres

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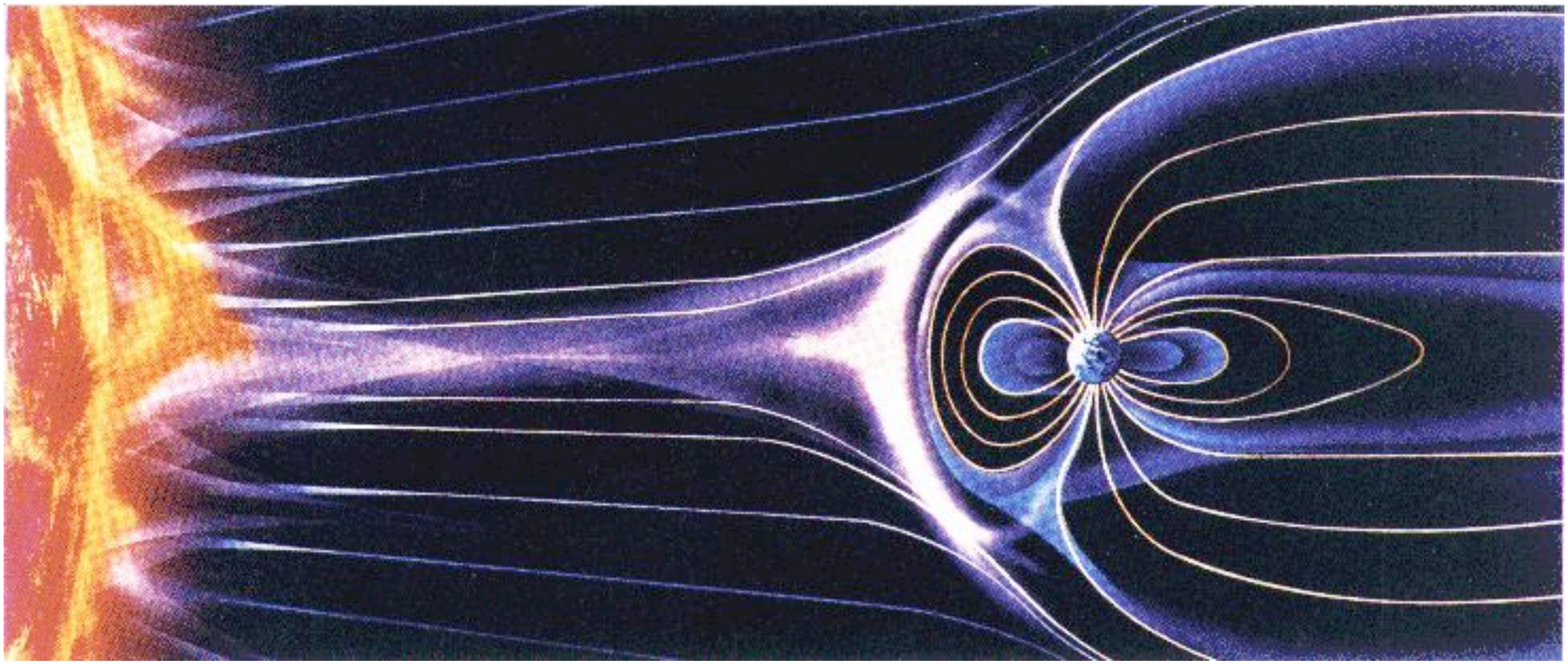
Third Canadian Solar Workshop, Montreal, 2006

Parker Solar Wind



- heat from sun core accelerates radial flow from subsonic to supersonic
- bow shock at the earth

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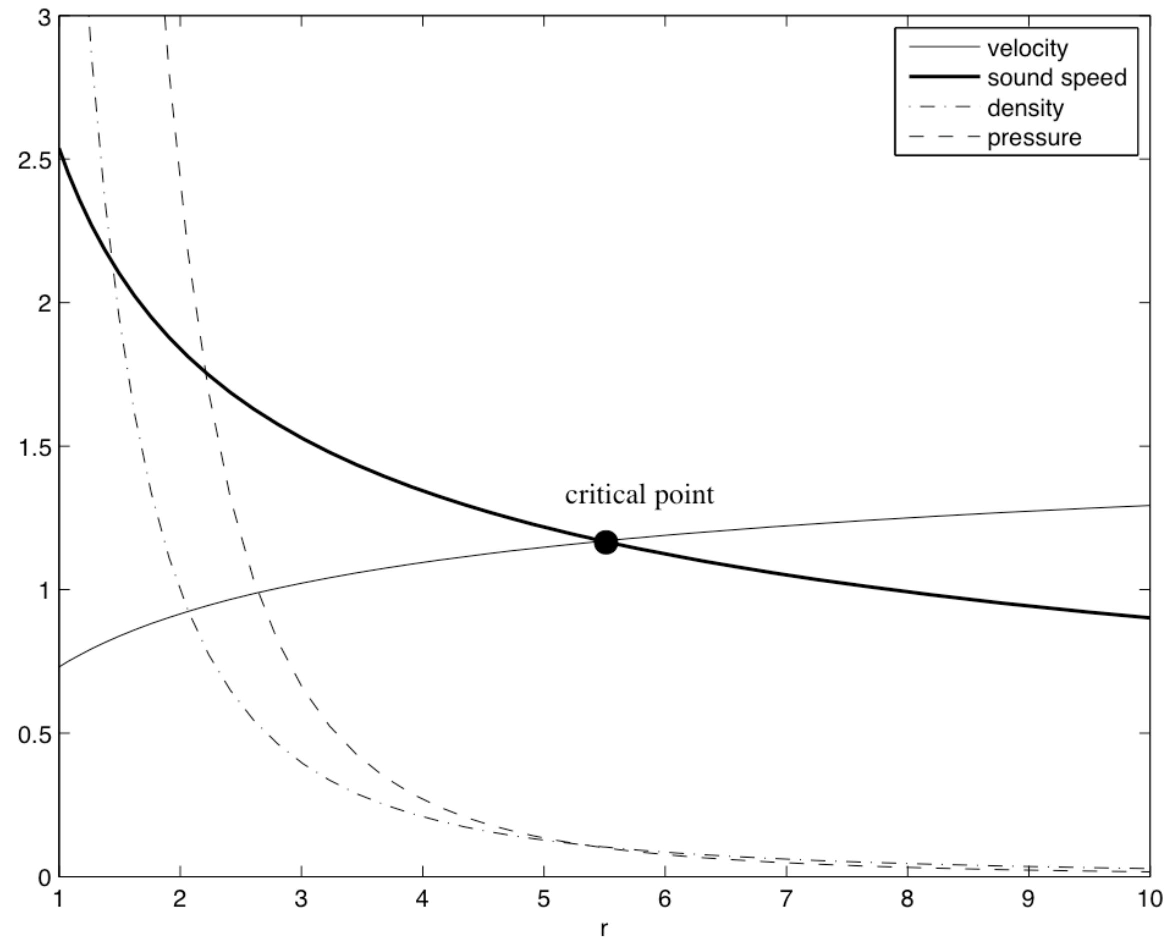
transonic radial outflow solution of Euler equations of gas dynamics



