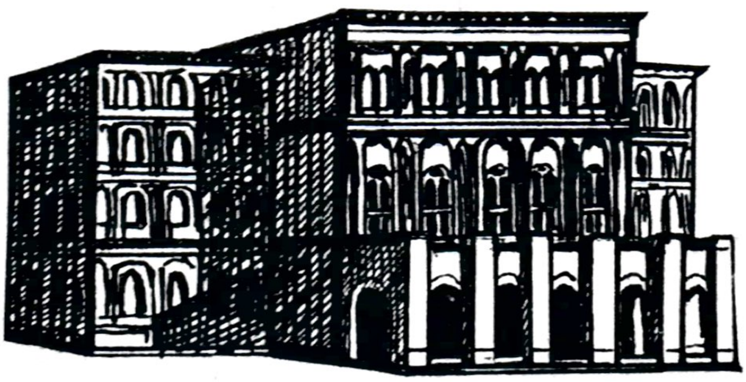
MATEMATICAL PROGRAMMING SOCIETY

ABSTRACTS

IX.
INTERNATIONAL SYMPOSIUM ON MATHEMATICAL PROGRAMMING

BUDAPEST - HUNGARY

August 23-27, 1976



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I.M. Stancu-Minasian: ON THE MULTIPLE MINIMUM RISK PROBLEM.

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A.B. Templeman: OPTIMALITY CRITERIA AS MATHEMATICAL DUALS.

J.Terno: BRANCH AND BOUND AND SHORTEST PATHS.

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L.Toint - F.M.Callier: ON THE ACCELERATING PROPERTY OF AN ALGORITHM FOR FUNCTION MINIMIZATION WITHOUT CALCULATING DERIVATIVES.

J.A.Tomlin: A SURVEY OF RECENT ADVANCES IN MATHEMATICAL PROGRAMMING SYSTEMS.

K.Tone: ON OPTIMAL PATTERN FLOWS.

L.E.Trotter - D.B. Weinberger: SYMMETRIC BLOCKING AND ANTI-BLOCKING RELATIONS FOR GENERALIZED CIRCULATIONS.

Hoang Tuy: IMPROVED ALGORITHMS FOR CONCAVE PROGRAMMING UNDER LINEAR CONSTRAINTS.

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S.Vajda: CONTROL OF MANPOWER SYSTEMS BY LINEAR PROGRAMMING.

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Béla Vizvári: GENERALIZED LAGRANGE MULTIPLIERS IN INTEGER PROGRAMMING.

R.V.Valqui Vidal: DESIGN OF A PIPELINE ROUTE. AN OR APPROACH TO AN ENGINEERING DESIGN PROBLEM.

A.Volpentesta: FINITE GROUPS AND INTEGER LINEAR PROGRAMS. AN UNIFYING SURVEY AND SOME NEW PROPERTIES.

S.Walukiewicz - I. Kaliszewski: TIGHTER EQUIVALENT FORMULATIONS OF INTEGER PROGRAMMING PROBLEMS.

F.Weinberg: A NECESSARY AND SUFFICIENT CONDITION FOR THE AGGREGATION OF LINEAR DIOPHANTINE EQUATIONS.

D.de Werra: ON OPTIMIZATION IN EDGE-CHROMATIC SCHEDULING.

R.J.B. Wets: THE OPTIMAL RECOURSE PROBLEM.

J.Wijngaard: AN APPLICATION OF DYNAMIC PROGRAMMING TO A BUILDING GAME WITH DOMINOES.

H.P.Williams: THE ECONOMIC INTERPRETATION OF DUALITY FOR PRACTICAL MIXED INTEGER PROGRAMMING PROBLEMS.

A.B. Yadkin: A PARAMETRIC APPROACH TO THE SOLUTION OF LINEAR AND QUADRATIC PROGRAMMING PROBLEMS.

H.Yamashita: A DIFFERENTIAL EQUATION APPROACH TO NONLINEAR PROGRAMMING AND ITS APPLICATION TO GLOBAL OPTIMIZATION.

H.P. Young: POWER,PRICES AND INCOMES IN VOTING GAMES.

A.Zilinskas: ON ONE-DIMENSIONAL MULTIMODAL OPTIMIZATION.

S.Zionts: CHOOSING AMONG ALTERNATIVE DECISIONS INVOLVING MULTIPLE CRITERIA.

N.N. Abdelmalek, National Research Council, Ottawa, Ontario, Canada K1A 0R6. MINIMUM L-INFINITY SOLUTION OF UNDERDETERMINED SYSTEMS OF LINEAR EQUATIONS.

The problem of obtaining a minimum L-infinity solution of an underdetermined system of consistent linear equations is reduced to a linear programming problem. A modified simplex algorithm is then described. In this algorithm no conditions are imposed on the coefficient matrix and minimum storage is required. The algorithm is an efficient and fast one. A numerical example is detailed.

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NONLINEAR PROGRAMMING : CONSTRAINT REGULARIZATION

This paper is concerned with the Kuhn-Tucker Necessity Condition in Mathematical Programming and its various constraint qualifications. The main question considered is : "Given that a set of constraints do not satisfy a given constraint qualification, is it always possible to modify the set by adding to it redundant constraint and thereby force it to satisfy the constraint qualification?"
The answer to the above question is shown to be "Yes" for some, but "No" for other constraint qualifications.
Also, necessary and sufficient conditions for the additional set of constraints to be finite are given.
Certain consequences of the work, including a new constraint qualification and a new necessity criteria which are extensions of the Kuhn-Tucker-Necessity Condition are given.

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A GRADIENT TYPE METHOD FOR SEARCH OF AUGMENTED LAGRANGIAN FUNCTION SADDLE-POINT

The convex program

$$f(x) \to \min, \quad Ax = b, \quad x \in Q \qquad (1)$$

is considered. Here $f: R^n \to R$ is a continuously differentiable function, A – $m \times n$ matrix, $x \in R^n$, $b \in R^m$, Q is a given closed convex set.

A gradient type method for solving the problem (1), based on augmented Lagrangian function

$$M(x,y) = f(x) + (y, b - Ax) + \frac{k}{2}|b - Ax|^2,$$

$k > 0$, $(x,y) \in Q \times R^m$ is discussed.

It is proposed to use the following iterative process for search of a saddle-point of $M(x,y)$ (and hence for solving the problem (1)):

$$x^{n+1} = P_Q(x^n - \alpha \nabla_x M(x^n, y^n)), \qquad (2)$$
$$y^{n+1} = y^n + k \nabla_y M(x^{n+1})$$

where $\nabla_x M(x,y) = \nabla f(x) - A^*(y + k(b - Ax))$, $\nabla_y M(x) = b - Ax$, and P_Q – projection operator on Q.

It is proved that under noise conditions the process (2) converges to a saddle-point of $M(x,y)$. Disturbances of gradients $\nabla_x M(x,y), \nabla_y M(x,y)$ as well as of projection operator at each step supposed to be less than a specified δ_n such that $\sum \delta_n < \infty$. Furthermore it is supposed that the step length α satisfies the following condition $0 < \alpha < \frac{2}{L + 2k\|A^*A\|}$ where L is a Lipschitz constant for $\nabla f(x)$.

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and
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MASTER SEMIGROUP POLYHEDRA

Let $(S,\tilde{+})$ be a finite commuticative semigroup and b an element of S. We define the Master Semigroup Polyhedron E (S,b) as the convex hull of the integer vectors $(t_s \in N : t_s \geq 0, s \in S)$ satisfaying $\tilde{\sum}_{s \in S} t_s . s = b$ /where $\tilde{\sum}$ is the iteration of $\tilde{+}$ and $t_s . s = \tilde{\sum}_{i=1}^{t_s} s$). This definition includes covering polyhedra: Convex hull of integer solutions $t \geq 0$ of $At \geq b$ where A is a matrix and b is a vector both with non-negative integer entries, the columns A_j of A are all the diferents vectors less than of equal to b. The semigroup here is $(\{S : 0 \leq S \leq b\}$, S integer vector$\}, \tilde{+})$ with $S \tilde{+} p = \min (S + p, b)$ (the minimum is taken component by component). It also includes Gomory's Master Group Polyhedra.

Given a polyhedron P, we call de β-polar of P de set $P^{\beta} = \{v: yx \geq 1$ for all $x \in P\}$. We say that P is β-closed when $P = P^{\beta\beta}$.

We obtain the following characterization of β-closed semigroup polyhedra:

<u>Theorem:</u> E (S,b) is β-closed if and only if there exists an integer $k > 1$ sluch that $b = k.b$.

<u>Definition:</u> $\bar{S}$ is a b-complementor of s when $s \tilde{+} \bar{s} = b$ and $s \tilde{+} \Gamma = h \tilde{+} s = b$ implies $\Gamma \tilde{+} h = b$. $(S,\tilde{+})$ is b-complementary when every element in S has a b-complementor /groups and covering semigroups are b-complementary and moreover the b-complementors are unigue/.

We present several theorems characterizing the facets of $E(S,b)$ as vertices of highly structured polyhedra e.g.

<u>Theorem:</u> Let $(S,\overset{+}{\sim})$ be a b-complementary semigroup. We have:

1. All the facets of $E(S,b)$ are of the form $x \geq 1$ or $x_s \geq 0$.

2. $\pi x \geq 1$ represent a facet of $E(S,b)$ if and only if π is a vertex of

$$\{\pi \geq 0 : \pi_b = 1;\ \pi_s + \pi_r \geq \pi_{s \overset{+}{\sim} r} \text{ for all}$$

$$s, r \in S : \pi_s + \pi_{\bar{s}} = 1,$$

$$\text{for all } \bar{S} \text{ a b-complementor of } S\}.$$

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DIRECT DIFFERENTIAL IDENTIFICATION OF COMPARTMENTAL SYSTEMS BY LINEAR PROGRAMMING

A new approach is presented to the direct differential method of identification /method originally proposed by R.Bellman/ that seems to overcome some of the main objections formulated against this estimator.
The characteristics of the approach are as follows:
1./ The derivatives are estimated by approximating the experimental data with cubic spline functions.
2./ The parameter estimation from the m set of n linear simultaneous equations is performed by linear programming, under the norm L1.
The main improvement, with respect to the original approach, seems to be the parameter estimation via linear programming, which easily permits the introduction of constraints on the parameters value, to avoid a meaningless solution, and the derivative estimation via base function, that avoid the error amplifications of the numerical derivation.
A numerical experiment and an analysis of data are given to show the reliability and the practical applications of the proposed technique.

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COMPUTATIONAL EXPERIENCE AND APPLICATIONS WITH THE GEOFFRION'S ALGORITHM

The paper describes an algorithm for the solution of descrete problems with zero-one variables.The algorithm is based on the Geoffrion's implicit enumeration technique.The author introduces several modifications reducing the computation time and storage requirements.Imbedded dual linear program generating surrogate constraints is incorporated within the iterative scheme.Sparsity techniques are employed using sophisticated compacting scheme for the non-zero elements in the integer tableau.Reinversion subroutine with static ordering of the inverted basis of the dual problem is developed.A reordering scheme of the variables in the partial solution according to the values of the coefficients of the current surrogate constraint is also constructed.The algorithm is tested with different in size and type integer problems and the computational experience reported.The results presented in the paper are obtained on ICL I906A and CDC-7600 computers.The program is written in FORTRAN IV language.

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MINIMIZATION OF CONVEX FUNCTIONS WITH ERRORS

Numerical methods are proposed for solving convex optimization problems where are known only approximatively the values of the objective function and its sub-differential. Applications of these methods are also given.

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SECOND ORDER CHARACTERIZATIONS OF PSEUDO-CONVEX FUNCTIONS

A function f(x) is called pseudo-convex in $C \subseteq R^n$ if for all $x, y \in C$

$$(y - x)^T \nabla f(x) \geq 0 \quad \text{implies} \quad f(y) \geq f(x).$$

For twice differentiable functions necessary as well as sufficient criteria for pseudo-convexity are derived. In contrast to those for convex functions these criteria involve not only second order but also first order derivatives. Special attention is given to quadratic functions.

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MATHEMATICAL MODELS FOR THE LOCATION OF GOVERNMENT

This paper describes some models to assist with locations of Government Departments developed for the Property Services Agency of the British Government.

The models are concerned with allocating Departments to locations to minimize the combined costs of accommodation, dislocation (i.e. moving Departments from their present location) and communication.

Our first model groups parts of Departments into clusters, using an extension of Euclidian Cluster Analysis, so that Departmental groups communicating extensively with each other are allocated to the same cluster as far as possible. We then use mathematical programming to allocate clusters to locations. The methods are an extension of those described by E.M.L.Beale and J.A. Tomlin (1972) "An Integer Programming approach to a class of combinatorial problems". Mathematical Programming 3 pp 339-344.

