STAT 333 Lecture Blog Fall 2011 (T/R)

Diana's Lecture Blog

©Back to Forums & Topics List

Image: Second second

I'm planning to use this space to post a brief summary of what we did in each class afterwards. You can use this space to ask questions to clarify anything that went on in class, or if I asked a take-home question, post your answers. You can also use it to see what you missed if you weren't in class for some reason. :)

 Review Lecture - Dec 5

 Diana Katherine Skrzydlo - Dec 9, 2011 2:54

 PM

 Mark Unread

 Reply]

 More actions...

We reviewed the main concepts in the whole course, up to and including birth and death processes.

 PLecture 24 - Dec 1
 Diana Katherine Skrzydlo - Dec 9, 2011 2:53

 Diana Katherine Skrzydlo - Dec 9, 2011 2:53
 0 unread of 1 messages - 1 author(s)

 Mark Unread
 author(s)

 More actions...
 More actions...

Today we looked at several examples of birth and death processes, including the poisson process (pure birth), pure death process, single-server queue, a situation with three machines and two mechanics, and a linear growth model (without and with immigration)



More actions...

Today we started off by looking at the two-state chain in a fair amount of detail, including deriving the P(t) matrix family and the equilibrium distribution. More details about solving the DE are in the handout that was posted last week.

We saw another example of using the Q matrix to find out how a chain behaves (that question was on a past final of mine), and looked at an alternate interpretation of the elements of Q using the Alarm Clock Lemma.

Finally, we defined Birth-and-Death processes, described how to find the rates out of each state and the embedded chain probabilities, derived the structure of the Q matrix and solved for the equilibrium probabilities, which have a nice closed form. Next time we'll look in more detail at several examples. (And we'll choose the time for the review session, which I forgot to do today!)



0 unread of 2 messages - 1 author(s)

Today was all about the matrix \mathbf{Q} , or generator matrix for a continuous Markov Chain. We have the two types of randomness in a cts MC, the randomness of how long we spend in each state (which we showed must be exponential) and the randomness of where we go when we leave a state. The matrix \mathbf{Q} , which we defined to be the derivative of $\mathbf{P}(\mathbf{t})$ as t->0⁺, turns out to have the rows sum to 0 rather than 1. Also it has its off-diagonal elements $q_{ij} = v_i^* P_{ij}$ and its diagonal elements $q_{ii} = -v_i$. We can use \mathbf{Q} to solve for the equilibrium distribution $\boldsymbol{\pi}$, to recover the v_i 's and P_{ij} 's (if we didn't already have them), and to actually obtain the matrix family $\mathbf{P}(\mathbf{t})$ by using the Kolmogorov Forward/Backward Equations. Note: You will <u>not</u> have to solve matrix differential equations, but you should know how to set them up (given a \mathbf{Q} matrix, show the pointwise equations the $P_{ij}(\mathbf{t})$'s must solve.)

See <u>here</u> for an awesome video from last year's YouTube assignment explaining all about \mathbf{Q} . I also posted an example of finding $\mathbf{P}(\mathbf{t})$ for a two-state continuous MC <u>here</u>. We'll go over it a bit more next class.

Collapse Replies

Re: Lecture 22 - Nov 24 Mistake in using l'Hopital's rule 🔜 Diana Katherine Skrzydlo - Nov 24, 2011 1:11 PM Mark Unread [Reply] More actions...

0 unread of 1 messages - 1 author(s)

Sorry guys, when I was proving that the diagonal elements of Q are $-v_i$, I messed up the last step. When we apply l'Hopital's rule, we get $-v_i e^{-v_ih} / 1$, not $-v_ih/h$. The result holds.

📲 Lecture 21 - Nov 22 🔜 Diana Katherine Skrzydlo - Nov 22, 2011 4:34 PM 0 unread of 1 messages - 1 author(s) Mark Unread [Reply] More actions...

Today we finished off chapter 5 by talking about the Poisson process. We proved definitions 3 and 1 were equivalent to each other, which completes the proof that all 4 are equivalent, if you include 2<=>3 from STAT 230. We also looked at a few properties of the Poisson process: the "thinning" property (which you have proved in tutorial 4 and on the first midterm), and the conditional timings of events/number of events in a subinterval (which you will prove on the last assignment!)

