ACTSC 331 Lecture Blog Fall 2012

Lecture Blog

I'll use this space to post brief summaries of what we did in each lecture, shortly after the lecture ends. You can check here if you missed one, or if you're just reviewing the main concepts.

Lecture 36 - Dec 3

Diana Katherine Skrzydlo - Dec 3, 2012 10:39 AM
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Today we did a review of the course.

Good luck on exams everyone!

Lecture 35 - Nov 30

Diana Katherine Skrzydlo - Nov 30, 2012 1:48 PM
Mark Unread
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Today we finished up what we'll be covering in terms of profit tests.

We calculated the reserves based on the net premium policy values (on a more conservative basis) for each year, and then incorporated them into the table for calculating the surplus. The details are in the spreadsheet already posted (I just hid the rows, so unhide them and you'll see all the calculations). We add a "cost" to setting up next year's reserve at the end of each year, and an "income" of the reserve brought forward at the start of each year. That makes the surplus much more stable and better representative of what actually occurs.

From the surplus values (or profit vector), which are conditional on the contract being in force at the start of each year the way we calculated them, we can also obtain the profit signature by multiplying each element by the probability of being alive at the start of each year. Then we have unconditional surpluses, with which we can do any sort of analysis to evaluate the profitability of the contract.

On Monday we'll have a summary class (of the stuff that *will* be on the exam) and also your assignment 6 is due at the beginning. See you then!

Lecture 34 - Nov 28

Diana Katherine Skrzydlo - Nov 28, 2012 12:09 PM
Mark Unread
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Today we started talking about profit testing (chapter 11)

We can use a spreadsheet to easily project cashflows for a portfolio of contracts, and do useful analysis based on the results, including setting premiums that account for profit, setting reserves, identifying when profits will occur, etc.

The profit testing is done using the profit testing basis (yet another basis!) and includes the following assumptions:
- initial expenses happen before the first premium is received
- each period we are only looking at cashflow in that period
- each period we assume the contract is in force at the start of that period

We looked at an example of projecting the cashflows (and determining the surplus at each time period) for a 10 year discrete term insurance. The spreadsheet we used is posted. Next time we'll look at adding reserves to the profit test.

Lecture 33 - Nov 26

Diana Katherine Skrzydlo - Nov 26, 2012 10:50 AM
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Today we looked at the first of two advanced topics (which won't be on our final exam, but are on the MLC syllabus!), interest rate risk.

We reviewed yield curves, spot and forward rates, and saw how to apply the yield curve to insurance and annuity EPVs (just replace v^t with $1/(1+y_t)^t$).

We talked about the law of large numbers and found the mean and variance of the sum of N identical random variables X_i. This led us to the definition of diversifiable risk - a distribution X that has $SD(\sum X_i)/N$ go to 0 as N goes to ∞. With a diversifiable risk, we can increase the number of policies to reduce the variance. But with a non-diversifiable risk (such as interest rate risk), unfortunately that is not the case.

We won't spend any more time on this topic; we just touched on it briefly. If you want to learn more, check out Chapter 10.

Lecture 32 - Nov 23

Diana Katherine Skrzydlo - Nov 23, 2012 3:21 PM
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Today we derived expressions for EPVs of benefits under the common shock model. It turns out that for an annuity, we can just calculate the EPV of an annuity with independent * mortalities, but at a modified interest rate which takes into account both δ and λ. We can do a similar thing for insurances but we get a slightly more complicated result. I didn't show this, but it works the exact same way for discrete benefits as well. We would just replace v with $ve^{-\lambda}$.

In addition, we derived a useful result which works for any status u that is whole life or endowment insurance type (paying on the minimum of two times, no matter which one is first), possibly including any number of lives. We already know if for x and x:angle-n, but it holds more generally, and that is that $\tilde{a}_u = (1 - \tilde{A}_u)/\delta$, or equivalently $\tilde{A}_u = 1 - \delta\tilde{a}_u$. This would also hold in the discrete case with d instead of δ.

Finally, we looked at some examples of interpreting weird multiple life notation, including three lives and reversionary annuities that also have term limits.

Lecture 31 - Nov 21

Diana Katherine Skrzydlo - Nov 23, 2012 3:13 PM
Mark Unread
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Today we talked about the common shock model, which is one way of introducing dependence between lives, to make it more realistic.

Each life experiences their own individual mortality (now denoted with *'s), but also there is a force of accident/shock acting on them together that would cause them both to die at the same time. That force λ is assumed to be constant, so it does not depend on age. Thus, the time until death from shock has an exp(λ) distribution.

