Numerical simulation of supersonic escape from planetary atmospheres

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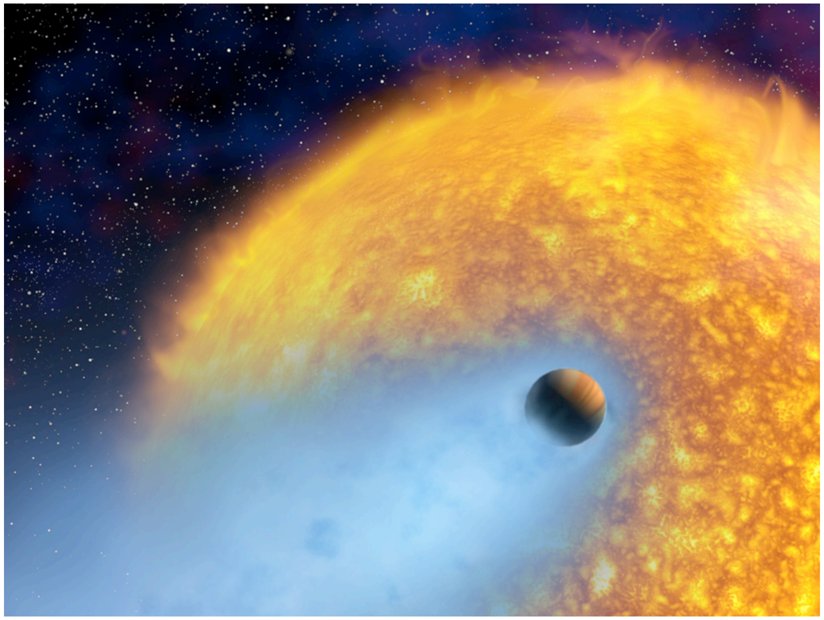
Outline

1. Supersonic gas escape from extrasolar planets
2. 1D numerical models
3. 2D numerical models
4. 3D numerical models
5. Supersonic gas escape from Early Earth

(1) Supersonic gas escape from extrasolar planets

- 173 extrasolar planets known, as of June 2006
- 19 multiple planet systems
- many exoplanets are gas giants (“hot Jupiters”)
- many orbit very close to star (~0.05 AU)
- hypothesis: strong irradiation leads to supersonic hydrogen escape



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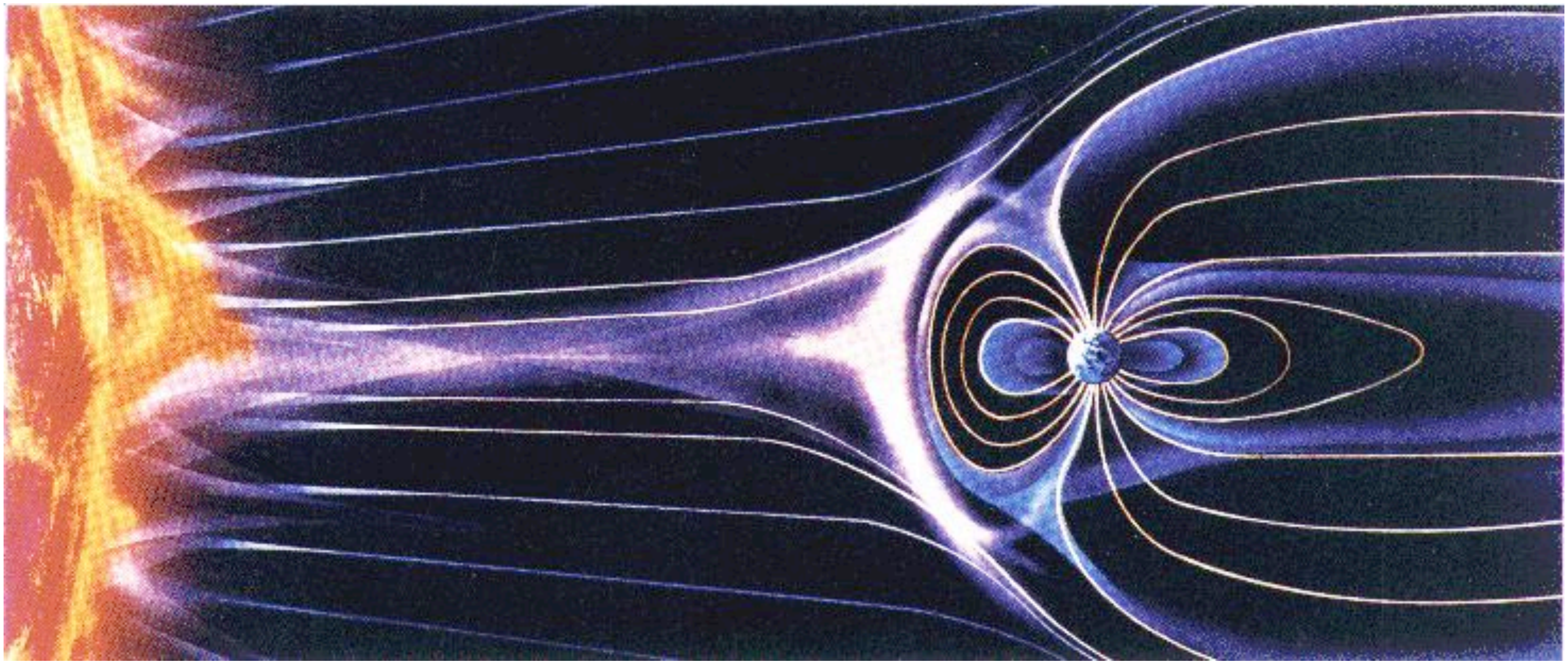
similar to solar wind