We then started Chapter 6 - continuous markov chains. We started by speaking in general terms about the movement in a cts MC, which is governed by both the time spent in each state (exponential) and the probabilities of where the process moves when it leaves a state (instantaneous tpm P). We defined the continuous version of the Markov property, and the infinite family of transition matrices P(t). The C-K equations hold just like they do in the discrete case, but unfortunately we can't truly capture the continuous movements with a single matrix. Looking at the long run behaviour, everything is the same as the discrete case (we keep all our definitions except nothing is periodic anymore) and the vector $\boldsymbol{\pi}$ exists uniquely if the chain is irreducible, but we don't have an easy way to solve for π .

Next time we'll see a matrix **Q** that solves all these problems!



0 unread of 1 messages - 1 author(s)

[Reply] More actions...

Today we started and got most of the way through Chapter 5 - Exponential and Poisson. We defined the exponential random variable, and looked at its memoryless property, as well as the "Alarm Clock Lemma" which tells us about the distribution of the minimum of a bunch of exponential rvs, as well as the probability that one in particular is the minimum. We defined a counting process, which is a special type of continuous stochastic process, and what it means for a process to have stationary and independent increments. Finally, we defined the Poisson process in 4 different ways! We have seen (or it's obvious) some pairs of definitions are equivalent, and we'll see a couple more next class.

🖻 Lecture 19 - Nov 10 🗐

More actions...

<u>Diana Katherine Skrzydlo</u> - Nov 10, 2011 1:59 PM <u>Mark Unread</u> [Reply]

0 unread of 1 messages - 1 author(s)

Today we finished off discrete Markov Chains. We continued to prove the property that all states in the same class have the same type, and then looked at when the equilibrium distribution is guaranteed to exist uniquely. Luckily it's in most cases, except when all states are transient/null recurrent, or when there are 2 or more closed classes. We saw a very detailed example with 7 states that had 4 classes (2 closed) and were able to derive the mini-equilibrium distribution in each closed class, the set of stationary distributions for the chain, and the absorption probabilities into each closed class from each transient state.

The midterm covers up to the end of this class. The review session on Sunday and the tutorial on Monday will have more examples to practice this material, and then the midterm is on Thursday. Good luck!



Today we looked at a few more examples of finding the equilibrium distribution of a Markov Chain. Then we saw how renewal theory applies to Markov Chains - there are two handouts on this material that you should look at in addition to the notes from class. We also defined accessibility and communication between states, and saw how that divides the states in a MC into classes, which are either closed (cannot leave) or open (can leave, but then cannot come back). We examined some examples and developed the rule that open classes contain transient states, finite closed classes contain positive recurrent states, and infinite closed classes can contain anything. We'll continue looking at this next time.

►Lecture 17 - Nov 7 → Diana Katherine Skrzydlo - Nov 8, 2011 11:02
AM
Mark Unread [Reply] More actions...

0 unread of 1 messages - 1 author(s)

Today we examined how Markov Chains behave in the short and long term. First we looked at the n-step transition probabilities, and were able to find them from the 1-step transition probabilities by deriving the Chapman-Kolmogorov equations. We have that **P** (the tpm) gives us all the information we need to look at where the process is at any number of steps in the future, by taking powers of that matrix. Even if we don't know exactly where the process is at time n, but just a probability distribution for X_n (the row vector \mathbf{p}_n), we can use it to find the distribution at time n+1. As n approaches infinity, the chain may approach an equilibrium, and we can find that the equilibrium vector $\boldsymbol{\pi}$ satisfies $\boldsymbol{\pi} \mathbf{P} = \boldsymbol{\pi}$ subject to the elements adding to 1. If $\boldsymbol{\pi}$ is unique, we can also see that \mathbf{P}^n approaches a matrix where all the rows are $\boldsymbol{\pi}$, which means the equilibrium doesn't depend on the starting state.

🖻 Lecture 16 - Nov 3 🗐

More actions...