subsonic ⇒|⇐ supersonic

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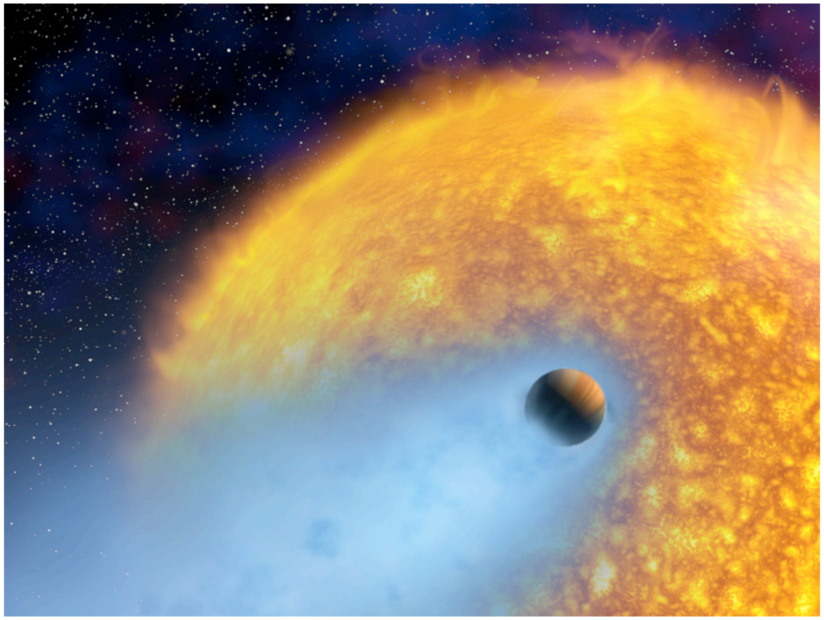
Supersonic gas escape from extrasolar planets

- http://exoplanet.eu
- 173 extrasolar planets known, as of June 2006
- 209 extrasolar planets known, as of November 2006
- 21 multiple planet systems



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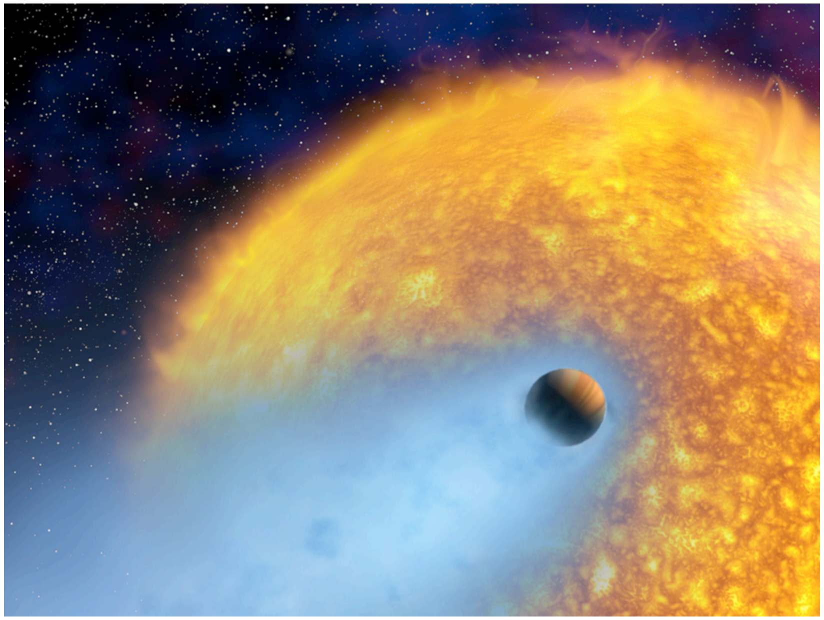
Supersonic gas escape from extrasolar planets

- many exoplanets are gas giants (“hot Jupiters”)
- many orbit very close to star (~0.05 AU)
- hypothesis: strong irradiation leads to supersonic hydrogen escape



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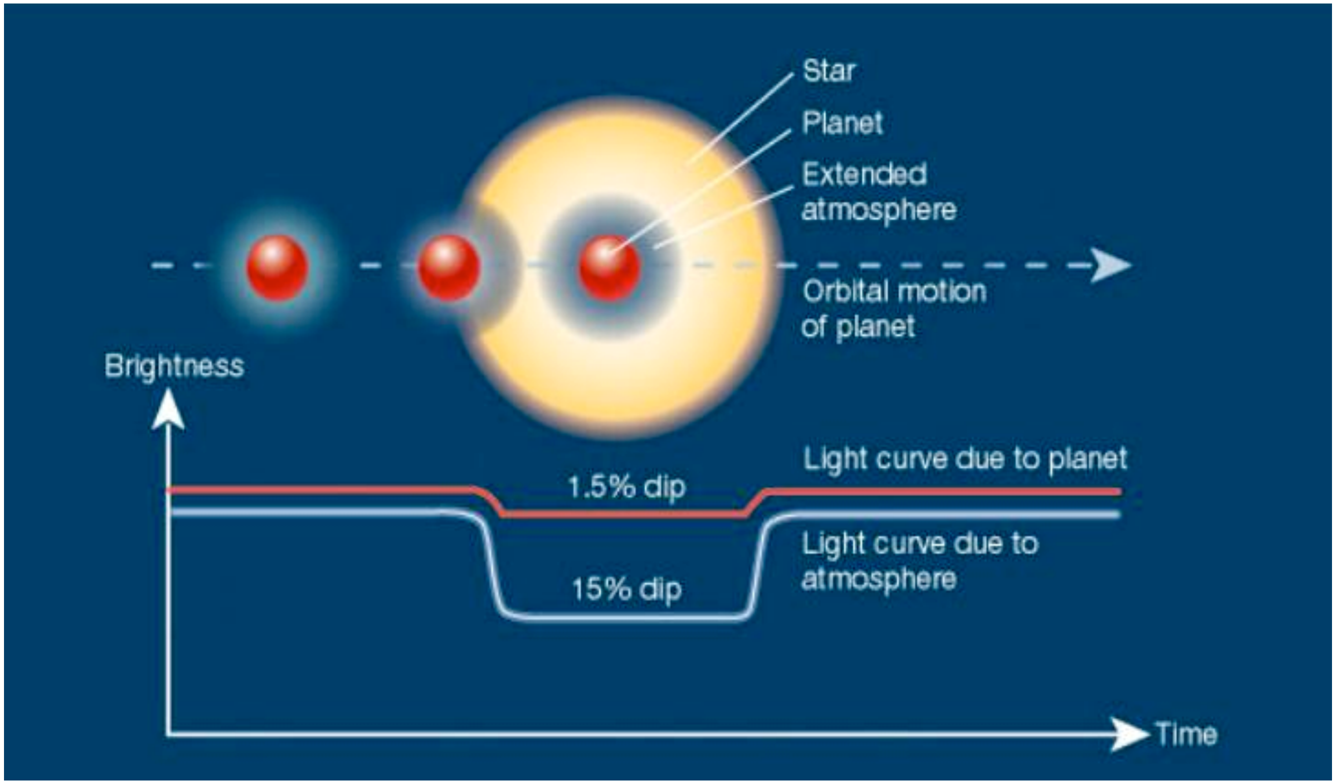
example: HD209458 (Vidal-Madjar 2003)