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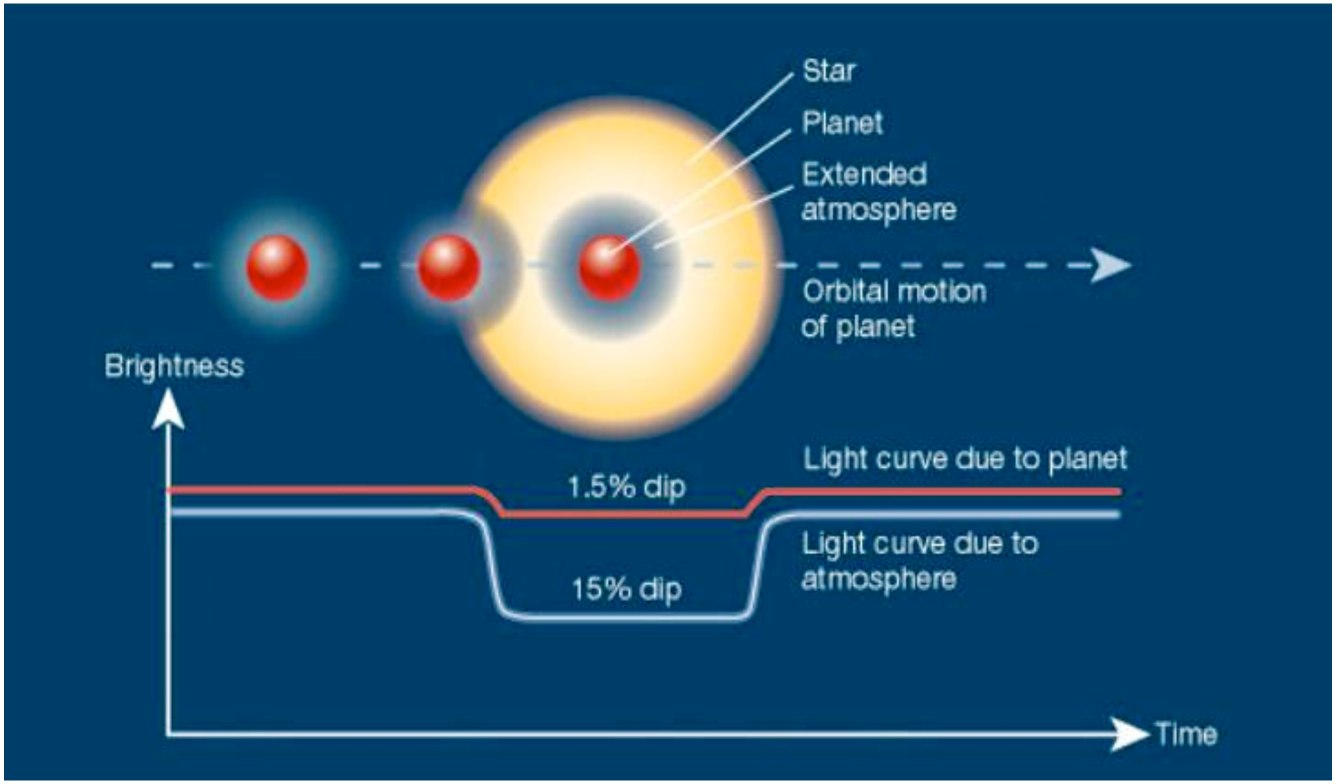
example: HD209458 (Vidal-Madjar 2003)



- 0.67 Jupiter masses, 0.05 AU, transiting
- hydrogen atmosphere and escape observed
- question: what is the mass loss rate? long-time stability of the planet?