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INTEGER LINEAR PROGRAMMING OVER CONES USING GENERALIZED FUNDAMENTAL POINTS

We consider the usual integer linear programming problem, relaxing some of the nonnegativity constraints, that is

$$\min \{c'x \;/\; A_I x \leq b_I\} ,$$

where A is an (m+n,n) integer matrix and A_I is a submatrix of A containing all rows whose indices belong to the index set I.

We show that this problem can be rewritten as

$$\min \{d'f(\lambda,\mu) \;/\; f(\lambda,\mu) = b,\ \lambda \in X(f),\ \mu \in \mathbb{N}\}$$

where $f \in F$ and F contains all functions f which are linear in μ and affine in λ, and X(f) is a set of special fundamental points corresponding to f. We show how $f \in F$ can be chosen so that the resulting problem attains its minimum for $\mu = o$, and characterize the corresponding fundamental points. An algorithm is presented along with computational comparisons with usual ILPC-methods, and it is shown, that this algorithm works best if (and only if) usual group methods are not applicable (because of the size of the determinant).
An extension is proposed for general ILPC-Problems using this algorithm as a subroutine.

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MULTICRITERIA DECISION MAKING IN DESIGN OF ELASTIC STRUCTURES

Though the design procedure as a whole is mostly conceived as an art, important parts can be formalized and treated by mathematical programming techniques. Such an application is design of elastic structures, whose main task is to carry loads with a least amount of materials, minimum cost of construction or high stiffness while satisfying certain constraints, such as strength and fabrication limits. In this vector optimization problem

$$\text{"minimize"} \; \{f(x) \,|\, g(x) \geq 0,\; h(x) = 0\}$$

the objective $f: R^n \to R^m$, and constraints $g: R^n \to R^p$, $h: R^n \to R^q$, are assumed to be twice continuously differentiable but not necessarily convex.

It is shown that multicriteria decision making not only reduces the artistic parts of designing and determines efficient solutions but that also quite general rules applicable to a broad class of problems can be derived. In this context the performance of the commonly used appoaches is discussed and suggestions for most appropriate techniques, such as interactive preference defining during the computational process, are made. The impact of multicriteria problems on optimization methods designed for single criteria problems is studied.

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ПРИНЦИП ИТЕРАТИВНОЙ РЕГУЛЯРИЗАЦИИ ПРИ ПОСТРОЕНИИ МЕТОДОВ ПРИБЛИЖЕННОГО РЕШЕНИЯ МОНОТОННЫХ ВАРИАЦИОННЫХ НЕРАВЕНСТВ

Многие экстремальные проблемы / минимизация выпуклых функционалов, нахождение точек равновесия бескоалиционных игр и т.п. / сводятся к решению так называемого вариационного неравенства, т.е. нахождения такого элемента $x \in Q \subseteq B$, что

$$< F(x), x-z > \leq 0 \ , \forall z \in Q \qquad (*)$$

Здесь Q выпуклое замкнутое подмножество банахова пространства B, F - оператор из B в его сопряженное B^*.

Минимально необходимое требование на F для построения содержательной теории $(*)$ - монотонность отображения F: $< F(x_1)-F(x_2), x_1-x_2 > \geq 0, x_1 x_2 \in Q$. Обычно при разработке приближенных итеративных методов решения $(*)$ требование монотонности усиливают тем или иным способом. Предлагается универсальный прием построения сильно сходящихся итеративных методов решения $(*)$ в том случае, когда F просто монотонен и никакого усиления этого свойства предполагать нельзя.

Этот прием / итеративная регуляризация/ состоит в следующем. Рассматривается вспомогательное неравенство

$$< F(x) + \varepsilon M(x), x-z > \leq 0 \quad z \in Q, \varepsilon > 0 \quad (**)$$

$M(x)$ сильно монотонный оператор из B в B^*.

Неравенство $(**)$ обычно обладает существенно лучшими свойствами по сравнению с $(*)$. В частности его уже можно решать "стандартными" итеративными методами.

Предлагается, назначив $\varepsilon = \varepsilon_1$, сделать один шаг к.л. итеративного метода решения $(**)$. Затем, заменив ε на ε_2 $(\varepsilon_2 < \varepsilon_1)$ сделать шаг итеративного метода с новым ε_2 и т.д.

Для большого числа итеративных методов решения $(**)$ можно так априорно назначить $\{\varepsilon_n\} \downarrow 0$, что полученая последовательность точек из Q сильно сходится к одному из решений $(*)$.

Эта общая схема порождает большое число новых методов решения задач математического программирования, нахождения седловых точек, точек равновесия и т.д., обладающих существенно большими возможностями, чем существующие.

Egon Balas, Carnegie-Mellon University

SOME RECENT DEVELOPMENTS IN ZERO-ONE PROGRAMMING

In recent years a number of interesting results have been obtained in the direction of characterizing or approximating 0-1 programming polytopes by a system of linear inequalities. We discuss some of these results and their practical significance. Particular algorithms are discussed for several classes of 0-1 programs.

Egon Balas, Carnegie-Mellon University

Nicos Christofides, Imperial College of Science and Technology, London

A NEW PENALTY METHOD FOR THE TRAVELING SALESMAN PROBLEM

We discuss an algorithm for the traveling salesman problem based on the approach of solving a sequence of assignment problems with the cost function successively modified so as to finally make the optimal assignment a tour. Computational experience will be discussed.

M.L. Balinski, International Institute for Applied Systems Analysis, Laxenburg, Austria,

and

H.P. Young, Graduate Center, City University of New York, USA

MULTIPLE OBJECTIVES IN APPORTIONMENT: AN AXIOMATIC APPROACH

The problem of apportionment is to find non-negative integers $a_1,\ldots,a_m$, $\sum_1^m a_i = h\ (>0)$ for all h "proportional to" the positive /"populations"/ integers $p_1,\ldots,p_m$ for any such p's.

Many objectives concerning what is meant by "proportionality" in this case can be set. Differing ideas of "fairness" are implied. This talk will report on some characterizations of approaches to the problem.

J.L.Balintfy, School of Business Adm.Univ.of Mass, Amherst

A STOCHASTIC PROGRAMMING MODEL OF THE SELECTIVE MENU PROBLEM

Food preferences are represented by preference state probability vectors and the incidence of choosing or not choosing an item are represented by two types of corresponding transition matrices. The first type is reducing the expected preference in one step while the second one is increasing it step by step on a discrete time scale. The choice probabilities can be computed at each stage of this process by Luce's axiom and it is proved that the expected preference of a choicegroup is always greater than the convex linear combinations of the items. The optimal selective menu is planned by maximizing the expected preference of a set of choice-groups while maintaining structural, cost and nutritional feasibility by a musltistage extension of the Armstrong-Balintfy algorithm of multiple choice programming with joint probabilistic constraints.

Richard H. Bartels, The Johns Hopkins University, Department of Mathematical Sciences, Baltimore, Maryland

Christakis Charalambous, The University of Waterloo, Department of Combinatorcs and Optimization, Waterloo, Ontario,

and

Andrew R. Conn, The University of Waterloo, Department of Combinatorics and Optimization, Waterloo, Ontario.

MINIMIZATION TECHNIQUES FOR PIECEWISE DIFFERENTIABLE FUNCTIONS: THE ℓ_∞ SOLUTION TO AN OVERDETERMINED LINEAR SYSTEM

An algorithm is presented for computing a vector x which satisfies a given m by n $(m > n \geq 2)$ linear system in the sense that the ℓ_∞ norm of the residual vector is minimized. That is, letting $a_1, \ldots, a_m$ be the columns of a matrix A, each column being of length n, and letting $\beta_1, \ldots, \beta_m$ be the components of a vector b, we wish to find a vector x which minimizes

$$\phi(x) = \| A^T x - b \|_\infty = \max_i | a_i^t x - \beta_i | .$$

The proposed algorithm minimizes the function directly in a finite number of steps. It is related to Cline's algorithm for the above problem as well as the algorithm due to Conn and Charalambous for nonlinear optimization.

The algorithm represents an improvement over others which proceed by directly minimizing the function ϕ in that it employs recent advances in the use of fast Givens' transformations to update orthogonal matrix factorisations. This permits the work per cycle in the algorithm to be reduced from $O(n^3)$ operations to $O(n^2)$ without sacrificing numerical stability.

The algorithm also incorporates the flexibility and simplicity of the approach taken to a related problem; that of finding the ℓ_1 solution to an overdetermined linear system by Bartels, Conn and Sinclair.

A.S.J. Batchelor and E.M.L. Beale, SCION Computer Services LTD, Milton Keyness, England,

A REVISED METHOD OF CONJUGATE GRADIENT APPROXIMATION PROGRAMMING

This paper describes a revision of the method described by Beale (1974) for solving large-scale nonlinear programming problems. The method was originally developed as a variant of the Method of Approximation Programming due to Griffith and Stewart (1961), but it is perhaps more illuminating to think of it as a variant of the Reduced Gradient Method, first introduced in Wolfe (1963) and generalized to nonlinear constraints by Abadie and Carpentier (1969).

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SPALTENGENERATION IN DER QUADRATISCHEN OPTIMIERUNG

1. Im Zusammenhang mit Dekompositionsmethoden vom Typ des Verfahrens der zulässigen Richtungen zur Lösung von großen linearen Optimierungsaufgaben entsteht beim Aufsuchen der lokal besten Richtung die quadratische Optimierungsaufgabe

$$\inf \{ \|a\|^2 : a \in \partial F(\bar{x}) \} \qquad , \qquad (1)$$

wobei

$\|.\|$ - die Euklidische Norm des R^n bezeichnet und

$\partial F(\bar{x})$ - eine konvexe polyedrale Menge des R^n ist, von der im allgemeinen nur ein Extremalpunkt und eine implizite Beschreibung der Menge $\partial F(\bar{x})$ gegeben sind. $\partial F(\bar{x})$ hat dabei im allgemeinen eine nicht sehr große Anzahl von Extremalelementen.

2. Im Vortrag wird die numerische Realisierung eines Algorithmus vorgestellt, der es erlaubt, Aufgabe (1) ohne die explizite Gewinnung von $\partial F(\bar{x})$ zu lösen. Der Algorithmus stellt eine Erweiterung des bekannten Dekompositionsverfahrens von Dantzig/Wolfe auf quadratische Optimierungsaufgaben dar und kann mit den meisten Verfahren vom Simplextyp zur Lösung quadratischer Optimierungsaufgaben effektiv gekoppelt werden.

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A COMPLEX OF OPTIMIZATION ROUTINES.

A complex of routines elaborated by the authors is described. The routines are intended for numerical unconstrained minimization of arbitrary functions and of functions being the sums of squares and for solution of linear, quadratic and nonlinear programming problems.

A zero-order and a first-order algorithms are suggested for the unconstrained minimization. The zero-order algorithm uses the coordinate descent, the conjugate-gradient method, the gradient being calculated by means of the function values, and the quadratic method of barycentric coordinates. The algorithm takes use of the advantages of each method, the switching of the methods being provided by the algorithm itself. The accuracy of the solution obtained is rather high. The first-order algorithm is based on the conjugate-gradient method that can be used for the solution of the large-scale problems. The estimations of the extremal eigenvalues of the object function Hessian are calculated according to the current information. A variable-metric conjugate-gradient algorithm is suggested to accelerate the process.

This algorithm requires large memory but its convergence is faster than that of the preceding algorithm. It is intended for solving of small size. A modification of Gauss-Newton's method is suggested for minimization of sum of squares. In the modified method a quadratic functional being the result of the original functional linearization is minimized instead of solving a system of normal equations used in the most schemes. The new algorithm is stable with respect to computational errors and provides convergence for wider domain.

Iterative algorithms suggested for the solution of linear, quadratic and nonlinear programming problems are based on the conjugate-gradient method applied to a modified Lagrange function. Here the two-side conditions on the variables are not involved in the penalty terms and they are taken into account by the conjugate-gradient method itself.

The compact representation of data enables us to solve the large scale problems. The performance of each algorithm is illustrated by test solutions results together with comparison between the methods.

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SOME RECENT APPLICATIONS OF NONLINEAR PROGRAMMING IN MUNICIPAL SERVICES

This talk reviews some of the mathematical models developed in the last few years to improve the delivery of labor-intensive public services, such as sanitation and fire protection. In each case one is faced with nonlinear and integer valued programs whose solutions have yielded significant insights into the operation of these services. Most of this work was performed for the city of New York, but the basic ideas are applicable in a number of contexts. Since much of it was also implemented, resulting in substantial cost benefits to the municipality, we believe that an acquaintance with these models is of interest to mathematical analysts who wish to apply their work to societally relevant problems.

In the two main examples to be discussed the mathematical program is developed from the basic fact that both men and equipment are limited and costly resources to be allocated in such a way that some net benefit be optimized. In actuality, the objective function is vector valued and the solution constitutes a trade off between conflicting goals. However, we formulate the problems in terms of equivalent scalar valued programs. For example, the work crews necessary to pick up uncollected refuse on certain days of the week, when demand is high, are such that other services are degraded as a consequence. The optimal allocation, spatially and temporarily, is therefore one which accommodates between inequities in different sectors of the city and at the same time maximizes service levels where they are most needed. The allocation also requires that a number of restrictions be met, based on labor practices and legal regulations.