Diana Katherine Skrzydlo - Nov 3, 2011 1:05

P M

Diana Katherine Skrzydio - N PM Mark Unread [Reply]

0 unread of 1 messages - 1 author(s)

Today we started Markov Chains (chapter 4 in the textbook if you have it). We defined the Markov property, which is that the future state of the chain depends only on the present state, not on the past. We defined the one-step transition probabilities P_{ij} and arranged them into a matrix **P** (the tpm), which has all elements non-negative and the rows sum to 1. Finally, we looked at lots of examples, including weather, win/loss of a team, random walks with various modifications, and the Ehrenfest model (for which a handout is posted.)

 Image: Diana Katherine Skrzydlo
 - Nov 1, 2011 5:06

 PM
 Mark Unread

0 unread of 1 messages - 1 author(s)

[Reply] More actions...

Today we finished up renewal theory. We looked at first passage time events in the random walk, which are delayed renewal, and developed results for lambda_01 using the same pgf approach as yesterday. We also looked at passage times to state k, which we can express as the combination of k steps from 0 to 1. Finally, we looked at the gambler's ruin problem, which is where we have a random walk with absorbing barriers at 0 and K, and a starting wealth of i. Two new handouts summarizing the material on the random walk and gambler's ruin.

Lecture 14 - Oct 31 Diana Katherine Skrzydlo - Nov 1, 2011 9:25 AM Mark Unread [Reply] More actions...

0 unread of 1 messages - 1 author(s)

Today (during the lecture in tutorial period) we continued work on the Random Walk. We looked at the renewal sequence and used it to find $E[V_00]$, and also to find the pgf of T_00, F_00(s) using the renewal relation. That gave us the result that lambda_00 (return to 0) is transient if p is not equal to 1/2, and null recurrent if p = 1/2. We can also find f_00 and $E[T_00]$ and $E[V_00]$ with our pgf approach.

 Diana Katherine Skrzydlo - Oct 27, 2011 1:53

 Diana Katherine Skrzydlo - Oct 27, 2011 1:53

 O unread of 1 messages - 1 author(s)

 Mark Unread [Reply]

 More actions...

Today we looked at the delayed renewal relation again (and how we can use the renewal relation on the associated renewal event lambda-bar to find inter-event waiting times) and an example.

We also defined the period of an event and developed the renewal theorem (both aperiodic and periodic) which gives us a quick way to find E[T_lambda] from the limit of the renewal sequence. We even saw how to apply it in the case of a delayed renewal event by breaking the waiting time up into pieces, each of which correspond to a renewal event.

Finally, we started talking about the random walk, and the periodic (d=2) renewal event "return to 0". We will see more on this in Monday's lecture. Have a good weekend!

Lecture 12 - Oct 25 Diana Katherine Skrzydlo - Oct 27, 2011 8:37 AM Mark Unread [Reply] More actions...

0 unread of 1 messages - 1 author(s)

Today we looked at a different way of finding out information about a renewal event - the renewal sequence. We can use it in two ways

- sum it up from n=1 to infinity to get E[V_lambda], which can tell us if the event is recurrent or transient, and can give us f_lambda;

- use it to obtain the pgf of T_lambda through the renewal relation

We also started developing a similar result for delayed renewal events, which we will revisit on Thursday.

Don't forget, the next two tutorial periods will be lectures.



Today we continued talking about renewal theory, starting with reviewing the definitions from last class. We defined recurrent and transient events, and further split recurrent events into positive recurrent (finite mean) and null recurrent (infinite mean). The breakdown of events in a stochastic process has the same pattern as the breakdown of a random variable (in particular, T_lambda) into improper/proper-null proper/short proper.

We also defined a new random variable V_lambda, equal to the number of times lambda occurs, and derived a few results for it, including the pmf and how to find f_lambda from $E[V_lambda]$.

The first assignment was handed back. Contact the TA who marked it if you have questions.

Don't forget the test is on Thursday. Next Tuesday Diana's section is back in MC 4021 just for that one day (since the class we switched rooms with has a midterm)

PLecture 10 - Oct 13 🗐

0 unread of 1 messages - 1 author(s)



Today we started by reviewing the uses of pgfs, as this is the last material covered on the first test.