- 0.67 Jupiter masses, 0.05 AU, transiting
- hydrogen atmosphere and escape observed
- question: what is the mass loss rate? long-time stability of the planet? ⇒ solve Euler equations!

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Euler equations of gas dynamics

- find $\rho(r,t), u(r,t), p(r,t)$ s.t.

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho\, r^2 \\ \rho\, u\, r^2 \\ (\frac{p}{\gamma-1}+\frac{\rho\, u^2}{2})\, r^2 \end{bmatrix} + \frac{\partial}{\partial r}\begin{bmatrix} \rho\, u\, r^2 \\ \rho\, u^2\, r^2 + p\, r^2 \\ (\frac{\gamma\, p}{\gamma-1}+\frac{\rho\, u^2}{2})\, u\, r^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\rho G M + 2\, p\, r \\ -\rho G M\, u + q_{heat}\, r^2 \end{bmatrix}$$

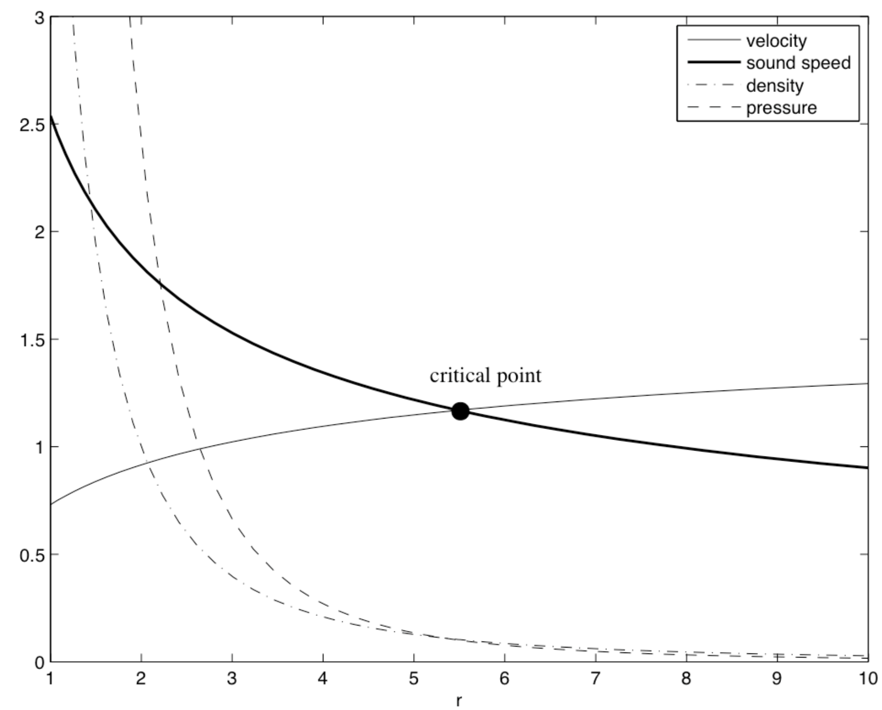
- nonlinear system of PDEs
- conservation of mass, momentum, energy

transonic radial outflow solution: problem definition

- Euler equations: 3 equations in three variables

$$\rho(r,t), u(r,t), p(r,t)$$

- lower boundary at planet surface: subsonic, needs two boundary conditions: density and pressure
- upper boundary: supersonic, needs no boundary conditions (all information flows out)



numerical method

- Euler Equations are conservation law

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial r} = S(U)$$

- solving the steady part alone is too hard (it is not known how to do that... more later!)

$$\frac{dF(U)}{dr} = S(U)$$

- engineers developed time-marching methods to steady state

numerical method

- hyperbolic conservation law

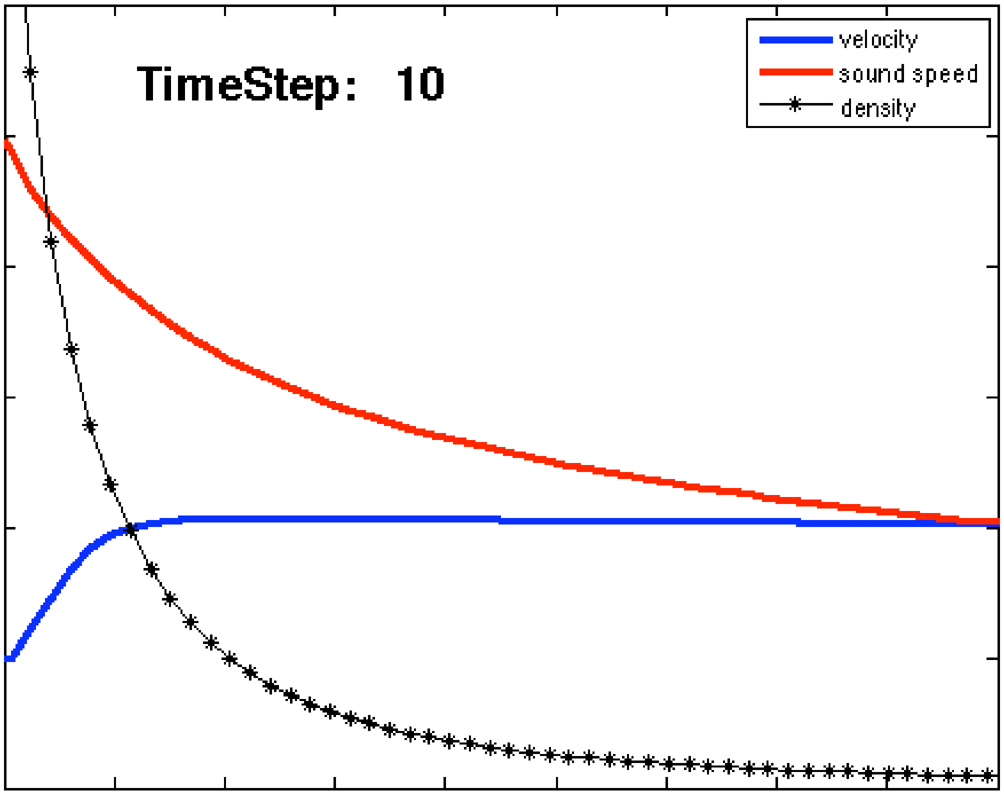
$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial r} = S(U)$$