Then we can express the probabilities of the joint life status "xy" surviving t years as the product of the probabilities that x survives, y survives, and the shock does not occur. That is, ${}_tp_{xy} = {}_tp^*_x\, {}_tp^*_y\, e^{-\lambda t}$. Also, we can easily see ${}_tp_x = {}_tp^*_x\, e^{-\lambda t}$ and ${}_tp_y = {}_tp^*_y\, e^{-\lambda t}$, which unfortunately means that ${}_tp_{xy} \neq {}_tp_x\, {}_tp_y$.

We looked at the same example from last time, where we assume the individual * mortalities follow the numbers we had last time, but in addition x and y are jointly subject to a common shock with parameter λ = 0.0005. We get the following results (again, if you can't get them or need more explanation, post here):
${}_2p_{xy} = 0.8935$
${}_2p_{xy\text{-bar}} = 0.9962$
${}_{1|1}q_{xy} = 0.0566$
${}_{1|1}q_{xy\text{-bar}} = 0.0027$

Lecture 30 - Nov 19

Diana Katherine Skrzydlo - Nov 19, 2012 1:50 PM

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Today we developed the expressions for the EPVs for insurance benefits for multiple lives (joint life, last survivor, and contingent) and saw the relationships between them.

Then we talked about the case where lives are independent, including what assumptions need to be made (and how realistic they are). If we assume independent lives, the math works out nicely because $\mu^{0.}_{xy} = \mu_x + \mu_y$ and hence ${}_tp_{xy} = {}_tp_x\, {}_tp_y$. We looked at an example and I gave you the numerical answers as follows:
If $q_x = 0.02$, $q_{x+1} = 0.025$, $q_{x+2} = 0.03$, $q_y = 0.03$, $q_{y+1} = 0.035$, $q_{y+2} = 0.04$, and x and y are independent lives, then ${}_2p_{xy} = 0.8944$, ${}_2p_{xy\text{-bar}} = 0.9972$, ${}_{1|1}q_{xy} =$ ~~0.0526~~ **0.0562**, and ${}_{1|1}q_{xy\text{-bar}} = 0.0022$. If you have any trouble getting to these numbers, reply here and I'll show more details.

Finally, we talked about how the independent lives assumption applies in the case of Gompertz and Makeham mortality models. With Gompertz, we can calculate an equivalent single age and then just use that age to calculate any benefit EPV or probability needed. With Makeham, we can find an equivalent equal age and then use a table of multiple life benefit EPVs (with two independent lives that are the same age) to value any benefit EPV even if the ages are different. The Illustrative Life Table is actually based on Makeham, and I posted the third and final page of it (with multiple life functions) in Content.

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Re: Lecture 30 - Nov 19

Student - Nov 19, 2012 8:10 PM
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Hi Diana,

Should the answer for ${}_{1|1}q_{xy}$ be 0.0562 instead of 0.0526? and may I now how can I compute ${}_{1|1}q_{xy\text{-bar}}$?

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Re: Lecture 30 - Nov 19

Diana Katherine Skrzydlo - Nov 19, 2012 11:49 PM
Mark Unread
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Yes, you're right, that was just a typo. It should be 0.0562 not 0.0526. I fixed it in the original post. Thanks!

As for ${}_{1|1}q_{xy\text{-bar}}$, there are several ways to think about it.
Probably the easiest is to use the relationship that we know for all xy-bar statuses: "xy-bar = x + y - xy" and then get ${}_{1|1}q_{xy\text{-bar}} = {}_{1|1}q_x + {}_{1|1}q_y - {}_{1|1}q_{xy}$

Another option is to express ${}_{1|1}q_{xy\text{-bar}} = p_{xy\text{-bar}} - {}_2p_{xy\text{-bar}}$

Or you can think about what it means to have the status xy-bar expire between time 1 and time 2 - there are three possibilities:
both are alive at time 1, both die between time 1 and 2
x dies before time 1, y dies between time 1 and 2
y dies before time 1, x dies between time 1 and 2
Then sum up the probabilities of each of those three cases:
${}_{1|1}q_{xy\text{-bar}} = p_{xy}\, q_{x+1:y+1\text{-bar}} + q_x\, {}_{1|1}q_y + q_y\, {}_{1|1}q_x$

All three methods will give you the same result, 0.0022. Can you think of any other methods I might have missed?