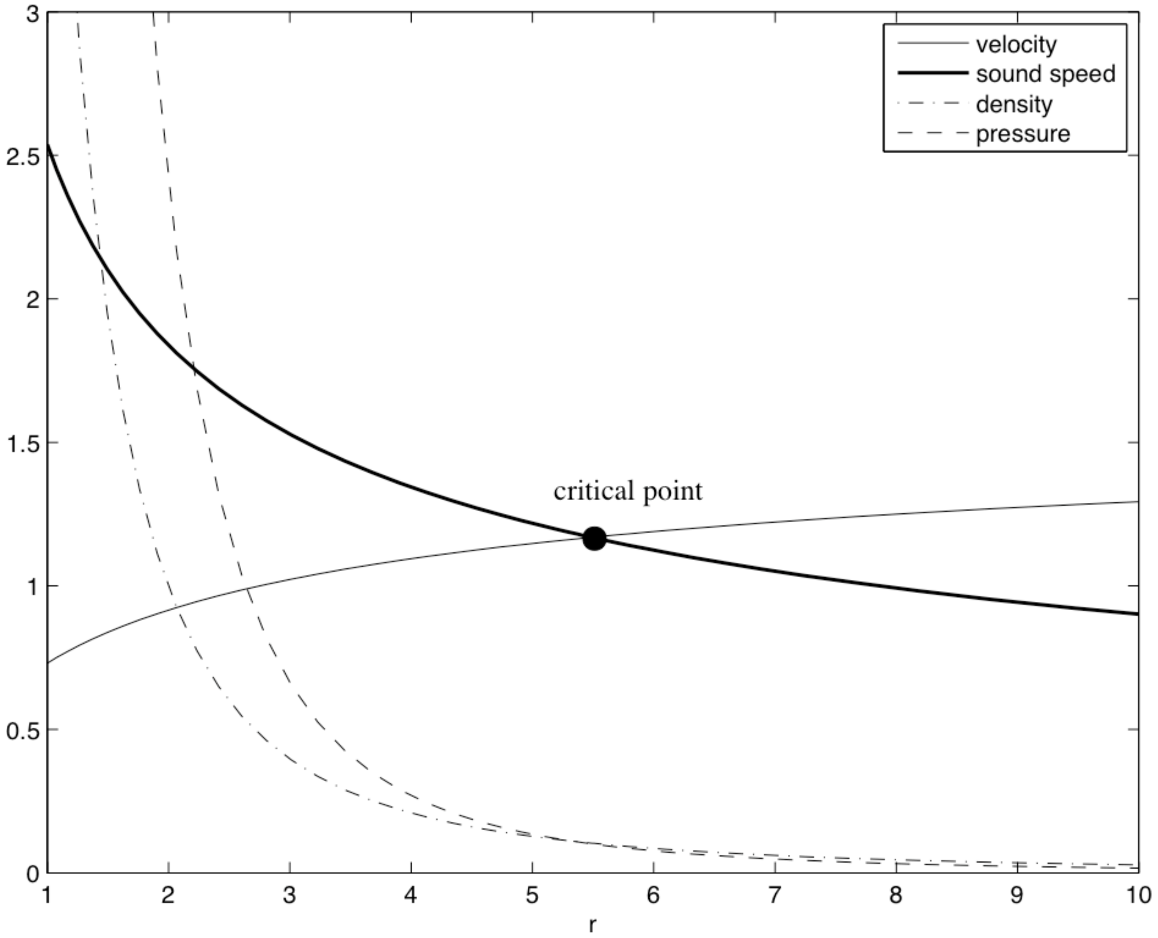
(2) 1D numerical models

- compressible gas dynamics (Euler equations)

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho\, r^2 \\ \rho\, u\, r^2 \\ (\frac{p}{\gamma-1}+\frac{\rho\, u^2}{2})\, r^2 \end{bmatrix} + \frac{\partial}{\partial r}\begin{bmatrix} \rho\, u\, r^2 \\ \rho\, u^2\, r^2 + p\, r^2 \\ (\frac{\gamma\, p}{\gamma-1}+\frac{\rho\, u^2}{2})\, u\, r^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -\rho G M + 2\, p\, r \\ -\rho G M\, u + q_{heat}\, r^2 \end{bmatrix}$$

transonic radial outflow solution



numerical method

- time-marching to steady state

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial r} = S(U)$$

- use standard CFD methods: finite volume method with Riemann solver

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} + \frac{F^*_{i+1/2} - F^*_{i-1/2}}{\Delta r} = S(U_i)$$

- very slow convergence to steady state!

results for 1D exoplanet simulations

- HD209458b:
  - lower boundary conditions $n=2.10^{15}$ cm^{-3} and T=750K
  - extent of atmosphere, outflow velocity, and mass flux consistent with observations (Vidal-Madjar 2003)
  - 1% mass loss in 12 billion years → HD209458b is stable

- A planet with 0.5 Uranus mass @ 0.4AU AND under 10X solar EUV radiation has 10% mass loss in ~850 million years (preliminary results) → Is Mercury the remnant of a giant planet?

- Tian, Toon, Pavlov, and De Sterck, Astrophysical Journal 621, 1049-1060, 2005

(3) 2D numerical models (Scott Rostrup)

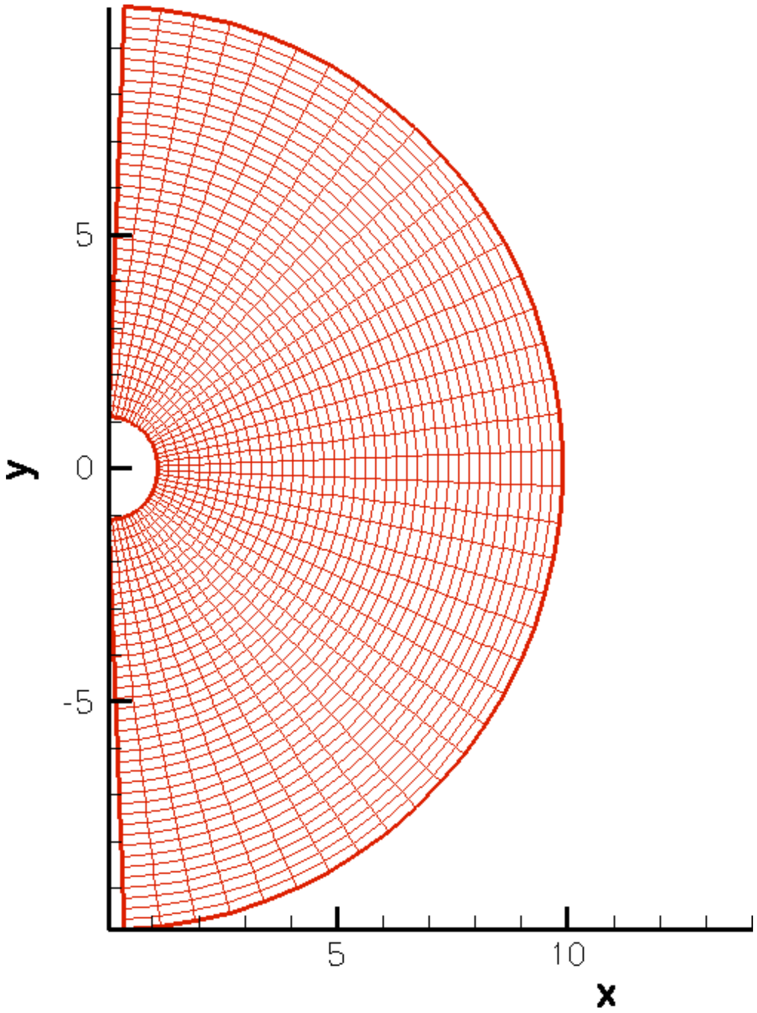
- Euler equations in multiple dimensions

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho\,\vec{v} \\ \frac{p}{\gamma-1}+\frac{\rho\,v^2}{2} \end{bmatrix} + \nabla\cdot\begin{bmatrix} \rho\,\vec{v} \\ \rho\,\vec{v}\vec{v}+I\,p \\ \left(\frac{\gamma\,p}{\gamma-1}+\frac{\rho\,v^2}{2}\right)\vec{v} \end{bmatrix} = \begin{bmatrix} 0 \\ \vec{F}_{ext} \\ \vec{F}_{ext}\cdot\vec{v}+q_{heat} \end{bmatrix}$$