- use Computational Fluid Dynamics methods: finite volume method

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + \frac{F^*_{i+1/2} - F^*_{i-1/2}}{\Delta r} = S(U_i)$$

- very slow convergence to steady state... (more later!)

Simulations of planet atmosphere



- n=50 points in space
- needs 1500 steps to converge

results for 1D exoplanet simulations

- HD209458b:
  - lower boundary conditions $\rho=7.10^{-9}$ g/cm^{-3} and T=750K
  - extent of atmosphere, outflow velocity, and mass flux consistent with observations (Vidal-Madjar 2003)
  - 1% mass loss in 12 billion years $\Rightarrow$ HD209458b is stable
- Tian, Toon, Pavlov, and De Sterck, Astrophysical Journal 621, 1049-1060, 2005

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Can we solve the steady Euler equations faster and more accurrately?

- yes!

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial r} = S(U)$$

- new approach: solve the steady equations directly

$$\frac{dF(U)}{dr} = S(U)$$

$$\frac{d}{dr}\begin{bmatrix} \rho\, u\, r^2 \\ \rho\, u^2\, r^2 + p\, r^2 \\ (\frac{\gamma\, p}{\gamma - 1} + \frac{\rho\, u^2}{2})\, u\, r^2 \end{bmatrix} = \begin{bmatrix} 0 \\ -\rho G M + 2\, p\, r \\ -\rho G M\, u + q_{heat}\, r^2 \end{bmatrix}$$

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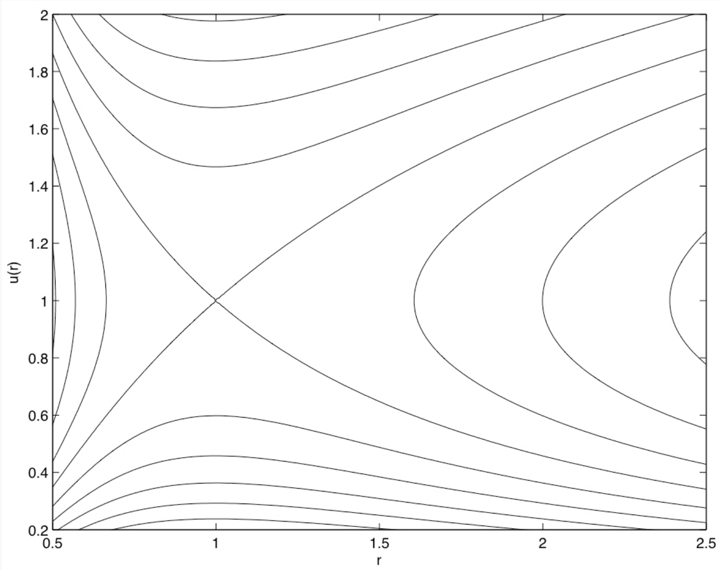
Solving the steady ODE system is hard...

- consider toy problem (isothermal Parker model): single ODE

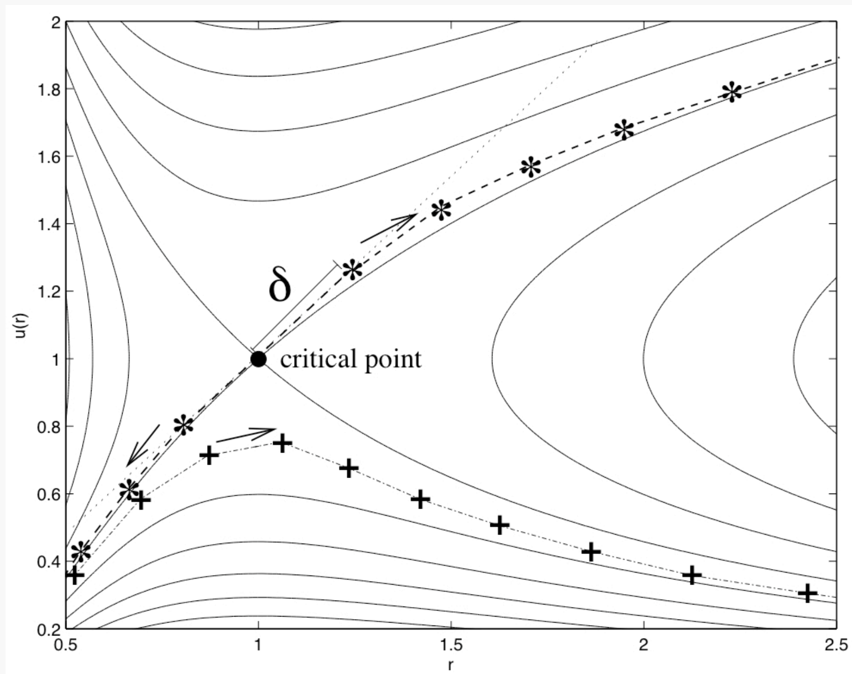
$$\frac{du}{dr} = \frac{2\,u\,c^2\,(r - r_c)}{r^2\,(u^2 - c^2)}$$



- normally need 1 boundary condition to determine solution
- transonic solution: no boundary condition needed!

Solving the steady ODE system is hard...

- solving ODE from the left does not work...



- but... integrating outward from the critical point does work!!!

Direct calculation of steady solution

$$\frac{du}{dr} = \frac{2\,u\,c^2\,(r - r_c)}{r^2\,(u^2 - c^2)}$$

1. Write as dynamical system...

$$\frac{dV}{ds} = G(V)$$

$$\frac{du(s)}{ds} = -2\,u\,c^2\left(r - \frac{GM}{2c^2}\right)$$
$$\frac{dr(s)}{ds} = -r^2\,(u^2 - c^2)$$

2. find critical point $G(V) = 0$

3. integrate outward from critical point

For the Full Euler Equations

$$\frac{d}{dr}\left[\begin{array}{c} \rho\, u\, r^2 \\ \rho\, u^2\, r^2 + p\, r^2 \\ (\frac{\gamma\, p}{\gamma - 1} + \frac{\rho\, u^2}{2})\, u\, r^2 \end{array}\right] = \left[\begin{array}{c} 0 \\ -\rho G M + 2\, p\, r \\ -\rho G M\, u + q_{heat}\, r^2 \end{array}\right]$$

- problem: there are many possible critical points!