Lecture 29 - Nov 16

Diana Katherine Skrzydlo - Nov 16, 2012 2:41 PM

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Today we reviewed the various statuses we can have with multiple lives, and some of the relationships between the probabilities.

We then talked about the different kinds of benefits that can be offered in a multiple life situation. For insurance, we have joint life, last survivor, and contingent; for annuities, we have joint life, last survivor, and reversionary. Each of those contracts can be whole life or have a term limit, and can be continuous, discrete, or 1/m-ly. The notation for the EPVs of the benefits is consistent with the single life notation.

We developed some formulas and relationships for all the types of annuities, and logical explanations for the relationships. Next time we'll do the same for insurances.

Lecture 28 - Nov 14

Diana Katherine Skrzydlo - Nov 14, 2012 5:42 PM

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Today we finished up talking about multiple decrement models by looking at what happens when we have decrements that occur discretely (usually at year end). We can either specify a separate row in the multiple decrement table for that exact age, or include it with the usual format, but only apply the decrement *after* all other decrements have taken effect for that year.

Then we began our discussion of multiple life functions. We looked at both the MSM model representing it, and the random variables T_x and T_y and their multiple life versions, T_{xy} = min(T_x, T_y) also known as the "joint life" and $T_{xy\text{-bar}}$ = max(T_x, T_y) also known as the "last survivor". We defined lots of probabilities (and some new notation) as well as how to relate those probabilities to the MSM probabilities we know.

Next time we'll see what kinds of benefits can be provided under this model and how to calculate their EPVs.

Lecture 27 - Nov 12

Diana Katherine Skrzydlo - Nov 12, 2012 10:43 AM
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Today we talked about the opposite of the last class - building a MDM from individual SDMs.

We got an expression for the dependent probabilities in terms of the independent probabilities (with a little trick of calculating $p^{00}_x = p'^1_x * p'^2_x * ... * p'^n_x$), which holds if we assume CFT or UDD in the MDM. Then we get $p^{0j}_x = \log(p'^j_x)/\log(p^{00}_x) * p^{0.}_x$.

But, we could also assume that each SDM has UDD hold (which makes a bit more logical sense). In that case, we get a more complicated result for the dependent probabilities - see the supplementary note for details.

We then did an example of taking apart a MDM, changing something, and putting it back together. The spreadsheet for it is posted.

Lecture 26 - Nov 9

Diana Katherine Skrzydlo - Nov 9, 2012 5:00 PM
Mark Unread
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Today we looked at an example of calculating premiums (and benefit EPVs in general) from a MDM using a multiple decrement table. Again, the same principles apply, and policy values are straightforward because there is only one active state.

Then we looked at how we can split a multiple decrement model into associated single decrement models and obtain independent probabilities of making a transition to each state j. Theoretically, the independent probability of going to j within t years is ${}_tq'^j_x = 1 - \exp\{-\int\mu^{0j}_{x+r}dr\}$, which just corresponds to the usual definition of ${}_tq_x$ in a 2-state model with only states 0 and j. This will always be different (larger) than the dependent probability of going to j within t years in the MDM, ${}_tp^{0j}_x$.

If all we have is a discrete table for our MDM, then we need a fractional age assumption to estimate the independent probabilities. We showed that if we assume CFT, we get the result that $q'^j_x = 1 - (p^{00}_x)\wedge(p^{0j}_x/p^{0.}_x)$, where each of the p's on the right side can easily be obtained from the multiple decrement table. Interestingly, we get the same result for the independent probabilities if we assume UDD in the multiple decrement table. The proof of that is in the supplementary notes.

Lecture 25 - Nov 7

Diana Katherine Skrzydlo - Nov 7, 2012 10:41 AM

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Today we talked about the fractional age assumptions we can make with a multiple decrement table. We get two slightly different results if we assume UDD (uniform distribution of decrements) or CFT (constant forces of transition), NOT the same result - I was thinking of what we're going to be doing on Friday.

For $0 \leq t \leq 1$ and $j = 1, 2, \ldots n$:
With UDD, ${}_tp^{0j}_x = t(p^{0j}_x)$
With CFM, ${}_tp^{0j}_x = (p^{0j}_x/p^{0\cdot}_x)(1 - (p^{00}_x)^t)$

We looked at an example of a double decrement table and calculating some probabilities. Next time we'll see how to calculate premiums and policy values using MDM, and how to split a MDM into individual models.

