PMath 464/764 – Homework 8
Due Monday, July 5

1. Let $C = V(F)$ be a projective plane curve, where F is irreducible. Show that a point P is a singular point of C if and only if $F(P) = F_x(P) = F_y(P) = F_z(P) = 0$. [Hint: Euler proved that if F is homogeneous of degree d, then $dF = xF_x + yF_y + zF_z$. Feel free to use that without proof.]

2. Find all the singular points of the projective curve $y^2z + yz^2 = x^3$.

3. Let P be a smooth point of an irreducible projective plane curve C with $I(C) = (F)$. Show that the tangent line to C at P is:

$$F_x(P)x + F_y(P)y + F_z(P)z = 0$$

4. Let R be DVR with fraction field K, and let S be a subring of K which contains R. Prove that $S = R$ or $S = K$.

5. Let P_1, P_2, and P_3 be three different points of $\mathbb{P}^1$. Prove that there is a projective change of coordinates T in P^1 such that $T(P_1) = [1 : 0]$, $T(P_2) = [0 : 1]$, and $T(P_3) = [1 : 1]$.

6. Let $\mathcal{O}_P(V)$ be the local ring of $\mathbb{A}^2$ at the point $P = (0, 0)$. Prove that $R = \mathcal{O}_P(V)$ is not a DVR.