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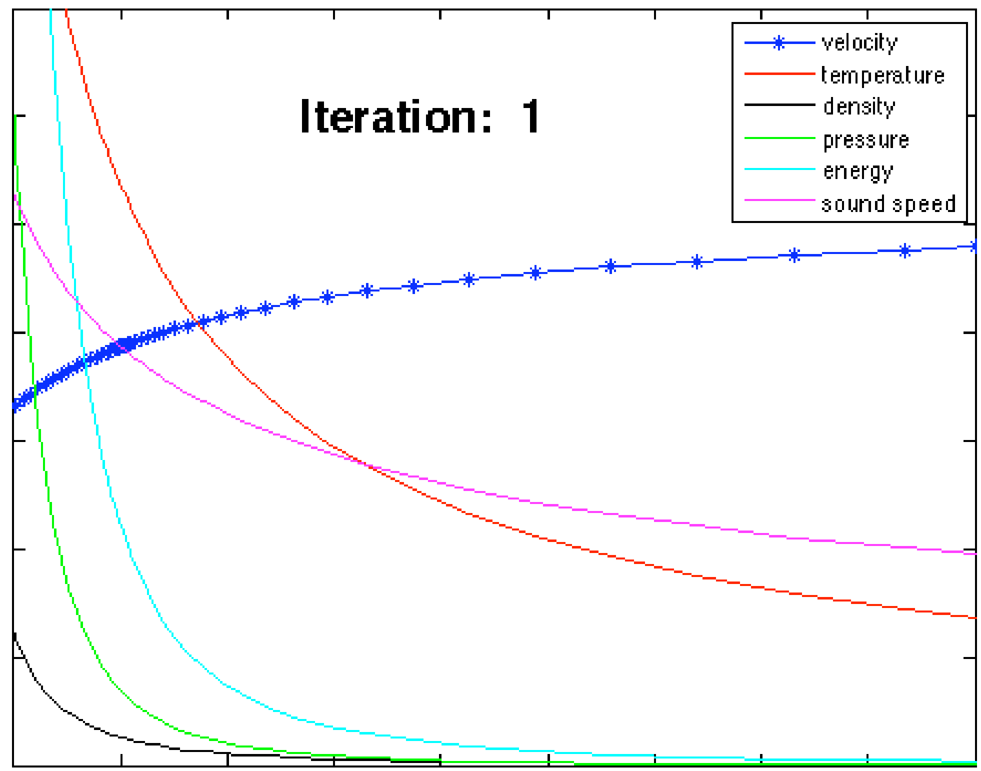
New algorithm for calculating steady transonic Euler outflows

guess initial critical point

1. use adaptive ODE integrator to find trajectory
2. modify guess for critical point depending on deviation from desired inflow boundary conditions (Newton method)
3. repeat



2D numerical models (Scott Rostrup)

- Euler equations in multiple dimensions

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho\,\vec{v} \\ \frac{p}{\gamma-1}+\frac{\rho\,v^2}{2} \end{bmatrix} + \nabla\cdot\begin{bmatrix} \rho\,\vec{v} \\ \rho\,\vec{v}\vec{v}+I\,p \\ (\frac{\gamma\,p}{\gamma-1}+\frac{\rho\,v^2}{2})\,\vec{v} \end{bmatrix} = \begin{bmatrix} 0 \\ \vec{F}_{ext} \\ \vec{F}_{ext}\cdot\vec{v}+q_{heat} \end{bmatrix}$$

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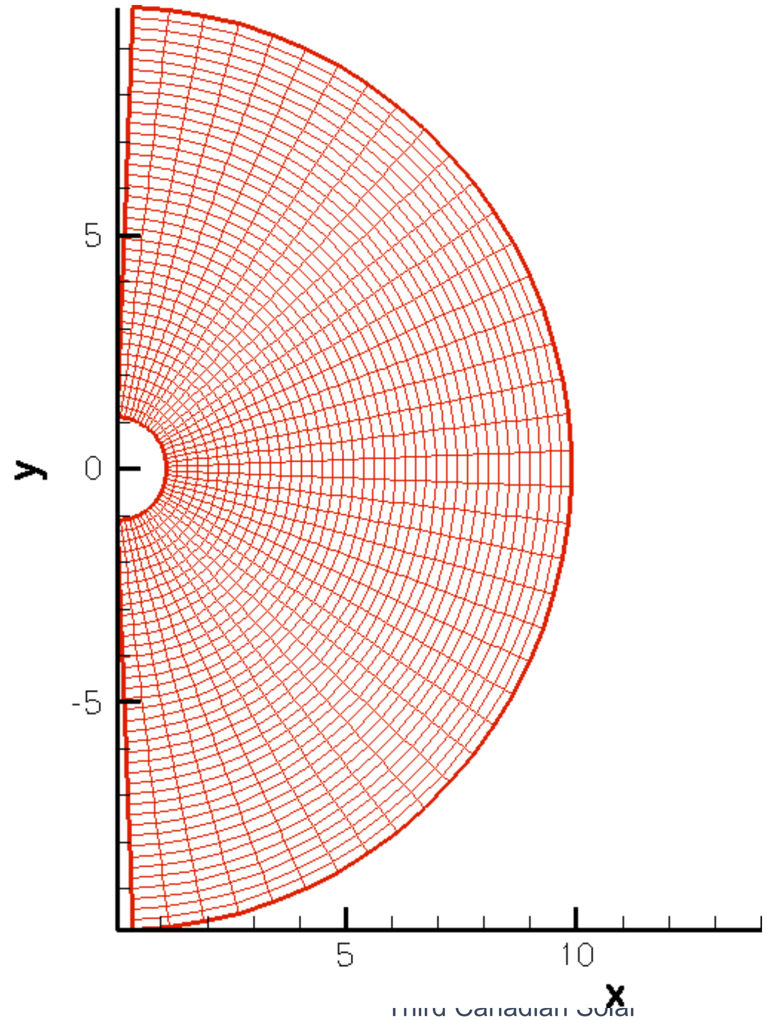
2D Simulations