We then started renewal theory, looking at definitions of a stochastic process, an event lambda, the first waiting time T_lambda, the probability of occurrence f_lambda, the interevent waiting times T_lambda^(k, k+1), and finally the definition of a renewal event and a delayed renewal event. We saw a couple of simple examples.



Today we talked all about power series and generating functions, and in particular, probability generating functions. A summary (and even more detail for some parts) of what we did can be found in the two new handouts posted <u>here</u> and <u>here</u>. Credit goes to Prof. C. Cutler for preparing the handouts.

Eccture 8 - Oct 6 Diana Katherine Skrzydlo - Oct 6, 2011 9:30 PM Mark Unread [Reply] More actions...

0 unread of 1 messages - 1 author(s)

Today we finished up chapter 3. We started with an example of using double averaging and the conditional variance formula for a case where the distribution of X|N=n was known. Then we talked about compound random variables (in general and the strict mathematical requirements for them) and derived formulas for the mean and variance of a compound rv S in terms of the means and variances of N (the compounding distribution) and the Xi's. We did a quick example of that, and then I summarized what we've done in chapter 3.

We will have a lot of new stuff up on this website for you in the next day or so, so keep

checking.

Remember there is no tutorial on Monday since it's a holiday. On Tuesday we start probability generating functions. Have a great long weekend!

 Image: Second system
 Diana Katherine Skrzydlo - Oct 4, 2011 1:40

 Image: Second system
 Diana Katherine Skrzydlo - Oct 4, 2011 1:40

 Image: Second system
 0 unread of 1 messages - 1 author(s)

 Image: More actions...
 More actions...

Today we started with an example of independent RVs being combined into another RV, and using conditioning, substitution, and independence (and then some somewhat tricky integrating) to find the cdf of that new RV. We also looked at an example of calculating a probability with conditioning - the "best offer" problem. This is also discussed in some detail in the textbook if you're interested on page 126.

We then defined conditional variance and looked at two ways of finding variance - developing a recursive equation for $E[X^2]$ and the conditional variance formula, which we proved.

Don't forget your assignments are due at the beginning of class on Thursday. You can ask questions during office hours or post on the discussion board if you need any help.



Today we continued talking about conditioning in the discrete case with an example of waiting for k successes in a row in Bernoulli trials. We then defined conditional expectation (etc) in the continuous case, and saw an example, using gamma functions as a shortcut to avoid integrating by parts. I gave you an exercise to prove that E[X] = 1 directly using the usual definition of expectation - feel free to post your answers here.

We looked at 4 properties of conditional expectation - linearity, double averaging, substitution, and independence, and saw how we can apply conditioning techniques to probabilities. Then we saw one more example, using a case where we know the distribution of X|U=u. X turned out to be a null proper random variable, which is a weird case where P(X=infinity) = 0 but the mean still diverges. The way to think about it in this case is that

there is a chance the coin is so biased against heads (arbitrarily close to 0) that it would take really really long to get one. But it will never actually take *forever*, since the chance the coin has P(H) = 0 is the chance that U=0, which is 0 since U is continuous.

Next time we'll have more examples of calculating probabilities and variance by conditioning.

□<u>Collapse Replies</u>



Today we continued talking about conditioning in the discrete case with an example of waiting for k successes in a row in Bernoulli trials. We then defined conditional expectation (etc) in the continuous case, and saw an example, using gamma functions as a shortcut to avoid integrating by parts. I gave you an exercise to prove that E[X] = 1 directly using the usual definition of expectation - feel free to post your answers here.

We looked at 4 properties of conditional expectation - linearity, double averaging, substitution, and independence, and saw how we can apply conditioning techniques to probabilities. Then we saw one more example, using a case where we know the distribution of X|U=u. X turned out to be a null proper random variable, which is a weird case where P(X=infinity) = 0 but the mean still diverges. The way to think about it in this case is that there is a chance the coin is so biased against heads (arbitrarily close to 0) that it would take really really long to get one. But it will never actually take *forever*, since the chance the coin has P(H) = 0 is the chance that U=0, which is 0 since U is continuous.