2D Simulations

- assume rotational symmetry about the y axis

➔ allows for non-uniform heating



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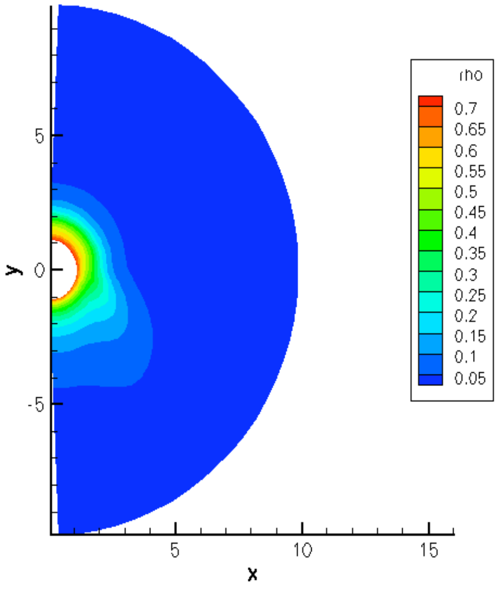
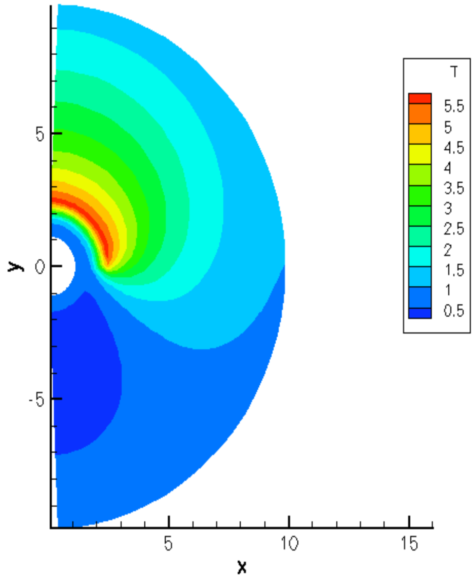
non-uniform heating

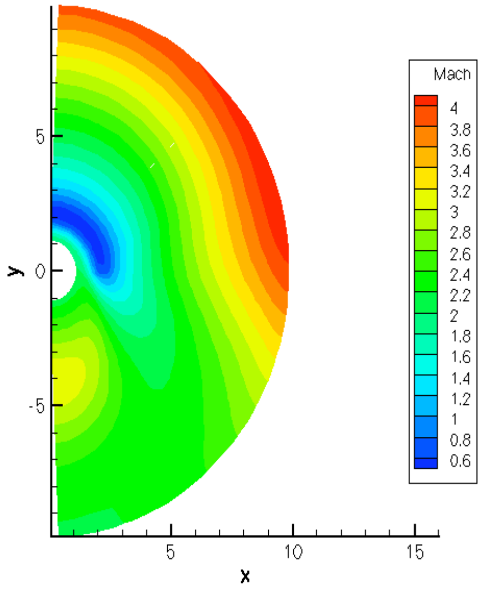
Density



Temperature



Mach Number



- heated by a thin layer in the northern hemisphere
- outflow mass flux similar to 1D case
- 1D gives reasonable approximation

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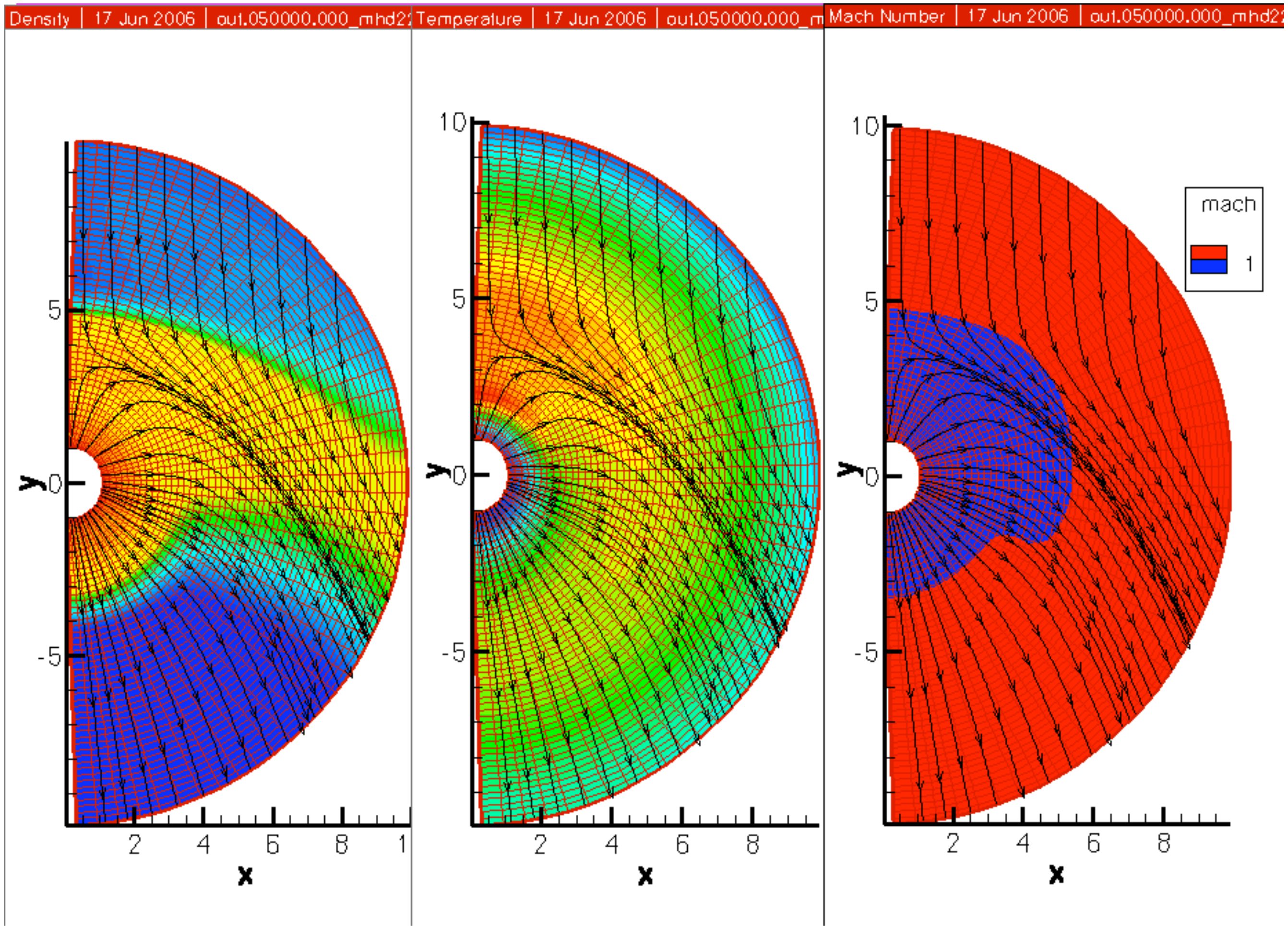
ongoing work: include stellar wind