Solution techniques are discussed for the models we present, including a way of converting into a linear format problems of the form

$$\min_{\{n_1\}} \Sigma\ f_j(n_1,\dots,n_k), \text{ where } f_{j+1}(n_1,\dots,n_k) = \max\ (f_j(n_1,\dots,n_k),0).$$

A.Ben-Israel, A.Ben-Tal and S. Zlobec, McGill University, Montreal, Canada

CHARACTERIZATIONS OF OPTIMALITY IN CONVEX PROGRAMMING WITHOUT A CONSTRAINT QUALIFICATION

Popular theories of convex programming (such as the Kuhn-Tucker and Fritz John theories) do not characterize optimality unless a certain hypothesis, known as "constraint qualification", is assumed. This talk presents the basic ideas of a recent theory of convex programming where the optimal solutions are characterized without any reference to a constraint qualification. The optimal solutions are characterized by a family of linear inequalities and cone relations.

The importance of these characterizations is demonstrated by examples from multicriteria optimization. Also a new class of feasible direction methods is formulated. Unlike the classical feasible direction methods, these new methods can not terminate at a nonoptimal point and they solve convex programs whether or not Slater's condition (or any other constraint qualification) is satisfied.

O.Benli, Middle East Techn.Univ.Ankara,Turkey
P.Nanda, Syracusa University,Syracusa,New York,USA

A SOLUTION PROCEDURE FOR LOCATION-ALLOCATION-PRODUCTION

A solution procedure based on decomposition principles is presented for location-allocation-production problems. The problem basically is to determine the location of plants /on a point-set/, and their production levels, which receive several raw materials at given proportions from a number of possible supply points and, subject to minimum and maximum production capacities, produce several commodities demended at /piecewise-linear/ production and transportation costs. The problem is formulated as a mixed-integer programming problem. A solution procedure is developed based on "projection, outer linearization/relaxation" /Bender's Partitioning /, whose subproblems can then be solved either by a produce based on "projection/piecewise" /Rosen's Partitioning/ or another procedure based on "projection, outer linearization/relaxation". Some computational results on steel plant location in the Soviet Union are given.
In addition to the applications of this model in plant location problems, this formulation is particularly suited to resource allocation problems in economic planning, interpreting plants as sectors of the economy, raw materials as inputs to sectors and commodities as outputs of vectors, with appropriate interpretation of the costs.

John M.Bennett, University of Sidney,Sidney, Australia

QUADRATIC PROGRAMMING AND PIECEWISE LINEAR NETWORKS, WITH STRUCTURAL ENGINEERING APPLICATIONS

Some linear network problems such as the elasto-plastic analysis of structures can be cast in the form of a quadratic programming optimizations problem. In such cases, the structure of the constraint matrix and of the quadratic form to be optimized can be exploited and lend themselves to a number of computational economies. The process can be looked upon as one which invokes restrictions only when they are relevant, and which does not require the updating of these restrictions. An efficient method for updating the triangular factors of a symmetric matrix which may be regarded as the equivalent of the basis matrix is used.

M. Benichou, J.M. Gauthier, G. Hentges, G. Ribiere, IBM,
Paris, France

THE EFFICIENT SOLUTION OF LARGE SCALE LINEAR PROGRAMMING PROBLEMS: SOME ALGORITHMIC TECHNIQUES AND COMPUTATIONAL RESULTS

First, this paper presents the results of experiments with algorithmic techniques for efficiently solving large scale linear and mixed integer programming problems. The techniques presented here are either original or recent. They are mainly:

- L/U format of the inverse,
- keeping of the L/U format during the primal iterations,
- inserting starting solutions defined by variable values rather than by their status (basic or non-basic). A specific algorithm is able to retrieve a basic solution,
- quickly building a starting basis by a new crashing algorithm,
- accuracy checking using new scaling algorithms and dynamic tolerances,
- choosing the incoming or the outgoing variable by Devex criterions,
- using pseudo-costs for efficiently solving MIP problems with a low proportion of integer variables,
- grouping the integer variables into Special Ordered Sets.

The solution of a great number of large scale problems have shown that efficient problem solving requires automatic adaptation of algorithmic techniques upon problem characteristics. For instance:

- use of L/U Primal for very dense problems and non-L/U Primal for sparse problems,
- use of distinct strategies for searching for integer solutions depending on the percentage of integer variables compared with continuous variable
- decomposition techniques for block models.

The second part of this paper describes an attempt to provide a powerful Mathematical Programming Language, allowing an easy programming of specific studies on medium-size models. In particular, this language permits:

- recursive use of L.P., i.e. the ability to analyze the results in program arrays and to accordingly modify the model before reoptimizing it,
- inexpensive build-up of Mathematical Programming algorithms based on the simplex method. Examples of use are given (Reduced gradient, Cutting plane additional constraints, ...).

All these features have been implemented in MPSX/370, a Mathematical Programming code able to solve large scale LP and MIP models. Extensive numerical results and comparisons on real-life problems are provided and commented upon.

B.Bereanu, Center of Math.Stat.Bucharest, Romania

STOCHASTIC - PARAMETRIC LINEAR PROGRAMS

In this paper it is developed the theory of stochastic-parametric linear programs and some of its applications. This theory leads to the generalized distribution problem of stochastic linear programming, i.e., the problem of finding the distribution and/or some moment of the random variables of a multiparameter stochastic process which represents the optimal value as a stochastic-parametric linear program. A special type of optimal control problem for such stochastic processes leads to various decision type problems of stochastic programming while the usual distribution problem becomes an essential post-optimal analysis.
Feasibility, existence and stability in this general framework are investigated together with various types of control. Computational problems are also discussed.

Bernau Heinz, Computer and Automation Institute of the Hungarian Academy of Sciences, Budapest, Hungary

UPPER-BOUND-TECHNIQUES FOR QUADRATIC PROGRAMMING

In quadratic programming the developing of the upper-bound-techniques is important mainly as computational method. The source of difficulty is the dimension of the tableaux to be transformed. In this subject only a very few publication has been written. Two methods are known;
a./ an application of the Dual-Method of Whinston and
b./ a Primal-Dual- Method of Concalves.

In this paper presenting 3 methods we show that the upper-bound-technique of linear programming can be incorporated directly into the procedures of Beale, Wolfe and Jagannathan for solving quadratic programming problems with upper bounds.

Furthermore, for these modified algorithms a comparison is presented according to some parameters, which are the following; The rank of the positive semidefinite matrix in the objective function; the lengths of intervals from which we choose the components of the vector in the objective function, of the right hand side vector and of the upper-bound vector uniformly distributed.

The test problems were solved on a CDC-3300 computer. In all the 3 methods the product form of inverse is used.

T.D.Berezneva. Central Economics Mathematical Institute, Academy of Sciences of the USSR, Moscow.

THE STRUCTURE OF EQUILIBRIUM LEVELS IN VON NEUMANN TYPE MODELS.

The purpose of this report is to investigate arrangement of equilibrium levels in the von Neumann type model. The von Neumann type model (or model M) is a linear closed dynamic production model determining by means of the pair matrices A, B (of the same dimension m n), which can contain negative elements, i.e.

$$Ax_t \leq Bx_{t-1}, \quad x_t \geqslant \theta, \quad t=1,\dots,T,$$

x_t - n-dimensional vectors.

Let us denote $S(\alpha) = \{ x \geqslant 0 : \alpha Ax \leqslant Bx \}$, $S_0 = \{ x \geqslant 0 : Ax \lneq \theta \}$; $W(\alpha) = \{ p \geqslant 0 : \alpha pA \geqslant pB \}$, $W_0 = \{ p \geqslant 0 : pB \lneq 0 \}$; $\mathcal{N}_0 = \{1,\dots,n\}$, $\mathcal{M}_0 = \{1,\dots,m\}$, $n(\alpha) = \{ i \in \mathcal{N}_0 : \exists x \in S(\alpha) \Rightarrow (Bx)_i > 0 \}$, $m(\alpha) = \{ i \in \mathcal{M}_0 : \exists p \in W(\alpha) \Rightarrow (pA)_i > 0 \}$.

The triplet ($\alpha > 0$, x, p) will be called equilibrium if

$$x \in S(\alpha), \; p \in W(\alpha), \; pBx > 0.$$

A scalar α is called an equilibrium level.

We shall consider the models satisfying the following conditions

1. If $x \in S(\alpha)$, $\alpha \geqslant 0$, then $Bx \geqslant 0$,

or

2. If $p \in W(\alpha)$, $\alpha \geqslant 0$, then $pA \geqslant 0$.

Let us note that if there exists such α_0 that $\alpha_0 = \max_{S(\alpha)\cap S_0 \neq \emptyset} \alpha > 0$ ($\beta_0 = \min_{W(\beta)\cap W_0 \neq \emptyset} \beta > 0$) and $n(\alpha_0) = \mathcal{N}_0$ ($m(\beta_0) = \mathcal{M}_0$) then there exists only one equilibrium level. Otherwise diferent levels exist. However the namber of equilibrium levels does not exceed n, if condition 1 holds, and m, if condition 2 holds.

Let us denote

$$\alpha_i = \sup_{S(\alpha)\cap S_i \neq \emptyset},$$

$$\mathcal{N}_i = \mathcal{N}_{i-1} - n(\alpha_{i-1}), \; S_i = \{ x \geqslant 0 ; (Ax)_k > 0 \text{ for some } k \in \mathcal{N}_i \}, \quad i = 1,\dots,r \leq n.$$

<u>Theorem</u>. If model M satisfies 1, then λ is the equilibrium level of M iff $\lambda = \alpha_i > 0$ for some i.

Analogic theorem is true if model M satisfies 2.

When M satisfies 1 and 2 we cannot require the existense $\max_{S(\alpha)\cap S_i \neq \emptyset} \alpha$, but we can construct a sequence of appropriate submodels maximal equilibrium levels of which define the equilibrium levels of the given model.

D.P. Bertsekas, University of Illinois, Urbana, Illinois, USA

MULTIPLIER METHODS: A SURVEY

The purpose of this paper is to provide a survey of convergence and rate of convergence aspects of multiplier methods for constrained minimization. Recent advances will be emphasized including global convergence and rate of convergence analysis of second order multiplier methods, treatment of two-sided inequality constraints, approximation methods for nondifferentiable optimization, and proximal point algorithms. Detailed expositions may be found in a survey paper appearing in Automatica, March, 1976 and in the literature quoted in this paper.

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A RELATIONSHIP BETWEEN METHODS OF CONJUGATE DIRECTIONS AND QUASI-NEWTON METHODS

In a recent paper McCormick and Ritter consider two classes of algorithms, namely methods of conjugate directions and quasi-Newton methods, for the problem of minimizing a function of n variables F(x). They show that the former methods possess an n-step superlinear rate of convergence while the latter are every step superlinear and therefore inherently superior. In this paper a simple and computationally inexpensive modification of a method of conjugate directions is presented. It is shown that the modified method is a quasi-Newton method and is thus every step superlinearly convergent. It is also shown that under certain assumptions on the second derivatives of F the rate of convergence of the modified method is n-step quadratic.

J. Bisschop and A. Meeraus, Development Research Center, World Bank, Washington, D.C. EFFICIENT UPDATING OF THE BASIS INVERSE IN LINEAR PROGRAMMING VIA PARTITIONING.

Matrix modifications, such as column interchanges, row interchanges or column and row additions, are viewed as matrix augmentations. Primal simplex iterations are treated as a special case, and an extremely compact updating procedure based on partitioning methods is developed. The method is especially suited for large sparse linear programs that are solved entirely in core. The procedure is not related to any of the methods based on the LU decomposition or the product form of the inverse. The new method has the distinct advantage that the growth of additional nonzero elements is not related to the size of the problem but only to the number of iterations following reinversion. It is shown that the representation of the updated inverse does not grow monotonically in size, and that it may actually contract during certain simplex iterations. Implementation of the new procedure is straightforward, and does not require any involved data manipulation activities. Computational requirements differ from iteration to iteration. There is every indication, however, that the method requires on the average more computations per simplex iteration than other existing methods. Stability properties of the new method are considered. Comparisons are made on the basis of real-world problems using a commerical code designed for large sparse linear programming problems.

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TRANSPORTATION NETWORK DESIGN

Different approaches to the design and analysis of transportation networks are discussed. A route selection algorithm is presented that balances fixed link construction costs and variable user costs in a network having a fixed set of nodes and a static service-dependent demand for inter-node service. The algorithm alternately applies link elimination and link insertion criteria which converge to a local optimum. Simplified representations of transportation networks, more amenable to rapid design and analysis, are also discussed.