Next time we'll have more examples of calculating probabilities and variance by conditioning.

Collapse Replies



Great job!

P.S. For some reason I can't see your third equation in your post, I have to click on the attachment to see it. If other people are wondering what it says, it's the integral from 1 to infinity of xe^{-xy} which turns out to be e^{-x} . This is just the pdf of an exponential RV with parameter 1, so the mean is 1.



When I was talking about the number of trials required to see 3 Heads in a row on a fair coin, I added 2 + 4 + 8 and somehow got 13. It should have been 14. :)

(That 2 + 4 + 8 comes from $1/p + 1/p^2 + 1/p^3$ where p = 0.5.)

🖻 Lecture 5 - Sept 27 🗐

Diana Katherine Skrzydlo - Sep 27, 2011 1:24



0 unread of 1 messages - 1 author(s)

Today we started Chapter 3 with a lot of definitions: the conditional RV X|Y=y, its conditional range, its conditional pmf P(X=x|Y=y), and its conditional expectation E[X|Y=y]. All of that assumes a fixed value y for the RV Y, but we can also have y vary as well, and that gives us a new RV E[X|Y], which itself has a pmf (the same as the pmf of Y) and an expected value E[E[X|Y]] which we proved turns out to just be E[X].

We also looked at two examples (one of which, the trials, we could have found E[X] without using conditioning, one of which, the mouse in the maze, would have been very hard to do that for!)

After class, someone mentioned a neat intuitive way to explain that E[X] in the mouse question turned out to be 10. We know the number of attempts to get out is a Geometric random variable with P(Success) = 1/4, so on average it will take 4 tries (3 failures and a success.) For the 3 failures, the average time in a tunnel is 2 minutes each, so 6 minutes of wandering on average, and then 4 minutes in the correct tunnel heading for the cheese!



0 unread of 1 messages - 1 author(s)

Today we started with an example of using indicator variables - the envelope matching problem (sometimes called the hat matching problem). It gave us a surprising result that the mean and variance of the number of matches was 1, for n>1 envelopes.

We then looked more closely at the meaning of a proper random variable, and defined short

proper and null proper as categories within proper. Improper random variables must have infinite means, but proper can have infinite or finite means. Finally we saw some quick examples of improper and null proper RVs, and got out early!

Lecture 3 - Sept 20 Diana Katherine Skrzydlo - Sep 20, 2011 1:23

 PM

 Mark Unread

 [Reply]

 More actions...

0 unread of 1 messages - 1 author(s)

Today we looked at one more discrete random variable (the Poisson) and talked about continuous random variables, including the pdf (which is not a probability), the uniform rv, and the exponential and gamma rvs that arise from the Poisson process. I did a summary chart of all the RVs we care about and their relationships.

We then talked about expectation, variance, and joint distributions for both continuous and discrete cases. We defined covariance and what it means for two variables to be independent. Finally, we looked at some neat properties of indicator variables, including their mean, variance, and covariance.

A tiny bit more about indicator variables - I mentioned right at the end that for them, independent <=> uncorrelated. It's the <= implication that is special for indicator variables, since indep => uncorr is always the case. Also I said they can be used to solve complex problems. You will see one of these on the first assignment! :)

PLecture 2 - Sept 15 Diana Katherine Skrzydlo - Sep 15, 2011 1:10 PM Mark Unread [Reply] More actions...

0 unread of 3 messages - 2 author(s)

Today we finished chapter 1 by defining conditional probability, the law of total probability, and Bayes' Rule, and did an example of Bayes' rule using the classic Monty Hall problem. (There are some neat sites relating to it at the bottom of this post.)

Then we started chapter 2 by defining a random variable, pmf, and cdf. We looked at Bernoulli trials, which can give rise to several different random variables, and we examined four of them: Bernoulli (or Indicator), Binomial, Geometric, and Negative Binomial. We also proved that the Geometric RV is proper. Someone asked a good question afterwards - why didn't I state the pmf of the geometric. The answer is, I forgot! It is $P(Y = k) = p^*(1-p)^{(k-1)}$. Notice that you can obtain this by letting r = 1 in a Negative Binomial RV. Also notice that the sum of r independent Geometric(p) RVs gives a Negative Binomial(r, p) RV, just like the sum of Bernoullis is Binomial!