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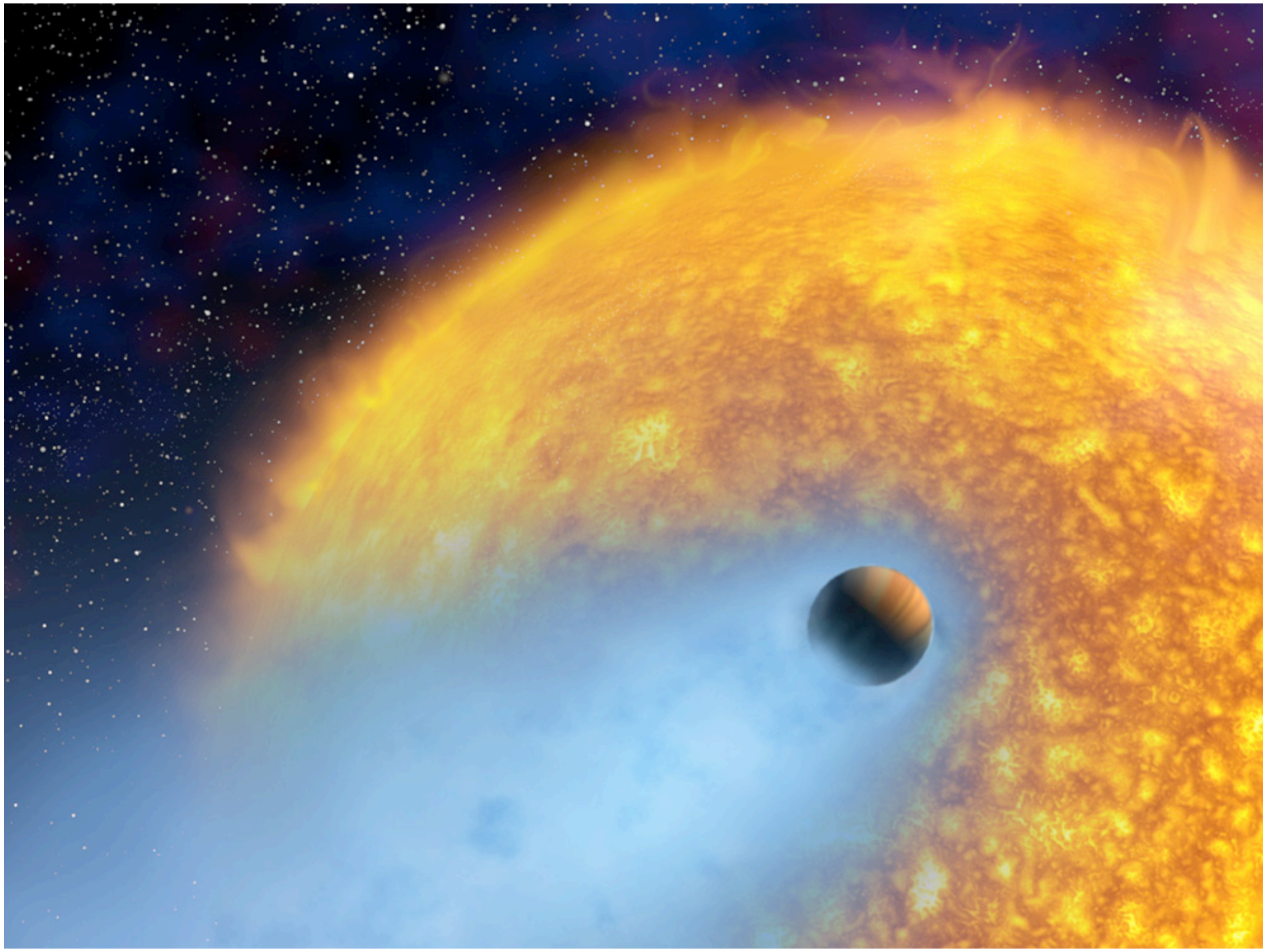
compare with artist's impression...