Lecture 24 - Nov 5

Diana Katherine Skrzydlo - Nov 5, 2012 2:54 PM

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Today we finished off MSM in general with the description of the Thiele differential equations for policy values. They are very similar to the Alive -> Dead case, but now we have to worry about benefits on switching states and also a change in the policy value we need to hold if we switch to another state.

Then we started talking about multiple decrement models (MDMs). These are special cases of MSMs where there is only one active state and n absorbing states. Because of this, there is a maximum of one transition that can occur, and we can easily obtain expressions for all ${}_tp^{ij}_x$'s (either directly or by using the KFEs, which are very simple). If the transition forces are simple, we can actually get closed-form expressions for all those probabilities. If not, we can use a multiple decrement table (the MDM equivalent of a life table) for integer ages, along with a fractional age assumption.

Next time we'll see how to use these tables and what results we get with the different fractional age assumptions.

Please note a correction has been posted for Assignment 2 question 2.

Lecture 23 - Nov 2

Diana Katherine Skrzydlo - Nov 2, 2012 1:22 PM
Mark Unread
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Today we talked about premiums and policy values in the multiple state model context.

We still use the same principles for calculating premiums and policy values, with one small change - now the policy value at time t depends on *which* state the policyholder is in, since there can be multiple ways a policy can be "in force" at that time.

We looked at the Healthy-Disabled-Dead model for an example, and were able to calculate the premium for a 20-year term benefit (paying a lump sum on death and an annuity while disabled), as well as the policy value at time 10 for both the healthy and disabled states. Note the significant difference in the value of ${}_{10}V$!

Next time we'll see how the Thiele DE works in the multiple state model case, and start on multiple decrement models. Have a great weekend!

Lecture 22 - Oct 31

Diana Katherine Skrzydlo - Oct 31, 2012 3:56 PM
Mark Unread
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Happy Halloween!

Today we talked about cases where the KFEs are actually useful (unlike the last class, where they were more complicated than we needed). If we have states that we can leave and re-enter, or forces of transition that depend on the time/age, then we often cannot get analytical expressions for the ${}_tp^{ij}_x$'s. One example is the Healthy-Sick-Dead model, where the policyholder can move back and forth between the Healthy and Sick states any number of times. The KFEs are still relatively simple, even though the expression for ${}_tp^{ij}_x$ is quite complex.

The other thing we can do is use the KFEs to approximate the probabilities by using the same trick we did with the Thiele DE. Choose a small time step h, and then the DE is approximately equal to $({}_{t+h}p^{ij}_x - {}_tp^{ij}_x)/h$. Then we can use our boundary conditions that ${}_0p^{ii}_x = 1$ and ${}_0p^{ij}_x = 0$ for $j \neq i$, and recursively build up values of ${}_tp^{ij}_x$ where t is a multiple of h.

Finally, we talked about the possible benefits payable under a MSM, and how to calculate their EPVs. We have
- annuities ãijx which pay continuously whenever (x) is in state j, given that they start in state i
- insurances Ãijx which pay on the instant of any transition into state j (not necessarily directly from i!) given that (x) starts in state i.
We can calculate the EPV by using the same principle as for the Alive->Dead model: sum or integrate over all possible payment dates t, the amount paid x discount x probability of payment.

Next time we'll look at how to calculate premiums and policy values using MSMs.

Lecture 21 - Oct 29

[Diana Katherine Skrzydlo](#) - Oct 31, 2012 3:46 PM
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Today we looked at the Healthy-Disabled-Dead model in more detail, with constant forces of transition.

We found that we could write the KFEs quite easily since many of the terms were 0, but that doesn't help too much since we get a system of DEs to solve.

Instead, with this model we can actually get explicit expressions for all of the 9 possible tpijx's in terms of t. We can use the main result for the cases where j = i (since in this model, none of the states can be re-entered once left, so the ii and ii-bar cases are the same). Then for the other probabilities we can condition on the time of first transition. Let that time be r, find the probability of the path from i to j with the first transition happening at r, and then integrate r from 0 to t. Finally, we use the fact that at any time t, the state must be one of {0,1,2} so certain sets of probabilities must add up to 1.

Then I had a bad cough so we ended class a bit early. Sorry, I hope no one catches my cold!

Lecture 20 - Oct 26

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Diana Katherine Skrzydlo - Oct 31, 2012 3:43 PM
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Today we proved our main result for MSM, which gives us ${}_tp^{ii\text{-bar}}_x = e^{-\int \mu i\cdot_{x+r} dr}$.