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Arnoldo C.Hax, Massachusetts Institute of Technology,Mass.USA

ON THE SOLUTION OF CONVEX KNAPSACK PROBLEMS WITH BOUNDED VARIABLES

In this paper we present a recursive method to solve separable differentiable convex knapsack problems with bounded variables. The method differs from the classical optimization algorithms of convex programming and determines at each iteration the optimal value of at least one variable. Applications of such problems are frequent in resource allocation and have recently shown to be useful in hierarchical production planning. Computational results are presented.

R.G.Bland, CORE, Belgium and State Univ.of New York,Binghamton, N.Y.

COMBINATORIAL GENERALIZATIONS OF LINEAR PROGRAMMING DUALITY THEORY

Matroids are a well-known combinatorial abstraction of real matrices. Recently a notion of orientability of matroids has been introduced. We will show that many familiar concepts from linear programming duality theory generalize in the context of dual pairs of oriented matroids. In particular we will discuss an algorithmic proof of a combinatorial generalization of the linear programming duality theorem; in the context of dual pairs of oriented matroids that are representable over the reals, this algorithm specializes to a new finite variant of the simplex method.

N.Bonde, Odense University, Odense, Denmark
J.Tind, University of Aarhus

BOUNDS IN SET PARTITIONING

In this paper we shall discuss different lower bounds for the optimal value of a set partitioning problem. These bounds have been implemented in an implicit enumeration algorithm for this kind of problems. Some experiments have been performed, and they show that the use of such bounds often reduces the total compution time substantially. Additionally a comparison of the relative computational influence of these bounds is given.

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COMPUTATIONAL RESULTS ON LARGE SCALE SET COVERING PROBLEMS USING A SEQUENTIAL GREEDY ALGORITHM

This paper discusses recrnt computational results on large scale set covering problems using a sequential greedy algorithm developed by the authors. The performance of the algorithm is analyzed according to differing cost structures, problem size for obtaining both the optimal solution and "good" approximations to the optimal solution.

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Carnegie-Mellon University, Pittsburgh, USA
and
T.C. Gleason, Wharton Business School, University of
Pennsylvania, Philadelphia, USA

NECESSARY AND SUFFICIENT CONDITIONS FOR PSEUDO-CONVEXITY USING EIGENVALUES

A necessary and a sufficient condition for pseudo-convexity for a twice continuously differentiable function are given in terms of eigenvalues of the Hessian of the function. The results are that the Hessian can never have more than one negative eigenvalue and that the negative eigenvalue is constrained in value by the gradient of the function.

S.P.Bradley - Naval Postgraduate School,Monterey,California USA
R.D.Shapiro - Massachusetts Institute of Technology,Cambridge, Massachusetts,USA
J.M.Mulvey - Harvard University,Boston,Massachusetts,USA

INNOVATIONS IN MATHEMATICAL PROGRAMMING EDUCATION

Although mathematical programming (especially linear programming) has been taught in colleges and universities for over 20 years, there has been little progress in designing and developing new pedagogy for teaching these subjects. The majority of courses still adhere to the age-old lecture system in conjunction with homework, quizzes and tests. In contrast, a number of progressive schools are in the process of introducing new techniques into the "classroom." These innovations take a variety of forms: (1) a learning center where the students utilize professional audio-visual aids; (2) a highly interactive linear programming timeshare system where students can formulate, display and update linear programming models without requiring previous computer programming experience; (3) a self-paced, individually tailored course in mathematical programming fundamentals; and (4) a highly portable audio cassette-based system which can be used anywhere. Each of these systems caters to the specific, and widely varying, needs of the students and the pedagogical styles of the faculty. In this paper, we survey these innovations as well as the effective use of cases in the classroom and other improvements to the more traditional lecture system.

Gordon H. Bradley, Gerald G. Brown, Naval Postgraduate School, Monterey, California, United States and Glenn W. Graves, University of California, Los Angeles, California, United States.

LARGE SCALE NETWORK OPTIMIZATION

The successful development of a large scale primal network computer program for the capacitated transshipment problem is described. The capacitated transshipment problem is the most general of the minimum cost flow models that include the capacitated and uncapacitated transportation problems and the personnel assignment problem. The mathematical development treats networks as a special case of large scale linear programming. Special emphasis is given to the choice of data structures that support efficient computations. Extensive computational results using the authors' widely distributed computer program, GNET, are described. Large scale (more than 25,000 variables) applications are discussed.

E.R.Brocklehurst and K.Dennis, National Physical Laboratory, Teddington,Middlesex, England.

COMPUTATIONAL EXPERIENCE WITH HEURISTIC ALGORITHMS FOR THE PURE AND MIXED INTEGER PROGRAMMING PROBLEMS

Recent experiments have shown that heuristic algorithms can find an optimal solution to an integer programming problem extremely rapidly. The improvements recently made to the algorithm proposed by BROCKLEHURST will be described. Computational experience on a variety of problems will be presented in order to compare the algorithms of BROCKLEHURTS, HILLIER, IBARAKI and TOYODA.

A.H.O.Brown, Rolls Royce Ltd.,Bristol, England

THE DEVELOPMENT OF A COMPUTER OPTIMIZATION FACILITY AND ITS USE IN PROJECT SUPERSONIC ENGINE DESIGN

Many design processes are seen to be ones of constrained optimization using functions developed empirically by sub-groups of an overall design team. The generalization of these processes in a form suitable for the application of non-linear constrained optimization techniques is examined. The Recursive Equality Quadratic Program algorithm due to M.C.Biggs of the National Optimization Centre,Hatfield,U.K. was selected and adapted for purposes of the investigation.
It is illustrated by application to the project design of a bypass engine replacement for the Olympus 593 in the Concorde supersonic aeroplane. A discussion follows, taking the form of a commentary on the development and running of this related problems making more general points as they arise.
Topics covered include permissible simplification of the User program,saving computing time in the numerical assembly of the partial derivatives, the application of the routine when regions exist in the field where the penalty function is incalculable, scaling,the effects of the inter-relation between step lenghts in the partial differencing routine and tolerances within the sub-programs,and the data and logic interface between controlling routine and function sub-programs.
Finally some discussion follows on the specification of the related sensitivity analysis problem as seen by the user. It is realised that particularly in the field of constrained optimization this provides a problem not necessarily met by the usual eigenvalue approach. Some experiments are detailed making use so far as possible of the original optimization routines.

P. J. Brucker, University of Oldenburg, Oldb; Federal Republic of Germany

SEQUENCING UNIT-TIME JOBS WITH TREELIKE PRECEDENCE ON M MACHINES TO MINIMIZE MAXIMUM LATENESS

The problem treated is one of job sequencing on m identical machines, where there is a precedence ordering between certain jobs, as given by a directed graph which is a tree or more generally a forest. The processing times of all jobs are assumed to be equal. Associated with each job i there is a due date d_i. A simple algorithm is given for finding a schedule which minimizes maximum lateness. It generalizes Hu's "Cutting the Longest Queue" algorithm which deals with the case in which $d_i = 0$ for all jobs i. However, the optimality proof developed for the more general algorithm is much shorter.

A second algorithm solves the problem of minimizing make-span under the restriction that no job is late.

For the two machine case it is shown that the schedule produced by the first algorithm also minimizes average completion time. Thus, to solve the problem of minimizing completion time under the restriction that no job is late a schedule with maximum lateness not exceeding zero has to be constructed.

The algorithms may also be applied to corresponding preemtive scheduling problems with jobs having processing times which are not equal.

R. E. Burkard, University of Cologne, Cologne, W. Germany

AN ALGEBRAIC APPROACH TO COMBINATORIAL OPTIMIZATION PROBLEMS

By an algebraization of the objective function is achieved that combinatorial optimization problems with different kinds of objective functions (e.g. sums or bottleneck objective functions) now occur as special cases of one general problem. The algebraization respects not only the structure of the underlying problems but also the structure of algorithms for solving these problems. Combinatorial optimization problems, which can be formulated without real variables (e.g. assignment problems), can be considered now in totally ordered semigroups, where $(S,*,\leq)$ obeys additionally a strong combatibility axiom and a divisibility axiom. For problems with real variables some additional combatibility axioms between the domain Ω of the variables and the semigroup S have to be fulfilled.
After discussing the structure of the systems $(S,*,\leq)$ and $(S,*,\leq;\Omega)$ general assignment problems and maximal flow problems in networks with generalized costs will be considered and algorithms will be given for solving these problems. To derive algorithms for the general problems "admissible transformations" will be introduced and will be characterized for the above mentioned problems.
The algebraic approach allows not only an insight in the structure of the problems but explains also the different numerical behaviour of the different kinds of objective functions. For example can be shown, why bottleneck problems are faster to solve than sum problems.

H.T.Burley, La Trobe University, Melbourne, Australia

PRODUCTIVE EFFICIENCY MEASURES

The efficiency of a firm is generally interpreted to be a measure of its success in producing as large as possible an output from a given set of inputs. If the production processes are linear, and all the inputs and outputs of the firm are specified in physical units, then the measure of this Farrell type productive efficiency is easily achieved within a linear programming framework.

In the primal estimation technique the columns of matrix F are the Leontief production functions of firms, and the right hand side is the factor use by firm t, and vector q details the outputs of each of the n firms in the set being studied.

$$Z_t^* = \max q'x$$

s.t.

$$\begin{bmatrix} F_{11}, & F_{12} & \ldots & F_{1t}, & F_{1n} \\ F_{k1} & & & F_{kt}, & F_{kn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} \leq \begin{bmatrix} F_{1t} \\ \cdot \\ \cdot \\ \cdot \\ F_{kt} \end{bmatrix}$$

and $x \geq 0$

The functional Z_t^* is clearly the maximum output with known technologies that can be produced from input $F_{.t}$. In game theory language the linear program determines if a mixed or pure strategy of other firms in x^* can dominate firm t strategy while using the same resources as firm t.

The measure is sensitive to the specification of inputs, outputs and the population of technologies or firms. In the paper a sensitivity analysis is applied to these three factors and the analysis has been developed to include non-linear and the Boles approach to von Neumann technologies. The measure is dimensionless, avoids index number problems, and does not assume firms face common input prices.

P. L. Buzytsky, Central Economics Mathematical Institute, Academy of Sci. of the USSR, Moscow, USSR

ON THE FINITENESS OF THE PRIMAL INTEGER PROGRAMMING ALGORITHMS

Consider the following integer programming problem:

$$\max\ (a_{00} - \sum_{j=1}^{n} a_{0j}x_j)$$

subject to

$$\begin{aligned} \sum_{j=1}^{n} a_{ij}x_j &\le a_{i0} & i=1,\dots,m \\ x_j &\ge 0, & j=1,\dots,n \\ x_j &- \text{integers}, & j=1,\dots,n \end{aligned} \qquad (1)$$

where a_{0j}, a_{ij}, a_{i0} are arbitrary real numbers and $a_{i0} \ge 0$, $i=1,\dots,m$. The primal algorithms' scheme of solving the problem (1) contains the steps as follows:

1. Select a $a_{0s} < 0$, $s=1,\dots,n$
2. Select a derivative row v and construct the cut

$$\sum_{j=1}^{n} \left[\frac{a_{vj}}{a_{vs}}\right] \le \left[\frac{a_{v0}}{a_{vs}}\right] \qquad (2)$$

3. Fulfil pivoting with pivot column s and pivot row (2) where the rules of selecting the pivot column and the derivative row are fixed.

Theorem. A primal algorithm of solving the problem (1) is finite if and only if it is finite for the following knapsack problem:

$$\max \sum_{j=1}^{n} c_j x_j$$

subject to

$$\begin{aligned} \sum_{j=1}^{n} a_j x_j &\le a_0 \\ x_j &\ge 0, \text{ integers}, \ j=1,\dots,n \end{aligned}$$

where $a_j > 0$, $j=0,1,\dots,n$; a_j, c_j are integers.

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SEQUENTIAL METHODS FOR MATHEMATICAL PROGRAMMING

The general convex quadratic programming problem /C.Q.P./ is considered, for which a lot of algorithms have been proposed.

Initially, such algorithms appeared to be efficient to solve real problems. This notwithstanding, during last years the number of real situatuons formulated as C.Q.P. has gone quickly increasing.

The main consequence of this fact is the necessity to be able to computationally handle large-scale structured C.Q.P.. Another consequence is that, generally, the number of constraints is not a priori known; more precisely, in every real problem the constraints are divided in classes of different importance.

Thus, to be able to treat real problems it is necessary to have methods of resolution which be able to take into account such aspects and the structure of the problem.

Here the idea is considered to obtain an optimal solution x^0 of a C.Q.P. by determining sequentially an optimal solution for a sequence of subproblems /having less constrains of the C.Q.P./, the latest of which has only the constraints, which are binding in x^0.

The particular case of linear programs is considered, and it is shown how to handle by a sequential method the structured cases.

Extensions to other nonlinear problems are discussed.