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(4) 3D numerical models (Paul Ullrich)

- we want to include effects of planetary rotation
- topology: grid between two concentric spheres
- if one wants uniform point density in all directions, one is naturally led to the use of unstructured grids (avoid the polar problem)

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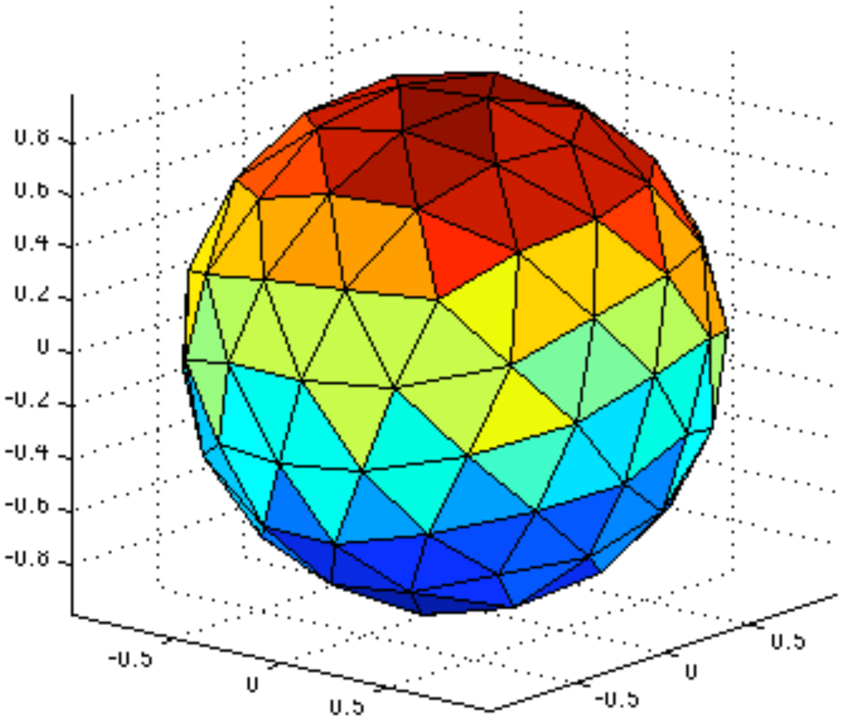
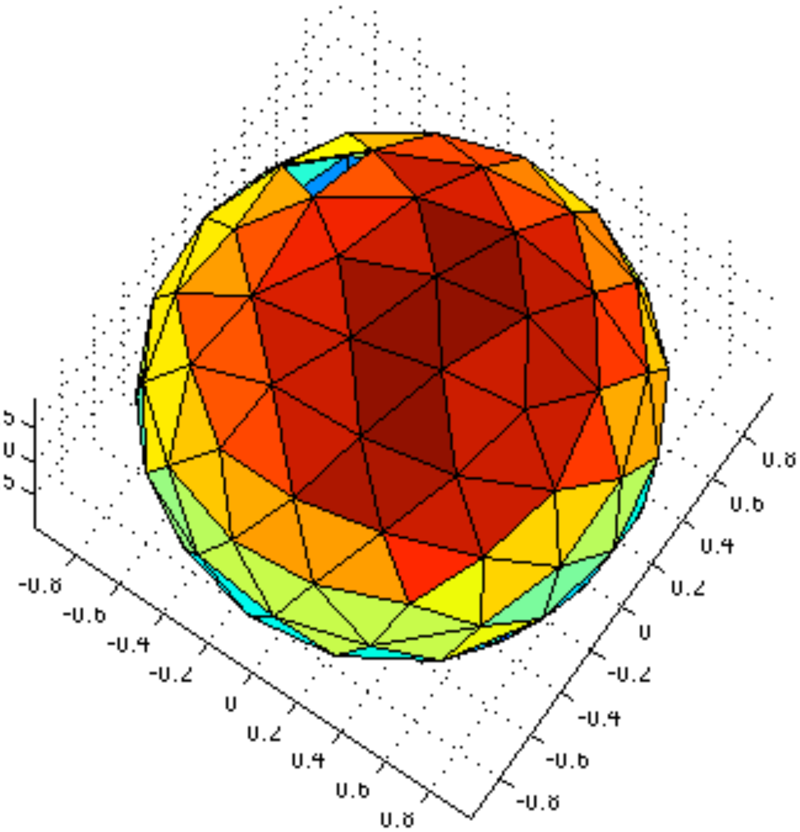
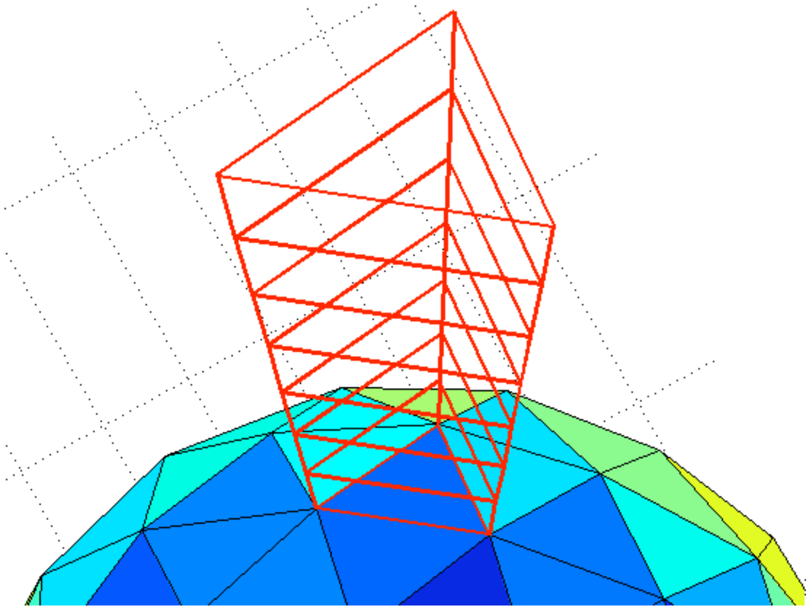
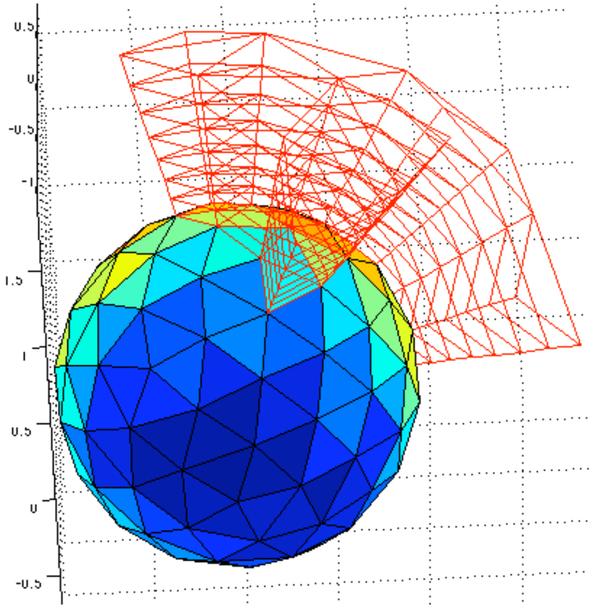
unstructured triangular grid on inner boundary