We also derived the Kolmogorov Forward Equations (KFEs), a set of differential equations for each ${}_tp^{ij}_x$ (where j can be equal to i, or not.) Logically, we can consider these equations as the rate of going into state j at exact time t minus the rate of going out of state j at exact time t, given that we started in state i at time 0. We get $d/dt\ {}_tp^{ij}_x = \sum_{k\neq j} ({}_tp^{ik}_x\ \mu^{kj}_{x+t} - {}_tp^{ij}_x\ \mu^{jk}_{x+t})$. In practice, many of the terms in the KFE will be 0.

Lecture 19 - Oct 24

Diana Katherine Skrzydlo - Oct 24, 2012 11:23 AM
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Today we introduced the notation that we need to work with multiple state models (MSMs), and saw how they relate to the Alive->Dead model notation. We have:
${}_tp^{ij}_x$ = prob of going from state i at age x to j at age x+t (where i can equal j, or not)
${}_tp^{ii\text{-bar}}_x$ = prob of staying in state i the entire time from age x to age x+t
μ^{ij}_x = force of transition from i to j at age x (where i cannot equal j)
$\mu^{i\cdot}_x$ = total force of exit from state i

Then we derived three preliminary results relating to these quantities. We'll use them to derive our main result next time, which is the multiple state equivalent of ${}_tp_x = e^{-\int \mu_{x+r} dr}$

Lecture 18 - Oct 22

Diana Katherine Skrzydlo - Oct 22, 2012 11:00 AM
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Today we started Chapter 8 - multiple state models

We looked at several examples of policies where the usual Alive -> Dead case would be insufficient to model the benefits, including accidental death benefits, critical illness insurance, disability income replacement, policies that depend on two lives, and an employee pension plan. We described the model that would be needed in each case.

We need to make some assumptions to actually use these models - some logical ones such as starting in state 0, and at any time being in exactly one of the listed states, and some technical ones including the Markov property, probability of two transitions in a short interval, and differentiability.

Next time we'll introduce the notation and start proving some results for these models.

Lecture 17 - Oct 19

Diana Katherine Skrzydlo - Oct 19, 2012 11:07 AM
Mark Unread
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Today we finished off the topic of policy values by talking about policy modifications. If the policyholder requests a change to their policy at time t, the insurer can use the cash value C_t (usually a % of ${}_tV$ or AS_t) to recalculate the modified benefits or premiums in the future, using the principle:
C_t + EPV@ t of future modified premiums = EPV@ t of future modified benefits + expenses.

Then we reviewed all the topics in the first half of the course, in preparation for Test 1 next Wednesday.

Lecture 16 - Oct 17

Diana Katherine Skrzydlo - Oct 17, 2012 11:17 PM
Mark Unread
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Today we looked again at the example of calculating asset shares. (It turns out that I had typo'd the calculation of the premium last class, so make sure you have the updated numbers for P, ${}_5V$, and AS_5. All the expressions are correct though.) The spreadsheet with all the details is posted.

We saw how to calculate AS_5 in detail, as well as how to break down the surplus (or loss) per policy into pieces coming from interest, mortality, and assumptions by changing one item at a time from assumed to actual. The order in which we do the calculations does matter, but not a huge amount.

Finally, we looked at how to deal with contracts where the policy value itself is given as the benefit. Sorry I ended up rushing a bit at the end, but if you went to the tutorial there was another example there in detail. Especially if the benefit is the entire policy value ${}_{t+1}V$, it becomes really easy to deal with.

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Re: Lecture 16 - Oct 17

Student - Oct 23, 2012 9:52 PM
Mark Unread
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I was reading the notes, and in the our spread sheet for ASt, there is an entry, (death + expenses paid).

When we calculate the entry, why do we include premium instead of death benefit in the calculation?

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Re: Lecture 16 - Oct 17

Diana Katherine Skrzydlo - Oct 24, 2012 8:51 AM
Mark Unread
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Because during the deferred period (which includes the first 5 years of the contract), the death benefit is the return of premiums paid. You'll notice that it's P in the first year, 2P in the second year, etc, because that's how much they would have paid by that point.

Lecture 15 - Oct 15

Diana Katherine Skrzydlo - Oct 15, 2012 5:26 PM
Mark Unread
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Today we started talking about some advanced topics. We started with asset shares.

The asset share at time t is similar to the retrospective policy value, but is based on the actual experienced mortality, interest, and expenses during the period (0, t). It represents the amount per policy that the insurer actually has (rather than the amount they need, which is the policy value).

