CO 351 Network Flows (Winter 2010)
Assignment 1

Due: Thursday January 21st, in class.

Policy. For any question, if you simply write "I don't know" you will receive 20% of the marks for that question.

Question 1: (20 marks)

Prove Theorem 3 in the notes. Here is Theorem 3:

Let s and t be nodes of a digraph $D = (N, A)$. There exists an st-dipath in D iff (if and only if) there is no st-cut $\delta(S)$ with $\delta(S) = \emptyset$. (i.e., $S \subseteq N$, $s \in S$ and $t \notin S$.)

Question 2: (15 marks)

Prove Corollary 5 in the course notes, <u>without</u> using Proposition 4. Here is Corollary 5:
Let u, v, and w be nodes in a digraph. If there exists a uv-dipath and a vw-dipath, then there exists a uw-dipath.
Hint: One way is by using Theorem 3.

Question 3: (5 marks)

Write the contrapositive of the statement:
"If there exists a uv-dipath and a vw-dipath, then there exists a uw-dipath."

Question 4: (20 marks) A consulting business runs out of two cities, Toronto and Vancouver. Each month, the manager decides whether to work in Toronto or in Vancouver, depending upon the demands in either city. In month i, the manager incurs a cost of T_i if she is in Toronto, and V_i is she is in Vancouver. She also incurs a fixed moving cost M whenever she switches cities. Given a sequence of n months, and sequences of operating costs $T_1, ..., T_n$ and $V_1, ..., V_n$, she has to find a work plan with minimum cost. In other words, she has to find a n choices of cities (each one either Toronto or Vancouver) for the n months, such that the sum of the operating costs for the n months plus the moving cost of switching cities is as small as possible. The plan may begin or end in either city.

Formulate this problem as a shortest st-dipath problem. Justify the correctness of your formulation.

More on next page.

Question 5: (10 marks)

Consider the linear program

$$\begin{array}{lll} \max & x_1 + 2x_2 - x_3 & \\ \text{s.t.} & -x_1 - x_2 - x_3 & \leq -1 \\ & -x_1 + x_2 + x_3 & \leq 1 \\ & x_1 - x_2 + x_3 & \leq 1 \\ & x_1 + x_2 - x_3 & \leq 1 \\ & x_1, x_2, x_3 & \geq 0 \end{array}$$

Write down the dual of this linear program, and all the complementary slackness conditions.