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POLYNOMIAL BOUNDING FOR NP-COMPLETE MATROID PROBLEMS

A large class of NP-complete combinatorial optimization problems may be reduced efficiently, i.e. by a polynomial algorithm, to k-parity matroid problems and these to the problem of finding an optimum intersection of three matroids, two of which are partition matroids [1] . Branch-and-bound or heuristically-guided-search [2] methods are currently used to solve NP-complete problems [3] and are particularly interesting for this class of matroid problems, since it is possible to use subgradient techniques in order to obtain possibly tight bounds to the optimum which are quite suitable to guide the search [4] . However subgradient methods are not polynomial bounded. In this work a polynomial bounded method is proposed for estimating the value of the optimum, that is the minimum weight of an intersection of three matroids, two of which are partition matroids. At the i-th iteration the method adds to the lower bound, initially set equal to zero, the value of a minimum weight intersection I of the two partition matroids. The cycles of I in the other matroid are then contracted, producing three new matroids over a smaller set of elements. This procedure generalizes the method presented in [5] for computing lower bounds to the lenght of a shortest hamiltonian cycle of a network.

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and

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BRANCH AND BOUND APPROACH TO A FIXED-CHARGE NETWORK EXPANSION

Planning the expansion of a transmission power system involves decisions on the location and specification of additional lines to the network, to satisfy fortcoming loads with the required degree of reliability.

Investment costs are high and the number of possible alternatives to be studied, often on a short time, are considerable.

The purpose of this paper is to suggest a simple model for solving capacity expansion problems in network, when there is a fixed-charge associated with the expansion of each are of the related graph.

Some alternatives have been tested for the choice of the nodes to be expanded in branch-and-bound methods used.

It is also presented some computational comparisons, concerning a given example.

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SEQUENTIAL METHODS FOR MATHEMATICAL PROGRAMMING. THE GENERAL CONVEX QUADRATIC CASE. APPLICATIONS TO STRUCTURED PROBLEMS.

The aim of this paper is first of all to briefly review the sequential method for strictly convex quadratic programs and to extend it to the general convex quadratic case. More precisely, it is shown how to handle the semi-definite case and degeneracy case. Some remarks are made about the linear case.

Moreover the linear and convex quadratic structured programs are considered. It is shown how the sequential method can be extended to such cases in order to take advantage of the structure. The decomposition algorithms are devised.

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MINIMAL RATION SPANNING TREES

The minimal spanning tree problem is well-known and efficient algorithms for solving it exist. One of them is called the "greedy" algorithm by Edmonds. In this paper we consider the following problem:

For a given undirected connected graph G: (N;E) with node set N and edge set E and given numbers C_e and D_e, $e \in E$ find a spanning tree T such that the ratio

$$\left(\sum_{e \in T} C_e \Big/ \sum_{e \in T} D_e \right)$$ is minimized.

An example is provided to show that an immediate generalization of the "greedy" algorithm will not work. The problem is solved parenthetically via a series of minimal spanning tree problems. The algorithm described for solving the above problem is polynomially bounded.

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DECOMPOSITION METHODS FOR MIXED INTEGER PROGRAMS. APPLICATIONS TO STAFFING PROBLEMS.

By means of Farkas' theorem the method of Benders for mixed integer programs is reviewed and used to generate cuts having particular properties. More precisely, a condition is given for a given hyperplane be a Bender's cut. It is shwn how to use such a condition in a resolution procedure.

An application is made to a staffing problem by an airline; such a problem is formulated as an all-integer nonlinear program, which can be decomposed into a set of linear all-integer programs, so that Benders' procedure may be applied.

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AN ALGORITHM FOR MIXED NORM MINIMAX LOCATION PROBLEMS

We consider locating n new points in the plane relative to m existing points such that the maximum weighted distance between any pair of points is minimized. The distance measure between pairs of points can be any ℓ_p norm, $1 \le p \le \infty$. We state the problem in the form $\min_x \max_i f_i(x)$ where $x = (x_1, x_2, \ldots, x_n) \in E^{2n}$, and the f_i are convex, not necessarily differentiable, functions involving ℓ_p norms. We prove convergence, to an ϵ-optimal solution, of a subgradient algorithm which relies on the special properties of the functions f_i.

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HYPERBOLIC OPTIMIZATION PROBLEM

In the present paper some properties of an hyperbolic optimization problem are discussed

$$\sup_{x \in \Omega} \left\{ H(x) = \frac{p_0 + (p, x)}{q_0 + (q, x)} \right\}$$

whoever

$$\Omega = \{ x \mid A'x = b', \; A''x \le b'' \}$$

On the basis of these properties an algorithm is designed. The algorithm is so arranged that to allow a full mathematical investigation of the problem and either an optimal solution or suboptimal and asymptotic solutions, if any at all, are found.

A distinction from the known algorithms is that no preliminery knowledge of additional information concerning the function $H(x)$ and its region Ω is necessary.

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EFFICIENT ALGORITHMS FOR THE MAXIMUM FLOW PROBLEM

This report presents algorithms for maximum flow problem. This problem was formulated in [1]. Also there was suggested algorithm for its solution called Ford-Falkerson algorithm (AFF). But in some cases AFF can lead us to computatinal difficalties. J. Edmonds and R.M. Karp suggested AFF-modification needs $O(np^2)$ primitive operations for solving this problem (n - number of nodes, p - number of arcs in the network). E. Dinic suggested another algorithm for solving maximum flow problem of $O(n^2p)$ operations. Dinic's algorithm decomposes the solving of the problem for sequence of steps. Auxiliary network called reference-network (RN) is found before each of these steps. Reference-network helps more efficient computations [2]. A. Karzanov suggested algorithm of $O(n^3)$ operations using a new method of flow computation in one RN. By using Dinic's and Karzanov's methods the auther succeeded in suggesting the new algorithm of $O(n^2\sqrt{p})$ operations. This boundary is the best of those known to the author. In the report are also the results of experimental computational comparison of these algorithms.

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OPTIMIZATION OF A WATER RESOURCES SYSTEM BY STOCHASTIC PROGRAMMING WITHOUT RECOURSE AND LINEAR RULES

Water resources problems have been of great interest to many researchers in various fields such as mathematical programming, hydraulic engineering, power engineering and economics. In each water resources project, there are three distinct phases: planning, allocation and operation. In this paper, we have studied the Khuzestan water resources system in southwestern Iran. Assuming that the first two phases have been completed for this system, the primary concern is then the optimization of its operation phase.

Modeling of such systems depends strongly on the characteristics of such systems and the data available. On one hand, due to the fact that water is allocated at various locations of such systems over fixed durations of time, a multi-period model is preferred over a continuous-time one. On the other hand, the inputs to such systems, i.e. river inflows, are stochastic by nature and would eliminate the use of deterministic models. Aside from many deterministic modeling attempts, few efforts have been made on the stochastic approach. The common scheme for stochastic programming models uses chance constraints and linear decision rules. However, we have proposed a stochastic programming model with fixed recourse, which is more adaptive to the observed data during the decision process compared with the chance-constraints model. In this approach, a multi-period model is considered for the problem. For the end of each period and for any reservoir, we consider two decision variables: target reservoir levels, and amount of water releases (recourse actions). The former will be determined at the beginning of the period based on forecast inflows, while the latter will be determined at the end of the period based on actual observations. For the inflow statistics of our particular system, we have found that it is appropriate to assume linear decision rules in determining the target reservoir levels based on observed data of the two previous periods. First, from the solutions of a set of linear programming problems, the cost function associated with the set of recourse actions is obtained along with corresponding distribution policies for the downstream areas of the system. Then, the resulting nonlinear deterministic problem can be solved by existing computer packages.

The Khuzestan water resources system consists of three major rivers, five reservoirs, five power plants, seventeen irrigation areas, and fourteen municipal and/or industrial centers. A great deal of data and information were gathered and received through the regional authorities. Among other data, the monthly historical water flows as far back as seventy-two years are used to estimate the relevant probability distributions of the river inflows. The mathematical model includes many constraints associated with power and irrigation demands, flood damage, recreation, navigation and other physical operating constraints. The cost criterion is formulated as functions of penalties and benefits corresponding to water shortages and surplusses, respectively.

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GLOBALLY AND SUPERLINEARLY CONVERGENT ALGORITHMS FOR NONLINEAR PROGRAMMING

In this paper a group of globally and superlinearly convergent algorithms are presented for the general nonlinear program

minimize $f(x)$ subject to $g_j(x) \leq 0$, $j = 1, \ldots, m$ and $g_j(x) = 0$, $j=m+1, \ldots, k$

where $f, g_1, \ldots, g_k$ denote real valued functions on the n-dimensional Euclidean space. We assume that the gradients of $f, g_1, \ldots, g_k$ exist and are Lipchitz continuous on the Euclidean space.

The algorithms minimize the exact penalty function which has been studied by Zangwill, Pietrzykowski, and Howe

$$vf(x) + \sum_{j=1}^{m} \max(0, g_j(x)) + \sum_{j=m+1}^{k} |g_j(x)|, \quad v \; 0$$

when the iterates are far away from the optimal solution. Gradient methods for the unconstrained minimization of differentiable functions are generalized to handle the nondifferentiable exact penalty function. The Armijo stepsize procedure, or the minimum stepsize procedure is used to determine the stepsize. Another contribution is a new method for updating the penalty parameter v of the exact penalty function.

The algorithm switches automatically to quasi-Newton algorithms which are closely related to the the quasi-Newton algorithms having been studied by Wilson, Robinson, Garcia, Mangasarian and Han when the iterates are near the optimal solution.

Fast numerical experience is obtained for the algorithms.

Acknowledgements. This material forms part of my Ph.D. thesis under the supervision of Professor O. L. Mangasarian. I wish to express my sincere appreciation to him for his guidance, encouragement and advice in the preparation of this material. I would like to thank Professor C. N. Cryer, Professor R. R. Meyer, and Professor S. M. Robinson for their valuable suggestions.

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THE APPLICATION OF BIVALENT NONLINEAR PROGRAMMING TO THE LINEAR ELECTRONIC NETWORK SYNTHESIS PROBLEMS

The basic steps in computer-aided electronic network design are: i) the modelling of components; ii) the topological synthesis in which an initial network is found; and iii) the numerical synthesis in which the designable parameters are adjusted so as to achieve the desired performance. If the design is unacceptable the process is repeated from either step (ii) or (iii) This paper deals with the topological synthesis problem . The solution is searched by a suitable nonlinear bivalent programming problem.

Any network found in step (ii) has to satisfy the following conditions: a) its graph is two-connected; and b) the actual transfer function of the network has non-zero coefficients if, and only if the corresponding coefficients of the desired transfer function are non-zero. The topological synthesis problem may be stated as follows: it is desired to find a class of the two-connected weighted graphs. In addition, the unknowns which are related to the edges of the graph have to satisfy several linear inequality constraints (condition (a)) and nonlinear constraints (condition (b)). The values of unknowns are restricted to 0,1 . If an optimization criterion is defined the problem of topological synthesis is converted to the typical nonlinear bivalent programming problem, and we solved it with a slightly modified version of the lexicographic enumeration algorithm of Lawler and Bell.

The main disadvantage of the method is the large storage needed. It is necessary to store very long expressions (condition (b)). If the designed network is allowed to have two types of components only e.g. R and C, then the problem of topological synthesis is greatly simplified. It is possible to substitute the checking of nonlinear constraints by the process of searching for trees and 2-trees with the maximum and minimum weight in a specially constructed family of graphs. The modified method requires only minimum storage.

The final section of the paper will present results of numerical computations.

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ON THE USE OF ZERO-ONE PROGRAMMING FOR FINDING AN OPTIMUM SOLUTION OF A SCHEDULING PROBLEM IN STEELWORKS ENVIROMENT

A mathematical formulation of a scheduling algorithm for the heat treatment problem of a high alloy steel manufacturer will be presented. The algorithm is based on the zero-one programming model with the criterion of maximizing production trhoughput of heat treatment furnaces.

The computer outputs of the developed algorithm will be analyzed and an alternative approach to the problem solution (simulation approach) will be shown.

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SOME PROBLEMS CONCERNING THE STRUCTURE OF MATROIDS

This paper deals with the structure of a matroid and contains some matroid construction problems.

The main result of the first part is a necessary and sufficient condition for the existence of a circuit with cardinality r+1 in a matroid of rank r. The condition found has connection with an exchange property of bases. In this condition some circuits constitute the family of bases of a matroid. The connection with the bases grapf of a matroid is put in evidence.

In the second part we investigate the problem if certain families of sets, given or constructed, form a matroid. Some quantitative estimations of the maximum number of bases of a matroid with rank r satisfying certain conditions are obtained.

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AN INTEGRATED SYSTEM FOR INTERACTIVE NON-LINEAR PROGRAMMING COMPUTATIONS

Existing optimization algorithms are highly formalized and do not allow much human intervention or interaction. This paper dealswith the design and development of software for nonlinear optimization using a collection of well-known nonlinear programming codes: SUMT (Sequential Unconstrained Minimization Techniques), GRG (Generalized Reduced Gradient), GPM (Generalized Projection Methods), and MCL (Method of Centers Linearized).