We started to look at an example which we will continue next time. We calculated the premium, time 5 policy value, and time 5 asset share. Next time we'll see how to split up the effects that interest, mortality, and expenses have on the profit/loss.

Lecture 14 - Oct 12

[Diana Katherine Skrzydlo](#) - Oct 12, 2012 4:09 PM
[Mark Unread](#)
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Today we looked at a couple more examples of identifying the contract from a Thiele DE. The boundary conditions often give us a lot of the key details, so watch for those. Someone asked a very good question after class - for the single premium contracts, why is ${}_0V = P$ instead of 0 like it should be. That's an excellent point, what I actually meant was that $\lim_{t\to 0+} = {}_{0+}V = P$. That is, the policy value just *after* inception is equal to the single premium, since it's paid at time 0 (which we usually consider to be just afterwards).

We then looked at how we can use the Thiele differential equation to get an approximation for policy values in small step sizes. We choose a small h (the smaller the better) and equate the DE with $({}_{t+h}V - {}_tV)/h$, then solve for what we want. We looked at an example of doing that in order to find the Premium with a continuous endowment insurance and a step size of h=0.01. The spreadsheet is posted and you can play around with the step size to see how that affects the premium.

Lecture 13 - Oct 10

[Diana Katherine Skrzydlo](#) - Oct 11, 2012 12:41 AM
[Mark Unread](#)
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Today we talked about the Thiele differential equation for policy values. It's basically the continuous equivalent of the recursions we developed for annual and 1/m-ly contracts. We can use it to determine how the policy value changes over time, and also use it as an approximation over small intervals.

We derived it by letting ${}_tV$ = EPV at t of future benefits - premiums (where both EPVs are integrals since everything is continuous), then doing a change of variable, applying the fundamental theorem of calculus on the right hand side, and using the product rule on the left hand side.

The differential equation we end up with is $d/dt\ {}_tV = \delta\ {}_tV + (P_t - e_t) - (S_t + E_t - {}_tV)\mu_{x+t}$. This can be interpreted as the change in policy value at time t is caused by (a) interest accruing at the instantaneous rate δ (b) premiums (minus expenses) coming in at a constant rate, and (c) NAAR being used to pay death benefits if a policyholder dies (which happens at instantaneous rate μ_{x+t}).

We looked at a couple of examples of identifying the contract details from the DE, and we can also identify the DE from the contract details. We'll see more examples on Friday.

Lecture 12 - Oct 5

Diana Katherine Skrzydlo - Oct 5, 2012 10:49 AM
Mark Unread
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Today we looked in more detail at calculating policy values between payment dates. I posted the Illustrative Life Table (from the Bowers textbook) and also a spreadsheet for the specific example we looked at.

We calculated ${}_{20.25}V$ for a whole life insurance for (40) exactly using both methods, and found the same result. We also talked about the shape of the graph, having discontinuities at payment dates, and specified in more detail what the policy value looks like over time for general whole life, endowment insurance, and term insurance contracts.

Finally, we saw how to use linear interpolation and linear interpolation with interest to easily approximate the policy value between payment dates. The key is to remember not to interpolate between ${}_tV$ and ${}_{t+1}V$, but between ${}_tV$+P and ${}_{t+1}V$, to take the discontinuities into account.

Have a great long weekend!

Lecture 11 - Oct 3

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Diana Katherine Skrzydlo - Oct 3, 2012 11:58 AM
Mark Unread
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Today we saw how to use the approach to finding the variance and distribution of Lt for an endowment insurance, and talked in a little more detail about what is required when the variables for benefits and premiums don't match.

We also started to look at calculating policy values between payment dates for non-continuous contracts. There are two approaches to calculating it exactly: start with the payment date just after the time we want, discount backwards, and adjust for the people who might have died; or start with the payment date just before the time we want, accumulate forwards, and adjust for the people who we know didn't die.

Next time we'll see both of those methods in action for a specific case, based on the illustrative life table (from the Bowers textbook) which is posted. We'll also look at methods for approximating policy values between payment dates. (That will be everything you need to do the assignment, which is also posted.)

Lecture 10 - Oct 1

Diana Katherine Skrzydlo - Oct 1, 2012 12:11 PM
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Today we talked about a few different things.

First, we looked at how we can use the NAAR to find the gain or loss due to mortality in a particular year, if we know the number of lives in force at the beginning of the year and the actual number who die during the year.