- assume rotational symmetry about the y axis

⇒ allows for non-uniform heating



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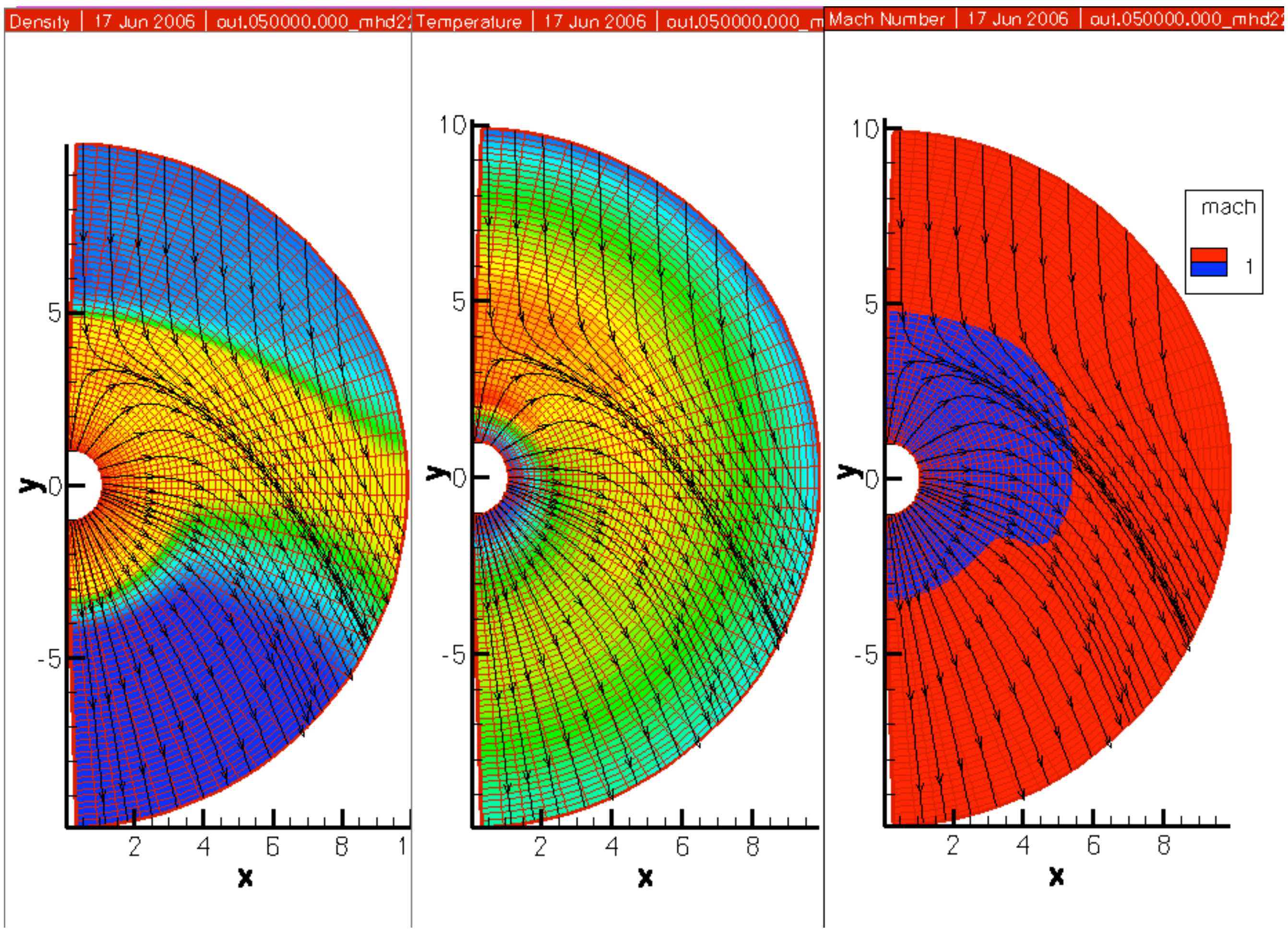
ongoing work: include stellar wind



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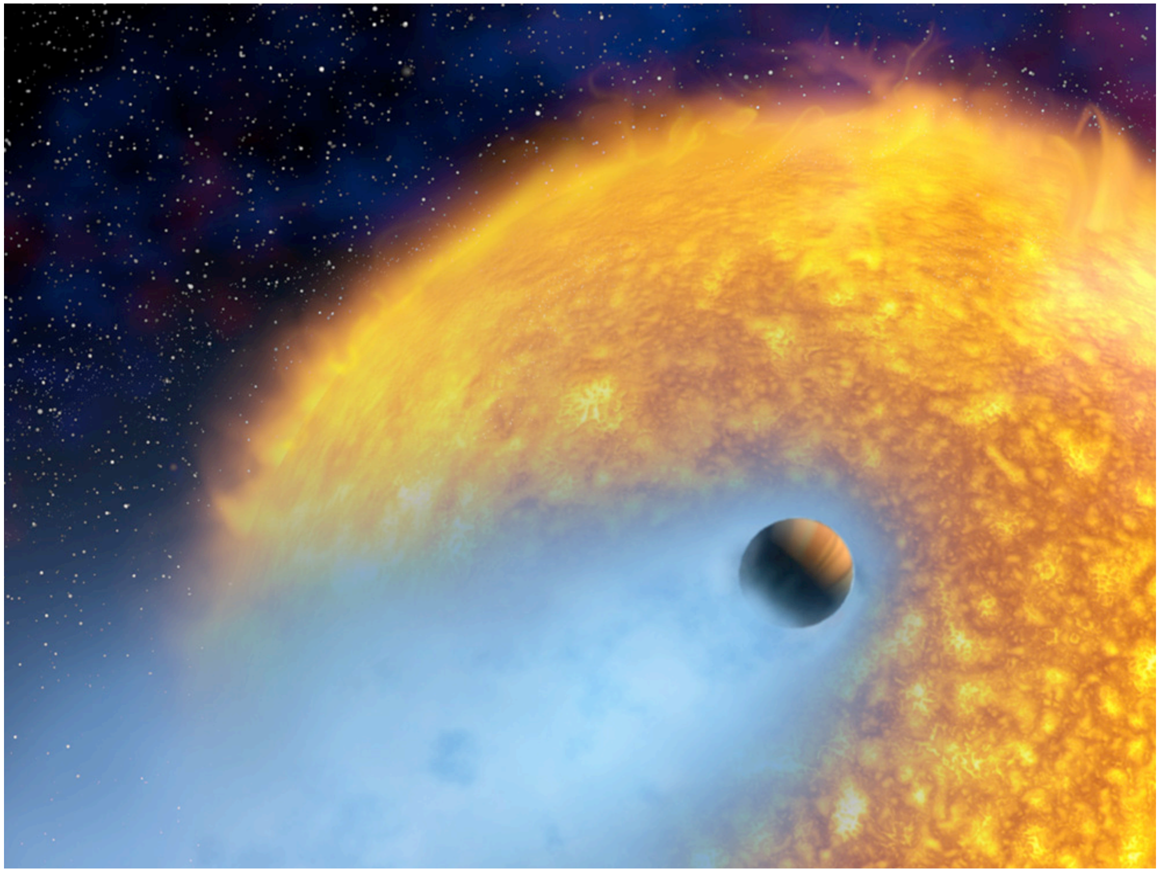
ongoing work: include stellar wind



