Topics in Computability:
A Meeting in Honor of

Richard A. Shore

January 21st and 22nd, 2007.
MIT, room 2-105

Program

Sunday

9:00am-10:00am registration/treats

10:00am-10:50 am Carl Jockusch "Complexity of chains and antichains in computable partial orderings"

11:00am-11:50am Robert Soare "The Foundations of Computability Reexamined"

12:00pm-2:00pm lunch

2:00pm-2:50pm Anil Nerode "Logic and Control"

3:00pm-3:50pm Stephen Simpson "Reverse Mathematics and Π^1_2 Comprehension"

4:00pm-4:30pm break/treats

4:30pm-5:20pm Julia Knight "Index sets for classes of high rank structures"

7:00pm Dinner at Legal Sea Foods

Monday

9:00am-10:00am treats

10:00am-10:50am Manny Lerman "The existential theory of the jump-usl of degrees with least element is decidable"

11:00am-11:50am Ted Slaman "Undecidability of the α-degrees"

Abstracts

ROBERT I. SOARE. *The Foundations of Computability Reexamined.* In this lecture we shall examine the foundations of computability and the implications for current research. The canonical wisdom presented in most computability books and historical papers is that there were several researchers in the early 1930s working on various precise models for a computable function and that they should all share approximately equal credit. This is incorrect. It was Turing alone who: (1) gave the first convincing model for an intuitively computable function; (2) gave a precise demonstration that this formal model captured the intuitive notion; and (3) defined the universal machine, which is of immense importance. In contrast, Churchs demonstration of "Churchs Thesis" was flawed (see Sieg [1994]). We present a number of quotes by Gödel in which he repeatedly gave credit to Turing and only to Turing, in particular not to Church [1936] and "Churchs Thesis" which Godel regarded as "much less suitable for our purpose." The November issue of the *Notices A.M.S.* is devoted to Turing, but the statements there and in other standard references are not entirely accurate and do not give Turing enough credit.

Turing went on to introduce the single most important concept in Computability Theory today for its applications in both theoretical and practical computability (not the Turing a-machine of 1936). It is surprising how few textbooks present this fundamental concept properly, if at all, and even then only very late in the book. This is like delaying the definition of a continuous or differentiable function to the end of a calculus textbook and then giving it confusing or misleading definition. We consider the question of what are the most important concepts in calculus and analysis and how are they studied. What are the analogous notions in Computability Theory and how are they developed? This helps us to recognize which concepts and questions are the most fundamental today and which are less essential in the broader mathematical spectrum.

MANUEL LERMAN, UNIVERSITY OF CONNECTICUT. *The existential theory of the jump-usl of degrees with least element is decidable.* In his retiring ASL President's address, Richard Shore surveyed the status of knowledge about degree structures. Among the results mentioned is the one we will be discussing; the paper mentioned was in preparation at the time, and is now complete.

A *jump uppersemilattice with least element* (jlusl) is an upper semilattice with least element which supports a jump operator, i.e., a unary function that is order-preserving and maps each element to one that is strictly larger. $\langle \mathbf{D}, \mathbf{0}, \vee, ' \rangle$, the usl of Turing degrees with least element $\mathbf{0}$ and the jump operator is a jlusl. **Theorem:** The elementary theory of the jlusl of Turing degrees is decidable. The proof proceeds by defining the concept of *finite support* for a jlusl, and showing that every jlusl with finite support can be embedded into the Turing degrees. This embedding is, in fact, into the REA degrees and hence into the arithmetical degrees, so the elementary theory of each of those jlusls is also decidable. We will present an outline of the proof, which uses the "iterated trees of strategies" framework developed by Lempp and the author, without going into the technical details of the framework.

THEODORE A. SLAMAN. *Undecidability of the α-degrees (jointly with Chong Chi Tat).* We will review the basics of interpretability within degree structures and how such techniques are applied. Attempting to adapt technology which was designed for the Turing degrees to the α-degrees, for a general Σ_1-admissible ordinal α, leads quickly to obstructions and impasses. We can adapt enough of the Turing degree technology to give an interpretation of finite structures within the α-degrees and thereby conclude that the theory of the α-degrees is undecidable. However, our approach makes essential use of finiteness and leaves open the question of general interpretability.

Anil Nerode. *Logic and Control*. We summarize past connections between logic and control and try to predict future developments.

Designing logic rules to control physical devices goes back to the 1900-1940 design of magnetic relay circuit controllers for elevators, subways, and telephones, and Howard Aitkins early relay computers at Harvard. Technical college students still learn digital circuit design, which is digital control based on logical primitives implemented in chips. The 1960s rule-based medical systems are logic-based transducers with physician symptoms as input and diagnoses as output which control the treatment of the patient. The 1970s-1980s PROLOG Planning systems extract PROLOG answer substitutions interpreted as plans to meet goals and constraints.

The 1950s saw the development of linear programming (Dantzig) and dynamic programming (Bellman), each designed to produce optimal plans, each widely used to this day. The connections of linear programming and integer programming to logic are extremely close.

The 1970s saw the development of logics of time for describing the evolution of computer states and for verifying that programs being executed had desirable properties. This has been a very productive long-term source of new logics and applications of logics to computers and networks.

Continuous linear controllers for continuous linear systems were developed at the MIT Radiation Laboratories for World War II radar based on earlier work of Bell Labs on feedback linear amplifiers in the 1930s. The main tools were linear differential equations and the Laplace transform to express characteristics in the frequency domain. This became the main methodology used for control of continuous processes to the present day. There were mathematical breakthroughs–Pontryagins optimal control for necessary conditions and Youngs measure valued control for sufficient conditions, both stemming from calculus of variations. Neither has yet had wide practical impact. The problems of extracting optimal controls for non-linear systems remain very challenging.