Then we revisited the 1/m-ly case, which is the same as the annual case for recursions, except we have to remember to adjust the recursion accordingly if the benefits and premiums are payable on different frequencies.

Finally, we started looking at the continuous case, where benefits are payable immediately on death and premiums are payable continuously. Here everything is again the same principles as annual, but even easier because we never have to worry about a particular date having a payment on it. We showed in the whole life case that we can express $L_t|T_x>t$ in terms of a simple function of the random variable T_{x+t}, and use that to get the mean, variance, and even the distribution function of L_t.

Lecture 9 - Sept 28

[Diana Katherine Skrzydlo](#) - Sep 28, 2012 12:41 PM
[Mark Unread](#)
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Today we revisited the recursive relationship for policy values. We can also express it in terms of the NAAR (net amount at risk), which is the difference between the benefit + expenses that would be paid at time t+1 and the policy value at time t+1. It measures the sensitivity of the contract in that year to mortality changes - if the NAAR is higher, more people dying is worse for the insurer.

We looked at an example of calculating the annual NAARs for both a 5 year endowment insurance and a 5 year term insurace, and not surprisingly found that the term insurance is more sensitive to mortality risk.

Finally, we looked at calculating policy values for contracts with non-annual payments. Everything is exactly the same, but recursions become even more useful because all we need are the probabilities of surviving each 1/m of a year and we can find the policy value at any multiple of 1/m years.

Next time we'll let m go to infinity and see what happens in the continuous case!

Lecture 8 - Sept 26

[Diana Katherine Skrzydlo](#) - Sep 26, 2012 10:34 AM
[Mark Unread](#)
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Today we talked about retrospective policy values and policy value recursions.

If everything happened exactly according to the assumptions in the basis (unlikely), then the policy value calculated retrospectively will equal the usual definition of the policy value. We won't be seeing a lot of retrospective policy values, but they can sometimes be useful when the prospective method (PV @ t of future benefits - premiums) is difficult. The retrospective formula is (${}_0V$ + PV @ 0 of premiums in (0,t) - PV @ 0 of benefits in (0,t))/${}_tE_x$.

Just like we have recursive relationships between A_x and A_{x+1}, we also have recursive relationships between ${}_tV$ and ${}_{t+1}V$. We proved the relationship in the whole life case, but the most general case is: $({}_tV + P_t - e_t)(1+i_t) = q_{x+t}\,(S_{t+1} + E_{t+1}) + p_{x+t}\,{}_{t+1}V$. This can be interpreted as the policy value at t plus net income at t, accumulated for one year, is equal to the expected amount required to pay death benefits (if the policyholder died) or provide the policy value for the next year (if the policyholder survived).

Lecture 7 - Sept 24


Diana Katherine Skrzydlo - Sep 24, 2012 1:52 PM
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Today we continued the example from last time, looking in detail at both the net and gross premium policy values at times 10 and 20 under two possible policy value bases: same as the premium basis, and same but with 5% interest instead of 6%.

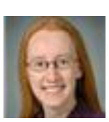
Not surprisingly, the policy values when we use 5% are larger than when we use 6% for the interest rate in the policy value basis. This is because the 5% interest assumption is more conservative, so we would need more money to meet our future obligations.

The one number I didn't have calculated was the net premium policy value at time 10 for when the policy value basis is the same as the premium basis. In that case, P" (the artificial premium calculated with no expenses) is P" = $10000A_{50}/ä_{50:15}$ = 3358.68/9.256917 = 362.83 and hence the net premium policy value is ${}_{10}V^n = 10000A_{60} - P"ä_{60:5}$ = 4568.085 - 362.83*4.22367 = 3035.61.

Summary:
When policy value basis is the same as the premium basis
${}_{10}V^n = 3035.61$, ${}_{10}V^g = 2989.97$
${}_{20}V^n = {}_{20}V^g = 5861.87$
When policy value basis is 5% interest instead of 6%
${}_{10}V^n = 3387.15$, ${}_{10}V^g = 3501.56$
${}_{20}V^n = {}_{20}V^g = 7687.99$

We then looked at why articificial premiums are even used, since they seem to be an irrelevant calculation. The reason is that if everything is on the same basis, we can greatly simplify the calculation of a policy value by using the relationships between A's and ä's.

Lecture 6 - Sept 21


Diana Katherine Skrzydlo - Sep 21, 2012 11:02 AM
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Today we saw where the money comes from to supply the policy value based on the example from last day. If the interest and mortality exactly follow the assumptions, the amount the insurance company has (per surviving policyholder) at time 1 is exactly the same as the amount that is needed for each policy value at time 1.