The system has evolved from "stand-alone" programs to an integrated package which allows the user to define objective function and constraints in a simple FORTRAN subroutine and call one of the algorithms to initiate the solution process. The user can also interact with the process by changing strategies in mid-stream through the use of different algorithms.

Because of this algorithm interrupt mechanism, one can study the structure of the problem or observe the solution process (for example, monitoring precision/convergence/number of function or gradient evaluations). As an experimental tool, it may also be used to study the interaction of algorithms and the best matching of an algorithm to a given class of problems.

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A GENERALIZED CONJUGATE GRADIENT METHOD FOR CERTAIN LARGE STRUCTURED PROBLEMS

We consider a generalized conjugate gradient method for finding the minimum of a non-quadratic function in finite dimensions that has a positive definite symmetric Hessian. The method is based on splitting off from the equivalent system of nonlinear equations for the vanishing of the gradient an approximating positive-definite system that is easily solvable, and then accelerating the associated iteration using (nonlinear) conjugate gradients. The method, which is an extension of our earlier one for the quadratic case, finds application in the solution of problems arising from the discretization of nonlinear elliptic partial differential equations. Its behavior is illustrated for the minimal surface equation, with splittings corresponding to symmetric extrapolated block Gauss-Seidel-Newton iteration and to separable elliptic operators amenable to fast direct methods.

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NASH'S EQUILIBRIUM IN N-PERSON GAMES AND IN ECONOMIC

Considering the equilibrium existence problem of the following systems:
The system consists of n agents. For i-th-agent, the strategie space x_i be a locally convex Hausdorff topological vector space. The allowed situation of the system $x=/x_1,x_2,\dots,x_n/$ will be limited by the requirement that x be selected from a convex,compact set $S \subset \bigtimes_i X_i$ .The objective function $F_i : S \rightarrow R^2$ be a real function and the allow law of movement φ_i : $S \longrightarrow S$ be a set-valued mapping. A situation $x^* \in S$ is said to be φ -equilibrium if

$$x^* = \bigcap_{i=1}^{n} \psi_i(x^*)$$

and for all $i=1,\dots n$ $F_i(x^*) = \max \{ F_i(x^* \| x_i) : (x^* \| x_i) \in \psi_i(x^*) \}$
where $x^* \| x_i = (x_1^*, x_2^*, \dots, x_{i-1}^*, x_i, x_{i+1}^*, \dots x_n^*)$.

This formation of equilibrium refomulated the concept of Nash's equilibrium in a way which is essentially related to that of stability to laws of movement Some equilibrium existence theorems are proved. The relation between the equilibrium in Arrow-Debreu economic model and -equilibrium is considered.
The results of Allingham M.G. [1], Berg, C. [2], Nash [4] and of author [3] are immediate corollaries of the present result.

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THREE ALGORITHMS FOR THE FERMAT-WEBER'S PROBLEM WITH GENERALIZED COST FUNCTIONS

To solve the Fermat-Weber's problem with generalized cost functions of euclidian distance, three convergent algorithms are proposed. For these algorithms an iterative function is defined for all points in $\mathbb{R}^n$, demand points included. The first one has no continuous iterative function, its convergence is proved with an Huard's statement. For the two others, the iterative function being continuous, the convergence is established with the well-known Zangwill's theorem.

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A COMPARISON OF SEVERAL AUGMENTED LAGRANGIAN FUNCTIONS

In recent years there has been increasing interest in studying both theoretical and practical properties of augmented Lagrangians for solving nonlinearly constrained optimization problems /see, for example, [1] ./

In [2], R. Fletcher presents a thorough study of the use of the Powell-Rockafeller-Hestenes Lagrangian in which computational techniques are presented for dealing with this Lagrangian via an unconstrained minimization algorithm. An alternative family of Lagrangians which are twice differentiable is given by O.L. Mangasarian in [3]. In this paper we develop a computational technique based on Mangasarian's Lagrangians and present a computational study in a common framework, of both families of Lagrangians.

The numerical study includes a sampling of test problems from, among other sources, a collection given by D.M. Himmelblau [4]. The testing approach is based on the development of modularized software so that major common steps /such as the unconstrained minimization/ are done by exactly the same software. This approach helps to assure the reliability of the results obtained. Further, the modules developed will eventually evolve into modules for augmented Lagrangians in the MINPACK mathematical programming software project under development at Argonne National Laboratory.

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ON SOLVING LINEAR COMPLEMENTARITY PROBLEMS AS LINEAR PROGRAMS

Recently, Mangasarian has demonstrated the possibility of solving a certain class of Linear Complementarity Problems by formulating them as equivalent linear programs and solving them as such. One of the key ideas in the above approach is an ingenious theorem giving both sufficient conditions for the LCP to belong to this class and a description of the equivalent LP.

The present paper begins with a sharpening of the aforementioned theorem of Mangasarian which is then used to show how all the complementarity problems described by Mangasarian are related to the theory of polyhedral sets having least elements.

This paper also discusses the question of whether the LP approach can be recommended for solving complementarity problems of the type for which it is intended. The discussion is illustrated with some computational experience.

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LAGRANGEAN CONDITIONS FOR QUASIDIFFERENTIABLE OPTIMIZATION

Lagrangean necessary conditions for optimality (of both Fritz John and Kuhn Tucker types) are extended to a class of constrained minimization problems, in finite or infinite dimensions, where (a) the objective and constraint functions are not necessarily linearly differentiable at all points, but have directional derivatives which are convex functions of direction, and (b) the objective function takes values in a partially ordered space. The class of functions in (a) is also characterized independently of derivatives; it includes the sum of a convex function and a linearly Gâteaux differentiable function. Both duality and converse duality theorems are obtained for the case of real valued objective, and all functions convex. Various known results are special cases, including those for problems involving square roots of quadratic forms.

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AN ALGORITHM FOR A PIECEWISE LINEAR MODEL OF TRADE WITH NEGATIVE PRICES AND INDIVIDUAL INFEASIBILITY

A model of trade is considered wherein prices may be positive, zero, or negative. Also it is not assumed that a trader has a feasible course of action, if he is isolated from the market. Existence of equilibria is shown under the assumption that each subset of traders does not become satiated by the goods producible or owned by the remaining traders. To prove existence we consider the aggregate production capability with weighted objective. An algorithm is given which either computes an equilibrium of the market or demonstrates that the nonsatiation condition fails.

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LAGRANGE MULTIPLIERS IN INTEGER PROGRAMMING

Given the linear problem P "maximize $b(x) = \Sigma b_i x_i$ subject to the constraint $c(x) = \Sigma c_i x_i \leq C$ with $x_i = 0,1$" - this is the well-known 0/1 knapsack problem -, we look for properties that relate an optimal point x° of P to an optimal point of the Lagrangean problem M_λ "maximize $b(x) - \lambda c(x)$".

The fact is that a solution to M_λ can be found in simple way: putting $l_i = b_i - \lambda c_i$, we restate the problem as "max $\Sigma l_i x_i$" which has the solution $x_i = 1$ if $l_i > 0$ and $x_i = 0$ otherwise.

If we have an optimal point x^* for M_λ which is also feasible for P, let $D = \lambda(C - c(x))$, the following properties hold

(1) $\qquad b(x^\circ) \leq b(x^*) + D$

(2) $\qquad \Sigma L_i y_i \leq D$, $\qquad$ where $y_i = |x_i^* - x_i^\circ|$ and $L_i = |l_i|$.

The first property can be used as a bound for the optimum of P and a measure for the goodness of the approximate solution x^*, the second one for reducing the size of P, in fact if $L_j > D$ then $x_j^\circ = x_j^*$.

A Branch and Bound algorithm is presented that utilizes property(2) for reducing the number of branches required by other algorithms, by considering the sum of the L_i relative to the components forced to a value different from that in x^*.

The algorithm is also modified to handle problems with more then one constraint, in fact the same properties hold if we consider the c_i and C as column vectors and λ as a vector of multipliers.

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ON LARGE SCALE UNDISCOUNTED MARKOV DECISION PROCESSES

The proposed algorithm attempts to obtain a recursive method for the evaluation of the sub-optimal policy and the corresponding relative values of the total expected cost vector for a completely ergodic undiscounted, discrete parameter, infinite horizon Markov decision processes. This method avoids matrix inversion and hence the error control problem due to round off which is very much pronounced in Howard's algorithm for large scale processes. Also the accuracy of the values of the cost vector, as evaluated by this method, are independent of the gain of the process. The upper and the lower bounds of the optimal gain are found out in each iteration cycle from the policy improvement routine using Odoni's method. The difference of these bounds is used as the stopping instruction.

The striking feature of the proposed algorithm is that it achieves a considerable amount of saving in computational time for both small and large scale problems compared to Howard's and even Odoni's algorithm. The computational time saving is more pronounced as the number of states increases.

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MONTE CARLO EVALUATION OF THE MULTIDIMENSIONAL NORMAL DISTRIBUTION FUNCTION BY THE ELLIPSOID METHOD

A computer algorithm is presented for computing the values of the multidimensional normal distribution function. The algorithm is based on a well known Monte-Carlo technique and makes use of the Ellipsoid Method for generation of normal random vectors. A corresponding FORTRAN program was made; computational results and computer running times are also presented.

The program works fast for great values of the distribution function /near to 1/, which are especially interesting in stochastic programming models. If the value to be determined is p then in almost 100 p % of the cases the generation of one normally distributed random vector reduces to the generation of one uniform random number from [0,1) and some logical comparisons. Thus 100 p % of the work becames independent of the number of the dimensions.

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ON THE NUMERICAL SOLUTION OF LARGE-SCALE GEOMETRIC PROGRAMMING PROBLEMS

For more than a decade now, researchers in the area of nonlinear programming have attempted to devise algorithms for the solution of geometric programming /GP/ problems. For the most part, these algorithms have been far from successful in solving even modarate sized problems. Recently, however, a number of algorithms appeared that are capable of solving problems of the order of 40 primal variables and 40 primal signomial constraints and up to 130 degrees of difficulty. Strangely enough, the most successful of these codes solve the primal problem and make no use of the elegant GP duality theory. The algorithms upon which these codes are based are not suitable for the solution of truly large-scale GP problems since they are not able to exploit sparsity in an efficient manner. Since large-scale applications /thousands of degrees of difficulty/ of GP abound in the area of optimal aircraft design and other important areas such as transportation, the question of how these problems can be solved using GP theory is an important one. To make use of sparsity, algorithms will have to concentrate on the linearly constrained dual problem. This paper discusses the difficulties associated with the solution of the dual problem and outlines possible ways in which these may be overcome. Also discussed is some ongoing research in attempting to adapt a highly successful large-scale optimization routine that solves general nonlinear programs with linear equality constraints and makes efficient use of sparsity, to the solution of dual geometric programs.

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NEAR OPTIMAL SOLUTION OF VERY LARGE STRUCTURED INTEGER PROGRAMMING PROBLEMS USING COMBINATORIAL ITERATION STRATEGIES

This paper describes a general practical approach to a class of 0-1 integer programming problems involving tens of thousands of variables and constraints which heavily utilizes their structure.The problems considered are scheduling problems;in order of increasing complexity of their constraint structure they are: school timetabling,job shop scheduling, trade training school scheduling and medical personnel scheduling. The philosophy of this approach is to generate easily a heuristic solution to an appropriately chosen set of primary constraints and then to use a combinatorial iteration strategy by which the current /partial/ solution may be permuted to optimize a suitable objective function and/or satisfy the remaining constraints. This approach easily allows the changes to problem specifications which occur frequently in real life to be handled interactively in real time. In practice, a balance must be struck in the design of the iteration strategy between complexity and exhaustiveness and cost considerations which include run time and precision of problem specification. These techniques have been implemented successfully in operating or developing systems for the problems mentioned above by Oxford Systems Associates Ltd. Practical experience regarding complementary information systems and interactive usage is described.

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and

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FINDING MAXIMAL RESTRICTED CHAINS OF DIRECTED GRAPHS FOR THE SOLUTION OF A GEOLOGICAL PROBLEM: SETTING UP STRATIGRAPHIC SUBDIVISIONS

In mathematical terms the problem can be shortly described as follows.

It is given a directed graph $G=[N; A]$, where N is the set of nodes and A is the set of arcs. Furthermore an nxn matrix C is known, the elements of which can be only 0 or 1 and n is the number of nodes in graph G. The directed chains

/1/ $$(i_1, i_2, \ldots, i_k)$$

of graph G, containing the maximal number of elements are to be found subject to the following condition:

/2/ for all $i_p, i_q \in \{i_1, i_2, \ldots, i_k\}$ if $p<q$ then $c_{i_p i_q}=0$.

The problem has occured in a geological research subject, the aims of the recent investigations are:

- to define the notion "stratigrapic subdivision" using the notions of formalized stratigraphy,
- fo find algorithms for determining optimal stratigraphic subdivisions regarding to the interest of the exploration of raw materials,
- to set up optimal stratigraphic subdivisions of the Eocene sediments of the Dorog basin using elementary poleontologic data.

