1. Let $G = \langle x, y \mid x^3 = y^7 = 1, xy = yx \rangle$ be a group presented in terms of generators and relations. How many elements does $G$ have? Prove your answer.

2. Let $G = GL_2(\mathbb{Z}_2)$ be the group of invertible two by two matrices with entries in $\mathbb{Z}_2$.
   
   (a) Find a faithful and transitive action of $G$ on a set $X$ with exactly three elements.

   (b) Show that $G$ is isomorphic to $S_3$, the group of permutations of the set $\{1, 2, 3\}$.

3. Let $X$ be a $G$-set, with associated homomorphism $\psi: G \to S_X$. Let $H = \ker \psi$. Define $\phi: G/H \to S_X$ by $\phi(g \mod H) = \psi(g)$. (The map $\phi$ is the homomorphism $\tilde{\psi}$ provided by the UPQ.) Prove that $\phi$ is a well defined homomorphism, and that the corresponding action of $G/H$ on $X$ is faithful.

   The moral of this is that you can make an action faithful by taking the quotient of $G$ by all the elements that act trivially on $X$. 