We also talked about the two different kinds of policy values - Net Premium Policy Value (NPPV) and Gross Premium Policy Value (GPPV). They are both calculated on the policy value basis (set of assumptions) but NPPV uses a recalculated premium and no expenses whereas GPPV uses actual premiums and expenses.

We looked at an example that we'll continue next time.

I made a mistake in the very last line of the lecture actually. What I wrote was "$L_{20} = 10{,}000\ A_{70}$" but what I meant was "$L_{20} = 10{,}000\ v^{K_{70}+1}$". 10,000 A_{70} would represent the expected value of L_{20}, or ${}_{20}V$. Sorry about that!

Lecture 5 - Sept 19

Diana Katherine Skrzydlo - Sep 19, 2012 11:12 PM

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Today we started Chapter 7 by talking about the basic idea of policy values.

We defined the time t future loss random variable L_t = PV at t of future benefits - PV at t of future premiums (similar to L_0), conditional on the contract being in force at time t. Benefit payments at time t are considered to have already been paid in the (very recent) past, and premium payments at time t are considered to be coming in the (very near) future.

We derived L_0 and L_1 for a 5-year endowment insurance, and noticed that even with the equivalence principle premium, the expected value of L_1 is not zero, which means the future premiums do not cover the future benefits, if the policy is in force at time 1. The amount of shortfall, $E[L_t|\text{in force at t}]$, is what we define as ${}_tV$, the policy value at time t.

Lecture 4 - Sept 17

Diana Katherine Skrzydlo - Sep 17, 2012 10:43 AM

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Today we finished reviewing ACTSC 232 by talking about annuities and premiums.

Annuities are quite similar to insurances in terms of the principles and techniques we can use to calculate EPVs and variances. We again described the methods to calculate and the relationships between EPVs both within and between payment timings. There are a couple of differences - with annuities we can have payments at the beginnings or ends of the periods, and for variance we cannot just calculate the second moment so easily by replacing v with v^2.

The way we calculate premiums depends on the Loss-At-Issue random variable L0 which is PV benefits - PV premiums. To calculate P based on the equivalence principle, simply set E[L0] = 0 and hence EPV premiums = EPV benefits. If we want to include expenses, we use the same technique with the gross loss-at-issue rv, and set P so EPV premiums = EPV benefits + EPV expenses.

Lecture 3 - Sept 14

Diana Katherine Skrzydlo - Sep 14, 2012 10:41 AM
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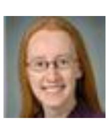
Today we reviewed Insurances (chapter 4) in more detail.

We talked about the present value random variable Z (as a function of T_x, K_x, or $K^{(m)}_x$) and what it looks like in the case of various contracts. We can always find the mean of Z (expected present value) by either first principles or taking amount x discount x probability of payment and summing/integrating over all possible payment dates. When we have a discrete model, the A_x (whole life EPVs) are often provided with the table, and we derived the recursive relationship between them. We'll see lots more recursions in this course! Lastly, we talked about the relationships between the benefit EPVs, both within a particular payment timing (term + pure = endowment ins, term + def whole = whole, etc) and between the different timings (using UDD).

Next time we'll talk about annuities (most of it is the same as this stuff) and premiums.

Some people have asked for some practice problems for the review quiz, and I will put some up soon. I'll try to do it today or over the weekend.

Lecture 2 - Sept 12



Diana Katherine Skrzydlo - Sep 12, 2012 10:36 AM
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Today we reviewed survival models in more detail.

We talked about the random variables T_0 and T_x and the functions associated with them (F, S, and f) and their relationship. We defined the force of mortality and its meaning and relationship to the other functions, actuarial notation, and (briefly) moments of T_x. Finally, we talked about life tables and how to use them to calculate survival or mortality probabilities for integer values, as well as the two different assumptions (UDD and CFM) if we want non-integer values. We also briefly mentioned select and ultimate tables.

Lecture 1 - Sept 10

Diana Katherine Skrzydlo - Sep 12, 2012 9:21 AM
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Today we went over the course outline.

We started off by brainstorming a list of the concepts from ACTSC 232. We'll continue to flesh these out in the next couple of classes. We had 4 major topics: Survival Models, Insurance, Annuities, and Premiums. If anyone took notes in electronic form and feels like posting the list here, that would be great.