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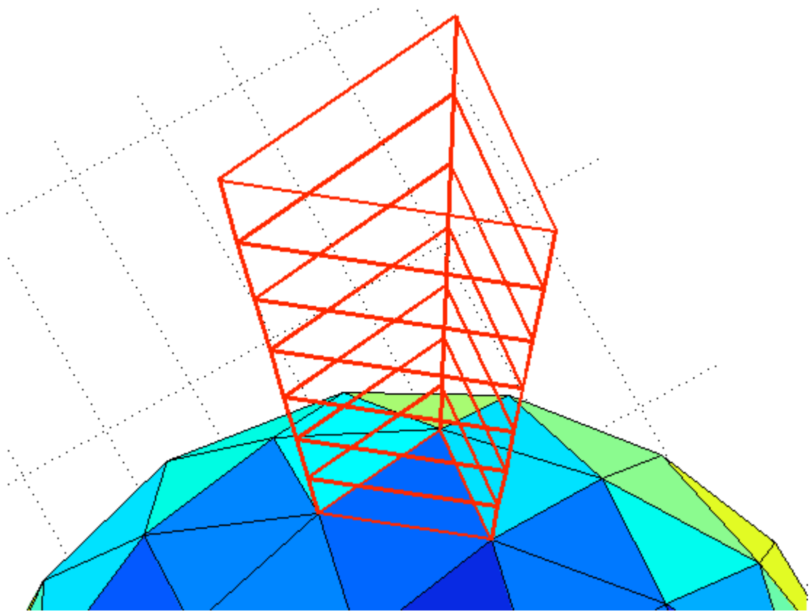
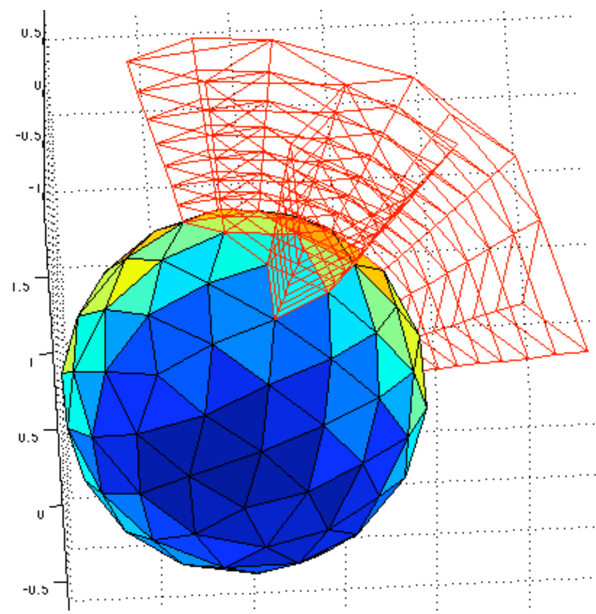
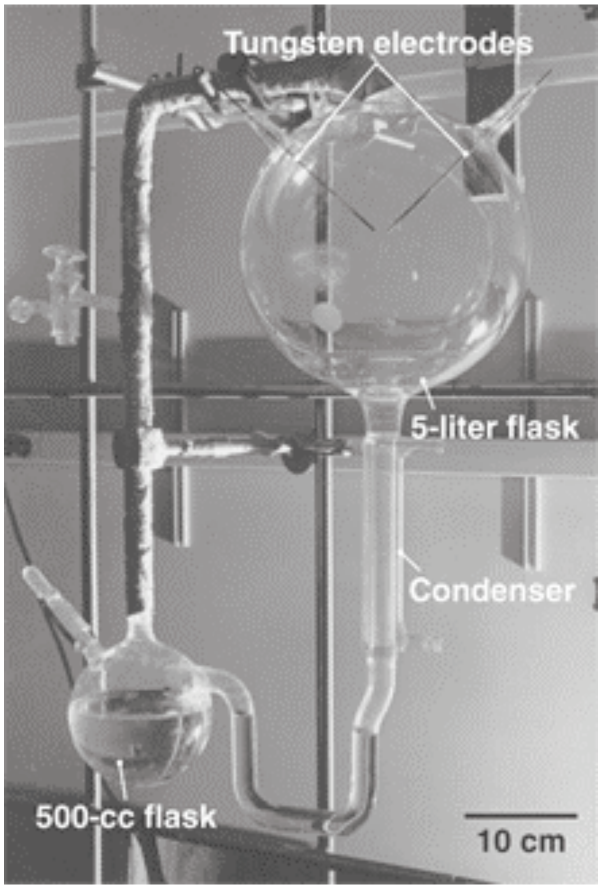
3D numerical models (Paul Ullrich)





- we want to include effects of planetary rotation

Primordial soup as the origin of life on Earth



- Stanley Miller (1953): formation of prebiotic molecules in a CH4-NH3 rich environment with electric discharge
  - -Problem: CH4-NH3 atmosphere unlikely
- later experiments show that prebiotic molecules can be formed efficiently in a hydrogen-rich environment
- alternative sources of organics: hydrothermal system, comet delivery