The above mathematical problem was solved by a special integer programming method. Some important features of this method can be used in solving some other specially structured large integer programming problem. The procedure has been coded for a CDC 3300 computer and applied for the stratigraphic subdivisions of Dorog basin. Problems up to n=2000 were studied.

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A NETWORK SOLUTION TO A GENERAL VEHICLE SCHEDULING PROBLEM

Dantzig and Fulkereson /3/, and later Bellmore et. al. /1/ have shown that certain vehicle /tanker/ scheduling problems can be formulated as minimum cost flow problems on a network. In this paper, the results of Dantzig and Fulkereson are extended to the case where more than one type of vehicle can be used in the determination of an optimal fleet. /In tanker scheduling terminology; how many small, medium and large tankers would form in an optimal fleet./.

It is seen how the problem can be formulated as a modified transportation problem where flow in some arcs is conditioned to there being flow on certain other arcs.

These "conditional" transportation problems were solved directly as linear programs and showed the peculiarity of terminating all integer in spite of not having a unimodular constraint matrix. We discuss how the model was implemented and the empirical results.

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ABOUT THE PROBLEM OF "CRITICAL CONNECTIONS" IN AN ORIENTED AND CONVEX GRAPH

This study is presents a method to determine all the "critical connections" in an oriented graph, by means of the associated matrix.
A critical connections is understood to be the only elementary path with a lenght equal to the path between two fixed vertices of the graph; such a path will be called "a path of type α",/t, α /.
The problem of determining all the paths /t, α / has a great practical importance when the bond between two vertices of a graph has to be ensured.
Thus, if the graph represents the network of a flow, and if one of the connections /t, α / is destroyed, it becomes evident that the traffic between the extremities of this path is stopped, a fact that might disturb the flow in some other sections of the net.
This study gives some methods to determine the connections, for three distrinct cases, such as:

i/ the study of external incidental arcs of the vertices of a graph
ii/ the study of internal incidental arcs between the vertices of the graph
iii/ the study of the use of the complementary graph and the nucleus of the graph in order to reduce the number of choices.

First, the notion of restricted graph is introduced for all the external /internal/ incidental arcs of a vertex, completed by adding a method for the construction of the introduced graph, after which the necessary and sufficient conditions for the existence of the connections /t, α / are given.
Based on the above notions, an algorithm with three steps is given, considering the external incidental arcs, and demonstrating at the same time, how to use it for the internal incidental arcs. A simplified form is also given.
The study is completed by extending it to two classes of problem such as:

1. the determination of the elementary path with a minimal number of connections /t, α /
2. the determination of the arc with a minimal number of connections /t, α /, from a set of paths having the same minimal value.

The fact that the studied problems extend the field of practical interest of the discussed problem is evident.

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FINDING THE OPTIMAL PATH IN THE CASE OF SOME MODIFICATIONS OF THE VALUES OF THE ARCS.

This study refers to the finding of the optimal solution solved between two fixed vertices of a connex graph when, compared to a preceding situation, there appear modifications of the values associated to the arcs, performing the least number of computations.

The following cases of modifications have been considered:

a. the value of the arc is reduced by a given number;

b. the value of the arc is increased by a given number.

In the case a. the method demonstrates how to get from the matrix of optimal values M assosiated to the initial graph G. to the matrix of optimal values $\tilde{M}$ associated to the modified graph $\tilde{G}$, and based upon the theoretical results /1 lemma and 3 theorems/ it is shown which are the elements in M and $\tilde{M}$ that remain unchanged, and the means to find the changed elements are given. In order to demonstrate the efficiency of the algorithm, a theorem giving the minimal number of computations is presented.

In the case b, the means to get from the matrix M to the matrix $\tilde{M}$ are two matrices $M^{(1)}$ and $M^{(2)}$, obtained as follows :

- $M^{(1)}$ is obtained through a labelling operation of the elements of M, that remain unchanged by considering the theoretical results;
- $M^{(2)}$ is the matrix obtained from $M^{(1)}$, by keeping the labelled elements from $M^{(1)}$ and including the other elements, by following certain given rules.

The operations imposed by Dantzig's algorithm are applied to the matrix $M^{(2)}$ considered as the matrix of the capacities of a new graph $G^{(2)}$, only for the unlabelled elements.

Finally the study considers the modifications appeared within certain continuous intervals.

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APPROXIMATIONS IN BAYESIAN DYNAMIC PROGRAMMING

Combining Bayesian analysis and dynamic programming only results in relatively efficient solution algorithms when the process is 'closed under sampling', i.e., when posterior and prior distributions belong to the same family of distributions. When this "closed family" property is not satisfied, little can be done in terms of getting exact solutions. In this paper it is shown that, whenever the posterior distribution can be expressed as a linear combination of distributions all belonging to the same class as the prior, efficient approximations can be developed that "sandwich" the exact solution to the underlying dynamic program, i.e., that provide upper and lower bounds to the optimal solution. Two special cases are analysed in detail: first, a class of problems is considered in which linear combinations of beta-densities arise, and second, we develop simple arroximations for processes where linear combinations of gamma-densities arise.

L.C.W. Dixon, The Numerical Optimisation Centre, The Hatfield
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AN ON-LINE VARIABLE METRIC METHOD

In two recent papers the author [1, 2] has indicated theoretical reasons why variable metric methods which are efficient in deterministic situations have undesirable properties when the function value $f(\underline{x})$ can not be evaluated consistently at $\underline{x}$. In ref. [1] it was shown theoretically that under these circumstances OPVM should converge to a region where $\|g(x)\| < \epsilon_0$ provided $\epsilon_0 > \epsilon_0^*$ some critical limit. In ref. [2] the properties of alternative procedures are discussed.

This paper will compare numerical evidence with these theoretical results

1 DIXON L.C.W. On the convergence of the variable matric method with Numerical Derivatives the Effect of Noise in the Function Evaluation - Paper 4 in Oettli & Ritter Eds "Optimisation & Operations Research - Oberwolfach 1975" - Springer Verlag.

2 DIXON L.C.W. Optimisation of Industrial Processes. Presented at the Royal Statistical Society Conference on Process Analysis in Industry - Leeds 1975.

L.C.W. Dixon, The Numerical Optimisation Centre, The Hatfield Polytechnic, Hatfield, Hertfordshire, England.

P. Mazzoleni, University of Venice, Venice, Italy.

MIXED INTEGER CONVEX PROGRAMMING

In this paper the Thiel-Van de Panne algorithm for quadratic programming is generalised for the solution of mixed integer convex programming problems.

Let us first consider the convex programming problem

$$\text{Max} \quad f(x)$$

$$\text{s.t.} \quad g_i(x) \geq 0 \qquad i=1, \ldots, m.$$

If $\hat{x}$ is the solution to this problem, $I(\hat{x})$ the index set of active constraints at $\hat{x}$, and V any subset of $I(\hat{x})$ then we may define x^V as the solution of

$$\text{Max} \quad f(x)$$

$$\text{s.t.} \quad g_r(x) = 0 \qquad r \in V$$

The generalisation is obtained through the following theorem:-

<u>Theorem</u>

$$x^V = \hat{x}$$

if and only if

$$g_j(x^{V_j}) < 0 \text{ for any } V_j = V - \{j\}, \quad j \in V.$$

This enables the branch and bound procedures of linear integer programming to be applied to the integer variables of the convex problem.

As the theorem implies that the additional constraint must be active, the variable reduction technique can be used to reduce the computation required to obtain the solution of the next subproblem.

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ALGORITHMIC EQUIVALENCE IN QUADRATIC PROGRAMMING

It is demonstrated that Wolfe's algorithm* for finding the point of smallest Euclidean norm in the convex hull of a given finite point set generates the same sequence of points as does the van de Panne-Whinston "symmetric" algorithm applied to the associated quadratic programming problem.

Furthermore, results on algorithmic equivalence in the context of general /i.e., not necessarily convex/ quadratic programming problems are presented.

* Philip Wolfe, "Algorithm for a least-distance programming problem," Mathematical Programming Study I /North-Holland, Amsterdam, 1974/, pp. 190-205.

S. Dolecki, Institute of Mathematics, PAS, Warsaw, Poland

GETTING RID OF THE DUALITY GAP

Let U be an abstract set and $\{U_p\}_{p\in P}$ a family of its subsets. Consider a class of problems

$$\underset{u\in U_p}{\text{minimize}}\ Q(u) \qquad p\in P \qquad /1/$$

where Q is a real function on U. Such the formulation includes all known kinds of constraints.

We perturbe a problem p_o of /1/ in order to obtain a substitute problem without constraints:

$$\underset{u\in U}{\text{minimize}}\left\{Q(u) + \inf_{u\in U_p}\varphi(p) - \varphi(p_o)\right\} \qquad /2/$$

where $\varphi\in\Phi$, a class of real functions on P. When P is a topological vector space the Lagrange functionals method may be applied. But the use of Lagrangians in the study of minimization problems leads to a so-called duality gap, as soon as we quit convex lower semicontinuous cases.

In other words, we risk to obtain

$$\inf_{u\in U_{p_o}} Q(u) > \sup_{\varphi\in\Phi}\ \inf_{u\in U}\left\{Q(u) + \inf_{u\in U_p}\varphi(p) - \varphi(p_o)\right\} \qquad /3/$$

Many efforts have been done to remove the discrepancy in /3/. Instead of using linear continuous forms Φ a variety of augmented Lagrangians was introduced.

We study a notion of convexity with respect to an arbitrary class Φ and we show that the equality holds in /3/, if and only if the primal functional given by

$$f(p) = \inf_{u\in U_p} Q(u) \qquad /4/$$

is Φ-convex/ at p_o/ .

It turns out that every lower semicotinuous function which is Φ-bounded is Φ-convex ,whenever Φ is appropriately chosen. We give then a characterization of lower semicontinuous problems.

Of special interest is the strong duality: not only equality holds in /3/,but also the supremum is attained. In connection we develop a theory of Φ-subgradients and provide results on dense and everywhere Φ-subdifferentiability.

For good classes Φ the strong duality implies the comlete equivalence of /1_{p_o}/ and /2/ for some φ: all minimizers,if exist, are the same for both the problems. This, of course, is the most desirable situation — specially for computational procedures.

[1] Dolecki S.,Kurcyusz S.,On Φ-convexity in extremal problems, to appear.

I.Dragan, University of Iasi, Romania

TWO DIMENSIONAL CONCEPTS OF SOLUTION FOR COOPERATIVE N-PERSON GAMES

In this paper we introduce two-dimensional concepts of solution for cooperative n-person games. The main idea is that each player can belong to several coalition of the game, therefore the pay-off of the game is given by a n x m matrix, where m is the number of coalitions, subject to a set of conditions.

The two-dimensional concepts corresponding to the customary concepts have been introduced, they have been called bimputation, bicore, Shapley bivalue, and their properties have been studied. Another concept of solution based on a network flow model have been introduced, too, and the problem of finding such a solution by means of an algorithm have been solved in some cases.

Two-dimensional concepts of solution for cooperative n-person games.

In a previous paper we introduced the "so-called" two-dimensional concepts of solution for cooperative n-person games. The properties of customary concepts of solution like imputation, core, stable set, suggested the definitions of the new concepts. A network flow model for defining another concept of solution have been introduced, too, and this concept have been further studied in a second paper.

The aim of the present paper is that of introducing a two-dimensional Shapley value, called the Shapley bivalue, the departural point being a set of axioms similar to those used by L.Shapley in his definition of the value.

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APPLICATION OF SPARSE MATRIX TECHNIQUES IN LARGE SCALE NONLINEAR PROGRAMMING

Computational experience has shown, that the Generalized Reduced Gradient (GRG) Method for Nonlinear Constrained Optimization is a very efficient method. However, for large problems, e.g. problems with more than 100 constraints and 150 variables, the core storage requirements of the GRG method are very large. The core storage is mainly used for the Jacobian of the constraints and for the inverse of the basis matrix, a square submatrix of the Jacobian. The core storage requirements can be reduced considerably if the Jacobian is stored as a sparse matrix, and if the inverse of the basis matrix is computed and stored, taking into account the sparseness of the basis matrix, both core storage and a large ammount of computing time can be saved.
The paper describes how it is possible to solve very large problems without any special structure with the GRG method. Storage schemes and inversion procedures are described in detail, and some computational results are given.

M.S.Dubson. Central Economics Mathematical Institute, Academy of Sciences of the USSR, Moscow.

THE NECESSARY MINIMUM CONDITION OF $1+\alpha$ -ORDER AND DUALITY IN NONCONVEX PROGRAMMING.