Monty Hall sites: <u>http://www.maa.org/devlin/devlin_07_03.html</u>

http://math.ucsd.edu/~crypto/Monty/monty.html - applet and explanation

http://probability.ca/jeff/writing/montyfall.pdf - some other variations

□ <u>Collapse Replies</u>



0 unread of 2 messages - 2 author(s)

Hi Diana,

I asked you this question right after the class. I'll restate the question here and please clarify it for me.

The main focus is on the Geometric distribution and weather can yield a result of infinite. You mentioned that it can be "arbitrary large" but not infinite, but I feel that these two concepts are the same. To my understanding (please correct me), a number is infinite iff it is larger than any real number. Take the opposite, if something is greater than any real number (or integer), it is infinite.

let x be any positive integer, for any p<1, P(Geometric(p)>x) > P(Geometric(p)=(x+1))>0, so P(Geometric(p)<=x)<1, and does not include the entire probability space. This is true for any x as integer, so the only solution is that Geometric(p) can take infinity for any p<1. (In this case the sum of all probabilities will be 1). On the other word, Y=infinite is meaningful.

On the other hand, as you said in the class, P(Y=infinity) = 0 when we take the limit, so the waiting time variable is a proper RV.

For now seems like there is no conflict: Y=infinite exists and the probability is 0 for p>0. However, does it mean the range of Geometric distribution can reach infinity?

Thank's for the help, and sorry for the confusion.

Bob

<<< Replied to message below >>> Authored by: Diana Katherine Skrzydlo Authored on: Sep 15, 2011 1:10 PM Subject: Lecture 2 - Sept 15

Today we finished chapter 1 by defining conditional probability, the law of total probability, and Bayes' Rule, and did an example of Bayes' rule using the classic Monty Hall problem. (There are some neat sites relating to it at the bottom of this post.)

Then we started chapter 2 by defining a random variable, pmf, and cdf. We looked at Bernoulli trials, which can give rise to several different random variables, and we examined four of them: Bernoulli (or Indicator), Binomial, Geometric, and Negative Binomial. We also proved that the Geometric RV is proper.

Someone asked a good question afterwards - why didn't I state the pmf of the geometric. The answer is, I forgot! It is $P(Y = k) = p^*(1-p)^{(k-1)}$. Notice that you can obtain this by letting r = 1 in a Negative Binomial RV. Also notice that the sum of r independent Geometric(p) RVs gives a Negative Binomial(r, p) RV, just like the sum of Bernoullis is Binomial!

Monty Hall sites: <u>http://www.maa.org/devlin/devlin_07_03.html</u>

http://math.ucsd.edu/~crypto/Monty/monty.html - applet and explanation

http://probability.ca/jeff/writing/montyfall.pdf - some other variations

□<u>Collapse Replies</u>



Perhaps the confusion comes from my choice of the infinity symbol to denote the event never occurring. I could just as easily have used X = -1 to indicate that we never obtain a Success.

When an event happens with probability 1, officially we say the event happens "almost surely." So with the Geometric, we will obtain a Success in a finite number of trials almost surely. In fact, the average time until a Success occurs is actually finite too. There are variables that are proper (0 chance of never occurring) but that have an infinite mean. We will see one this thursday and also some later on in the course. They are really non-intuitive, since we are saying it will definitely be finite, but on average, it's infinite!

PLecture 1 - Sept 13 🗐

Diana Katherine Skrzydlo - Sep 13, 2011 1:15

PM Mark Unread [Reply]

[Reply] More actions... 0 unread of 1 messages - 1 author(s)

Today we went over the course syllabus, and started Chapter 1. We defined sample space, event, probability, and used the 3 axioms of probability to prove some properties. Finally, we defined what it means for events to be independent.

A question came up after class: Can an event be the empty set? Yes, and the probability of the empty event would be 0.