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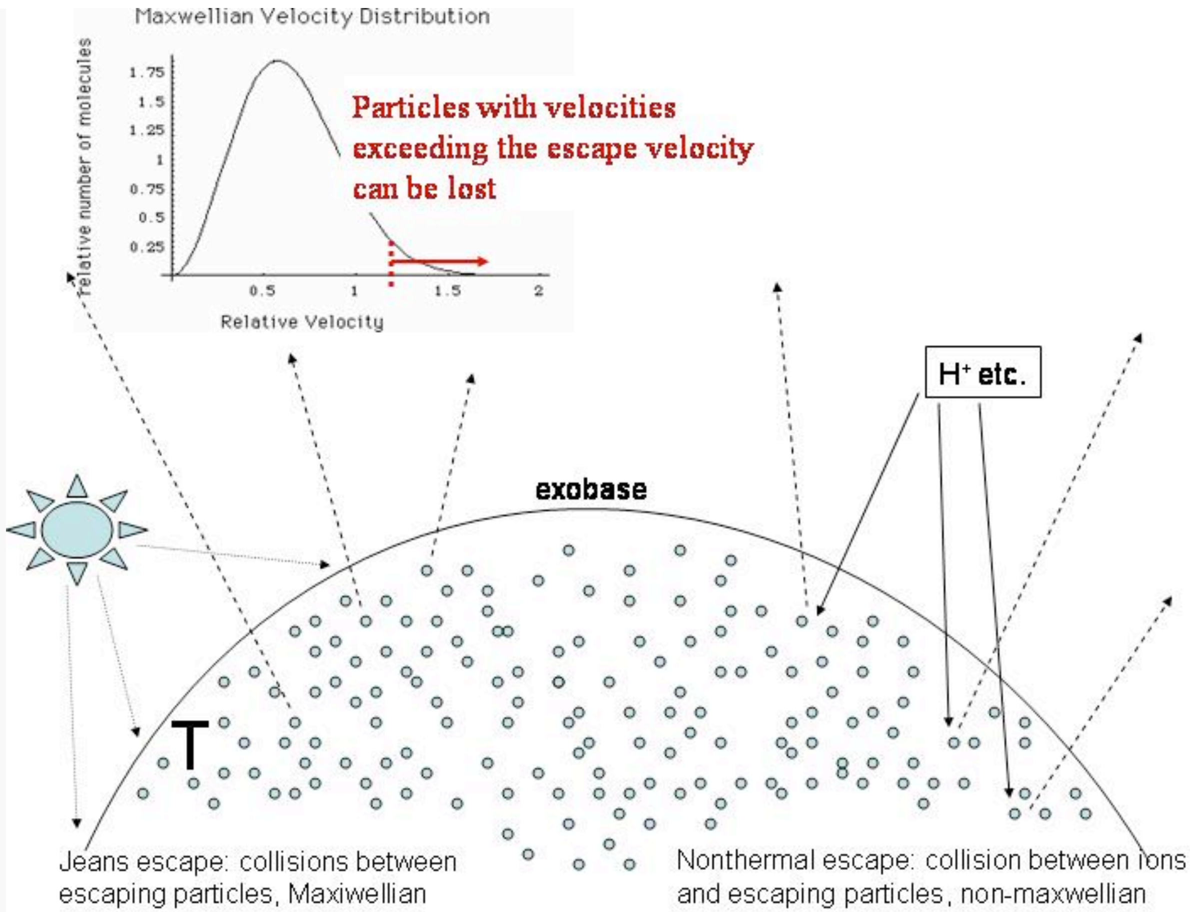
Supersonic gas escape from Early Earth

- there is no supersonic hydrodynamic escape from present-day Earth
- exo-base temperature is high: collisional, thermal escape dominates



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hydrogen content in Early Earth atmosphere

- hydrogen content: balance between volcanic outgassing and escape from atmosphere
- existing theory: static atmosphere with high temperature at top $\Rightarrow$ fast thermal escape $\Rightarrow$ hydrogen content was very low
- formation of prebiotic molecules in a hydrogen-rich atmosphere was thus discarded as a theory

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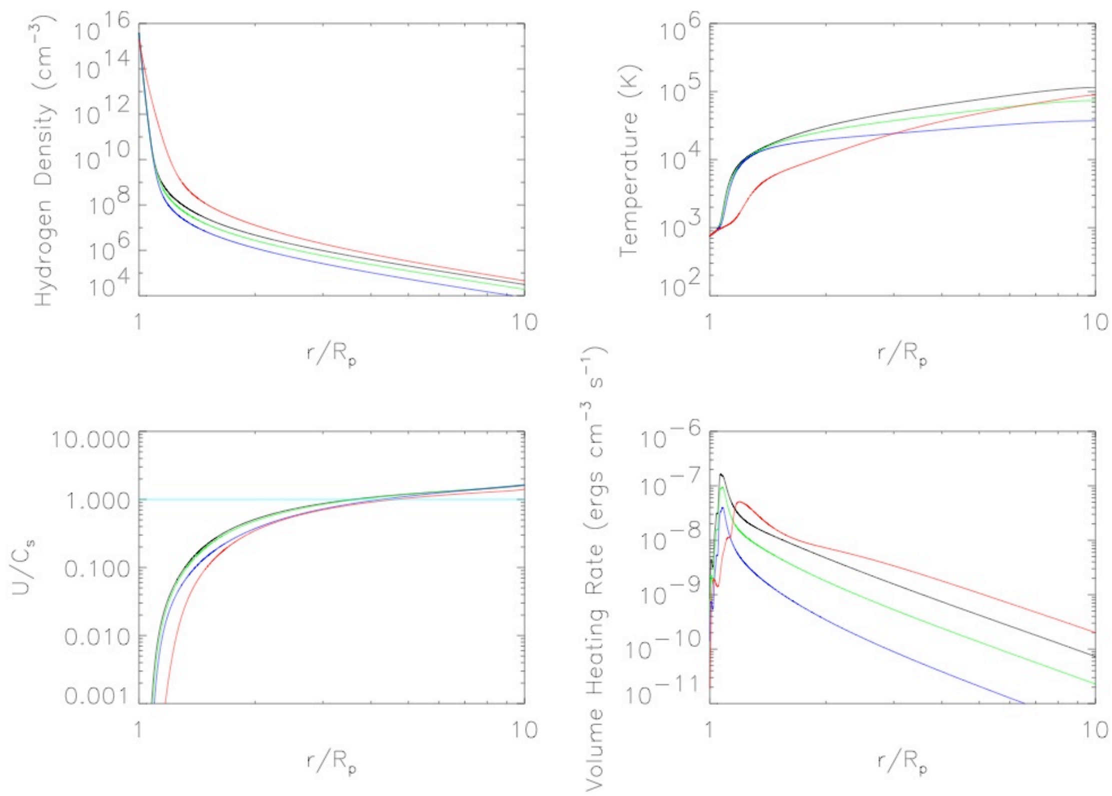
new theory: hydrogen content in Early Earth atmosphere

- our results: hydrogen escape was probably supersonic, with low temperature at top (no thermal escape), and total escape rates were low



hydrogen content in Early Earth atmosphere

- our results: hydrogen concentration in the atmosphere of Early Earth could have been as high as 30%
- formation of prebiotic molecules in early Earth's atmosphere could have been efficient $\Rightarrow$ primordial soup on early Earth is possible
- no need for hydrothermal vents, cometary delivery
- Tian, Toon, Pavlov, and De Sterck, Science 308, 1014-1017, 2005

The End

Questions?

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