In the 1970s and 80s PROLOG (also fuzzy control chips and state charts) were used to control highly non-linear systems for which there is no linear controller, using piecewise linear controls. But no methodology was offered to extract such programs from the system description and its goals.

The 1990s saw the systematic development of theories of digital controllers for physical systems (Hybrid Systems, Discrete Event systems). We regard the fundamental problem of hybrid systems as finding algorithms which extract digital controllers. These controllers must cause the underlying system to meet its constraints and its goals.

W. Kohn and I developed a methodology based on the calculus of variations on Finsler manifolds, which we have used in applications for ten years. It extracts near-optimal piecewise linear controls which are implemented by a finite automaton.

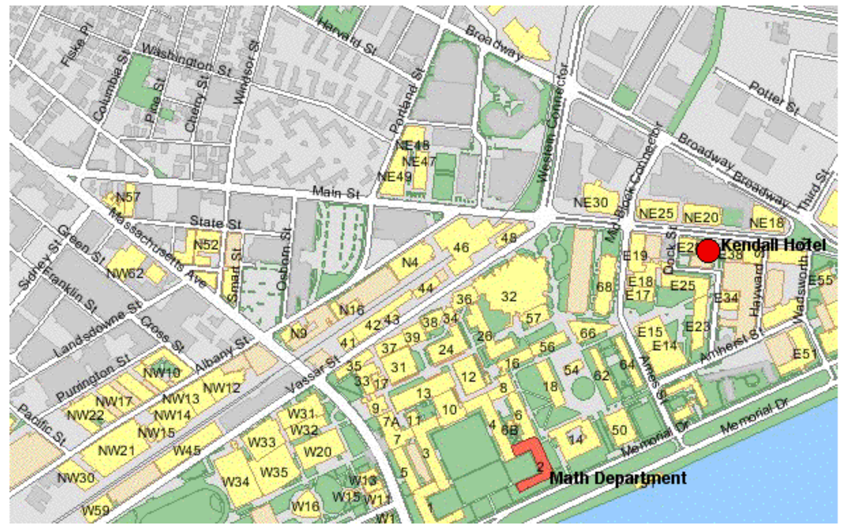
There are two separate problems. One is extraction, the other is verification. We will discuss the significance of each. We will argue that all subjects mentioned above should be parts of the same investigation.

Although by now a large number of logics have been proposed for hybrid systems, none yet have the clarity of PROLOG as described above, where answer substitutions are the plans (control programs). We outline the problems involved in developing languages and semantics which cover all the applications and technology above, and are actually useful.

Julia F. Knight. *Index sets for classes of high rank structures*. I will describe joint work with Wesley Calvert, Ekaterina Fokina, Sergey Goncharov, Oleg Kudinov, Andrei Morozov, and Vadim Puzarenko. We calculate in a precise way, the complexity of the index sets for three classes of computable structures: the class K of all structures of non-computable Scott rank, the class K_1 of structures of Scott rank ω_1^{CK}, and the class K_2 of structures of Scott rank $\omega_1^{CK}+1$. We show that $I(K)$ is m-complete Σ^1_1, $I(K_1)$ is m-complete Π^0_2 relative to Kleene's $\mathcal{O}$, and $I(K_2)$ is m-complete Σ^0_2 relative to $\mathcal{O}$.

Carl Jockusch. *Complexity of chains and antichains in computable partial orderings*. I will outline a simplified version of the proof of Herrmann's theorem that there is a computable partial ordering of ω with no infinite Δ^0_2 chains or antichains. I will also discuss recent joint work with Julia Knight and Valentina Harizanov. For example, we show that a computable analogue of the compactness theorem fails for computable chains and antichains in computable partial orderings. In addition, I will discuss some results from the recent paper "Combinatorial principles weaker than Ramsey's Theorem for pairs", by Hirschfeldt and Shore (to appear in JSL).

Stephen G Simpson. *Reverse Mathematics and Π^1_2 Comprehension*. This is joint work with Carl Mummert. We initiate the reverse mathematics of general topology. We show that a certain metrization theorem is equivalent to Π^1_2 comprehension. If P is a poset, let $\mathrm{MF}(P)$ be the space of maximal filters on P. Here $\mathrm{MF}(P)$ has the obvious topology generated by basic open sets $N_p = \{F \in \mathrm{MF}(P) \mid p \in F\}$, $p \in P$. An MF space is defined to be a topological space of the form $\mathrm{MF}(P)$. If P is countable, we say that $\mathrm{MF}(P)$ is countably based. The class of countably based MF spaces can be defined and discussed within the subsystem ACA_0 of second-order arithmetic. One can prove within ACA_0 that every complete separable metric space is regular and is homeomorphic to a countably based MF space. We show that the converse statement, "every regular, countably based MF space is homeomorphic to a complete separable metric space," is equivalent to Π^1_2-CA_0. The equivalence is proved in the weaker system Π^1_1-CA_0. This is the first example of a theorem of core mathematics which is provable in second-order arithmetic and implies Π^1_2 comprehension.



- Conference venue: Math Department 2-105.
- Conference hotel: Kendall Hotel, 350 Main Street, Cambridge, Tel: 617 577-1300.
- Registration/treats: Math department 2-290
- Banquet: Legal Sea Foods, 2 Cambridge Center (across the street from the Kendall Hotel).