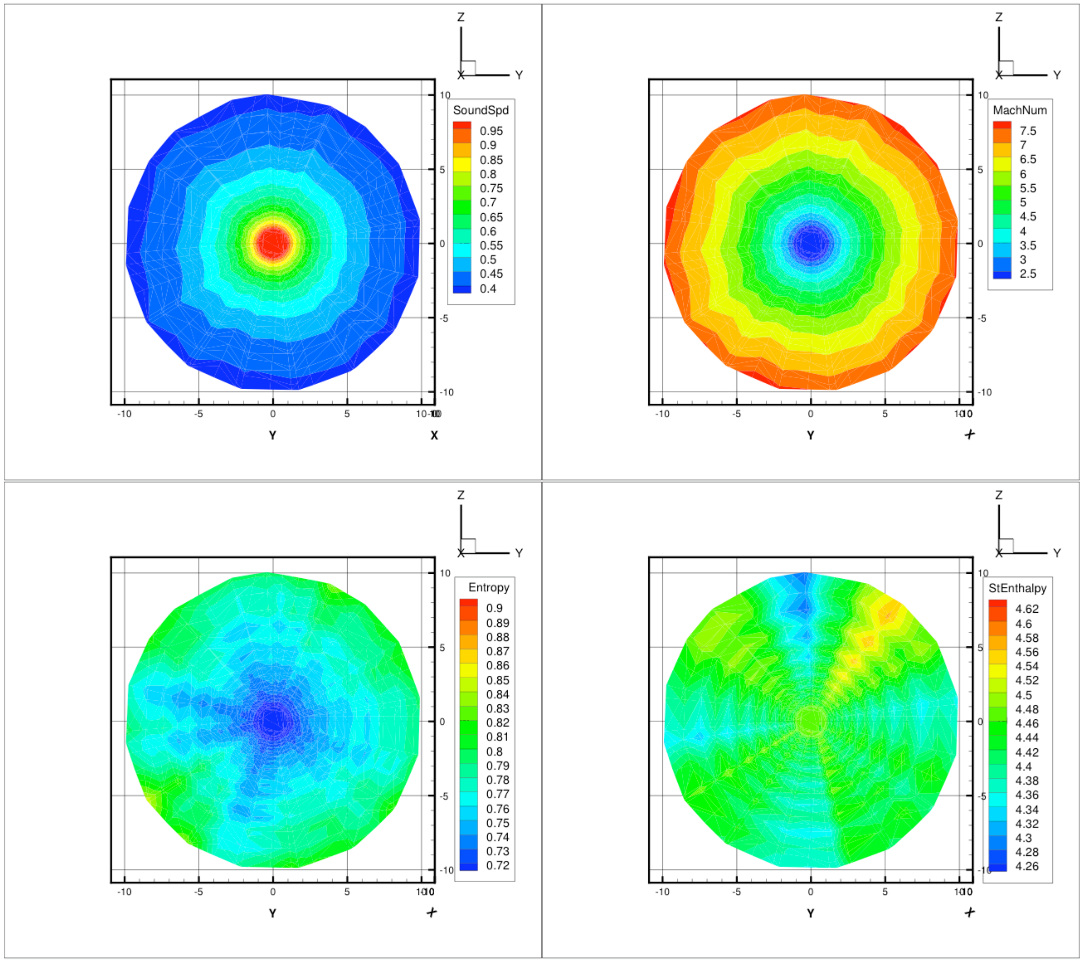


build stacks of triangular prisms out from the inner boundary





preliminary 3D finite volume results



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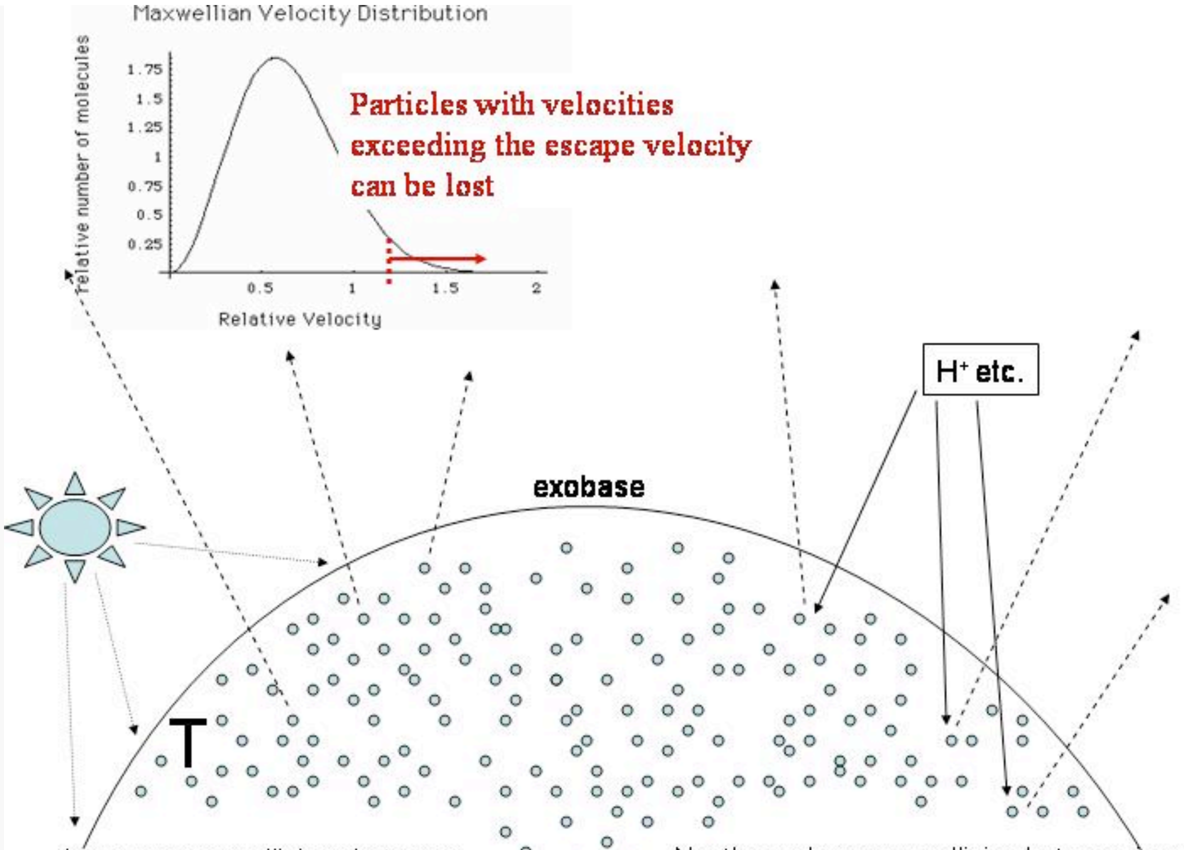
(5) Supersonic gas escape from Early Earth

- there is no supersonic hydrodynamic escape from present-day Earth
- exo-base temperature is high: collisional, thermal escape dominates



Jeans escape: collisions between escaping particles, Maxiwellian

Nonthermal escape: collision between ions and escaping particles, non-maxwellian

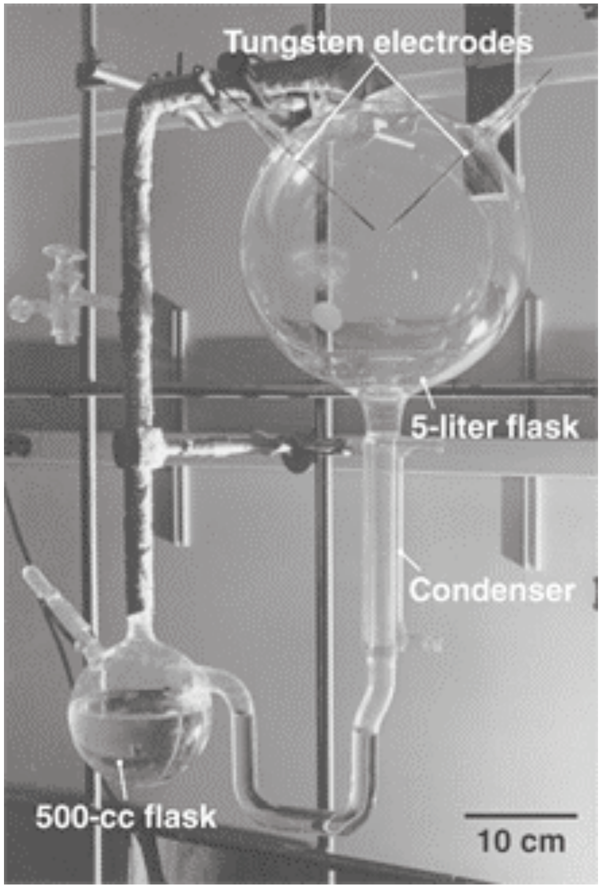
Supersonic gas escape from Early Earth

- hypothesis: when the Earth was young, the exo-base temperature may have been low, and supersonic hydrodynamic escape may have been ongoing
- test of hypothesis: do 1D simulation, find exobase temperature, and outflow flux

➔ our simulations confirm cold exobase and hydrodynamic escape with small mass flux

this finding also has implications for hydrogen content in Early Earth atmosphere!

primordial soup as the origin of life



•Stanley Miller (1953): formation of prebiotic molecules in a CH4-NH3 rich environment with electric discharge

➔Problem: CH4-NH3 atmosphere unlikely

•later experiments show that prebiotic molecules can be formed efficiently in a hydrogen-rich environment.

•alternative sources of organics: hydrothermal system, extraterrestrial delivery

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hydrogen content in Early Earth atmosphere

- hydrogen content: balance between volcanic outgassing and escape from atmosphere
- existing theories claim that hydrogen content was very low, as hydrogen escape rates through thermal processes were assumed very large
- formation of prebiotic molecules in a hydrogen-rich atmosphere was thus discarded as a theory

hydrogen content in Early Earth atmosphere

- our results: hydrogen escape was probably hydrodynamic, and escape rates were low
- our results: hydrogen concentration in the atmosphere of Early Earth could have been as high as 30%
- formation of prebiotic molecules in early Earth's atmosphere could have been efficient → primordial soup on early Earth is possible
- no need for hydrothermal vents, extraterrestrial delivery
- Tian, Toon, Pavlov, and De Sterck, Science 308, 1014-1017, 2005