$C^{1,a}$ is a space of differentiable functions $\{f\}$ having a Hölder-condition gradients with power $\alpha\in(0,1]$:

$$|\nabla f(x') - \nabla f(x'')| \leq L\,|x'-x''|^{\alpha} \quad \forall x',x''.$$

A certain function $\varphi_x(z)$ determined on a unit sphere can be associated with every $f\in C^{1,\alpha}$ and every x. The function $\varphi_x(z)$ being continious on all z from unit sphere and being involved in the expansion

$$f(x+\theta z) = f(x) + \theta\langle\nabla f(x),z\rangle + \theta^{1+\alpha}\varphi_x(z) + o(\theta^{1+\alpha}),\ |z|=1,\ \theta>0 \quad (1)$$

The expansion (1) is an analog of the Taylor expansion for functions from $C^{1,\alpha}$. For example, if $\alpha=1$ and $f\in C^2$, then $\varphi_x(z)=\frac{1}{2}(z,\nabla^2 f(x)z)$.

New necessary and sufficient minimum conditions are possible to formulate using the expansion (1) for the following mathematical programming problem:

$$\min\{ f_0(x) \mid f_i(x)\leq 0 \text{ for } i=1,2,\dots m \} \quad (2)$$

Let the constraint qualification be satisfied in a feasible point $\bar{x}$, $f_i\in C^{1,\alpha}$ for $i=0,1,\dots m$ in some neighbourhood of $\bar{x}$ and $\phi_{\bar{x},\bar{y}}(z)=\varphi^0_{\bar{x}}(z)+\sum_{i=1}^m y_i\varphi^i_{\bar{x}}(z)$ **where $\{y_i\}$ are the Lagrange multipliers and** $\bar{y}_i f_i(\bar{x})=0,\ i=1,\dots m$, $\nabla f_0(\bar{x})+\sum_{i=1}^m \bar{y}_i\nabla f_i(\bar{x})=0$ **together with** $T_{\bar{x}}=\{ z \mid |z|=1, \langle z,\nabla f_i(\bar{x})\rangle=0\ \forall i\in[1,2,\dots m]:\ f_i(\bar{x})=0\}$

Then the inequality $\phi_{\bar{x},\bar{y}}(z)\geq 0$ for all z from $T_{\bar{x}}$ is the necessary condition and the strict inequality is the sufficient condition for $\bar{x}$ to be argmin (2). The functoins $\varphi^0,\varphi^1,\dots\varphi^m$ **are related to functions $f_0, f_1, \dots f_m$ by the expansions (1) in the point $\bar{x}$.**

The also have the modified Lagrange function

$$F_\alpha(x,y,r) = f_0(x)+\sum_{i=1}^m \min_{u_i\geq 0}\{\langle y_i, f_i(x)+u_i\rangle + r\,|f_i(x)+u_i|^{1+\alpha}\}$$

with the penalty-like terms of $(1+\alpha)$ -degree, and it is enable us to consider a new dual problem for (2) :

$$\sup_{y\in R^m,\ z>0}\ \inf_x F_\alpha(x,y,z)$$

If the sufficient condition $\phi_{\bar{x},\bar{y}}(z)>0\ \forall z\in T_{\bar{x}}$ are satisfied and if the perturbation function $\rho(u)=\inf\{f_0(x)\mid f_i(x)\leq u_i\}$ satisfied the growth condition of $(1+\alpha)$ -order then we can prove the duality theorem for a finite value of the penalty coefficient $\bar{r}$, and the triple $(\bar{x},\bar{y},r)$ is the global saddle-point of $F_\alpha(x,y,r)$ for all $r\geq\bar{r}$.

In the case of $\alpha=1$ the results are similar to those suggested by R.T.Rockafellar.

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EXPERIENCE IN MULTISTAGE STOCHASTIC PROGRAMMING MODELS

It is a well-known fact that optimal solutions of stochastic programs depend substantially on the assumed distribution of random coefficients. In many practical cases, it is possible to use previous information and to estimate this distribution in advance. We shall deal with the situation when such a possibility does not exist, or is too cumbersome or expensive. On the other hand, we shall suppose that the same stochastic program repeates sequentially and that the (unknown) distribution of random coefficients is known to be fixed in the whole series of problems. Roughly speaking, the problem is how to use experience based on past observations for to construct an optimal (in certain sense) sequence of decisions. This problem will be explored, the properties of the sequence of decisions optimal with respect to empirical distributions in distinct stages will be studied and some of possible generalizations will be discussed.

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MARKOV DECISION PROCESSES WITH CONTINUOUS TIME PARAMETER.

The problem under consideration is how to define a continuous time decision process, ruled by a non-memoryless policy. First a definition of a non-memoryless policy R is given. Then we make a sequence of discretizations of R and construct a sequence of discrete time decision processes ruled by these discretized strategies. Each of these processes can be viewed as a probability measure on $D[0,\infty)$ and the weak convergence of these measures on $D[0,\infty)$ is proved.
($D[0,\infty) = \{f: \; f:[0,\infty) \to \mathbb{N}$; right continuous and with left hand limits in every point$\}$).
Now, by definition, this limit measure is the process ruled by R. With this approach an analogy of a theorem of Derman and Strauch [1] for the continuous time case is proved, which states that to every non-memoryless policy and initial state of the process there exists a randomized memoryless policy such that the corresponding processes have the same marginal distributions.

[1]: C. Derman and R.E. Strauch (1966): A note on memoryless rules for controling sequential control processes. Ann. Math. Statistics 37, pp. 276-278.

R.G.Dyson, University of Warwick, Coventry, U.K.

MINIMAX SOLUTIONS TO STOCHASTIC PROGRAMS - AN AID TO PLANNING UNDER UNCERTAINTY

In seeking to solve the 'wait and see' stochastic program researchers have devised methods of obtaining the distribution of the optimum. To obtain this distribution, or the expected value of the optimum, it is necessary either to solve a multiple integration problem by some approximate means, or use Monte Carlo simulation methods directly on the problem. Either approach requires a considerable amount of computation so that we are limited either to very rough approximate solutions, or to including only a handful of random variables.

One of the purposes of the 'wait and see' model is to compare and evaluate via the objective function various proposed investments. For example in a production/marketing/supply situation such proposals might include the purchasing of new equipment, launching of a new product or contracting for the supply of raw materials. In order to analyse problems for which the expected value or distribution of the optimum cannot be obtained, the author suggests that bounds on the value of the objective function are computed as measures of the uncertainty to aid in the evaluation process. The bounds are the minimax and maximax solutions to the 'wait and see' problem. Experimentation is currently being carried out on a model of a cannery, consisting of 100 constraints, 200 variables and 20 random parameters.

In this paper the formulation of the minimax and maximax problems is outlined. The minimax (maximax) solution occurs when the stochastic parameters jointly take on their most pessimistic (optimistic) values. It is argued that these values cannot be obtained a priori. Consequently it is necessary to define a plausible region for the parameter values. The minimax solution occurs when the objective function is maximized with respect to the decision variables, subject to the stochastic parameters taking on their most pessimistic values within this region. This minimax problem is non-convex. The paper illustrates this method of analysis on a test problem; compares the method with ad hoc selection of optimistic and pessimistic values for the random parameters and discusses the benefits and difficulties of applying the method.

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AN ALGORITHM FOR THE QUADRATIC ASSIGNMENT PROBLEM

The Quadratic Assignment Problem has been a serious challenge to the workers in the field of Mathematical Programming. The difficulty of this problem is the fact that it is combinatorial in nature. The lower bounds normally developed within the context of a tree search algorithm tend to be rather weak and fathoming does not occur before a good part of any branch has been generated.

In this paper we describe the features of a tree search algorithm designed for solving the Quadratic Assignment Problem. The lower bounds developed within the algorithm are easily calculated and are stronger than the ones usually used. The algorithm takes advantage of the data structure in order to reduce the size of the tree to be implicitly investigated and hence speeds up the search significantly.

The algorithm was coded and solved problems of sizes up to 15x15 optimally in a reasonable amount of time. It also solved problems up to 30x30 and solved the 34x36 wiring problem due to Steinberg. Although optimality was not verified for the latter group of problems, the algorithm produced better conclusions than the ones published in the literature.

K.-H. Elster, Technische Hochschule Ilmenau, GDR

RECENT RESULTS IN THE THEORY OF CONJUGATE FUNCTIONS

The great importance of conjugate functions is reflected in numerous applications in several mathematical fields as well as, for instance, in the description of physical facts. In particular, the theory of conjugate functions play an important role in the duality theory of nonlinear programming. Approaches to the duality theory of nonlinear programming are obtained on the one hand by Fenchel's dyality theory and on the other hand by bifunctions and the notion of the Lagrangian. Both approaches are closely connected with conjugate functions. The statements about duality in nonlinear programming published by Fenchel in 1953 are founded on conjugate functions.
The latter were generalized in the following years in different manner:

a) Definition of conjugate functions in more general spaces (by Moreau, Brønstedt, Dieter, Rockafellar, Altman, Maury, Elster/Nehse),

b) Use of coupling-functionals instead of scalar-product (or bilinearform) in the definition of the conjugate functions (by Moreau, Vogel, Weiss, Elster/Nehse),

c) Intrduction of conjugate operators (by Raffin, Breckner/Kolumbán, Zowe, Elster/Nehse),

d) Generalization by utilization of geometrical properties of the Fenchel-conjugates (Deumlich/Elster).

The first part of the talk gives a review of the most important results with respect to the mentioned generalizations. In particular, we reveal that the generalization d) starts from the fact, that the Fenchel-conjugate is closely connected with the polarity with

respect to a special hypersurface of the order 2 (paraboloid). Taking more general an arbitrary nondegenerate hypersurface Φ of the order 2 we obtain the so-called Φ-conjugate functions containing the Fenchel-conjugates as a special case.
In the second part of the talk it is shown, that the generalizations of conjugate functions necessarily determine generalizations of duality theorems of nonlinear programming. The most general results obtained on this topics are given here and well-known (partly classical) results are abtained by specialization.
Beside duality statements in the theory of nonlinear programming those statements play an important role which are known as Kuhn-Tucker Theorem, separation theorems for convex sets, Farkas-Minkowski Theorem, Dubovicki-Miljutin Theorem, a subdifferential theorem. Moreover, one use often the Hahn-Banach Theorem and the Krein Theorem. Hoang Tuy investigated the equivalence of this statements for functionals in local-convex Hausdorff spaces.
In the third part of the talk are given some equivalences for operators by generalization of the problem above using results of Elster/Nehse. Statements of other authors are included as special cases. Thereby one obtaines relevant connections of nonlinear programming to other mathematical fields.

M. Hamdy Elwany, Alexandria University, Cairo, Egypt.

MATHEMATICAL PROGRAMMING AS APPLIED TO PRODUCTION SCHEDULING

In this paper, a procedure for developing an optimal production schedule which spans several time periods has been suggested. The production of different products can run in parallel while the capacity of the existing facilities limits the production level in each line. The suggested procedure offers a dynamic optimal schedule which copes with variable market demands, especially when the exact values of the demands are not available. It can fulfill the market demands within the technological, capacity and feasibility constraints. This can be achieved together with optimizing an effectivness function.

Both the setting and re-setting costs of the production facilities and the holding costs of the in-process and finished products inventories influence the optimal schedule. To optimize their effect they are introduced as cost elements in the effectiveness function.

Integer programming has proved to be an effective approach in developing the optimal schedule. The formulation has introduced the effect of in-process and finished products inventories on coupling the production stations and fulfilling the market demands. Introducing this effect helps in getting flexible and effective schedule.

I.I. Eremin, Institute of Mathematics and Mechanics, Sverdlovszk, SU

NONSTATIONAR PROCESSES OF MATHEMATICAL PROGRAMMING

The theory of iterative methods, step by step successive counting up to closer definition first one the another components of the model /determining infromations, goels, restrictions,.../ is developing in reference to models of mathematical programming. The methods can be given into fundation of the simulation model reflecting dynamics of the behaviour of complicated unstationary technological - economical and natural systems. Formal mathematical analysis of this kind of methods is given.

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A.Vazquez-Muniz, IBM Scientific Center, Madrid, Spain

A MATHEMATICAL MODEL FOR AIR POPULATION ABATEMENT,NUMERICAL RESULTS

Several mathematical diffusion models have been constructed which relate the concentrations of specified pollutants at any point on the ground to the emission rates of polluters. Hence it is possible to regulate the emission rates in such a manner that the concentrations are below prescribed levels at all receptor grid squares in a region. This paper presents a mathematical model that can be used to study control strategies for air pollution abatement.

The probability distribution of the pollutant concentration is estimated over the total range of different meteorological conditions that significantly affect the concentration.

The alternative emissions control policies are determined by reduction of the prescribed source emissions in amounts proportional to the effect they have on the pollutant concentration in the whole polluted area. This is done in order to achieve prescribed air quality goals, under the entire range of atmospheric conditions. The main constraint require the new probability, with which the prescribed pollutant concentration is exceeded at each polluted grid square, be no greater than the maximum frequency allowed for the given period of time.

This model is supported by mixed integer programming techniques, so that the branch and bound phase uses the SOS rows, the quasi-integer binary variables and the candidate nodes selection facilities to achieve and evaluate several alternative emissions reduction policies.

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AN ALGORITHM FOR INTEGER-NONLINEAR PROGRAMMING

An algorithm for solving integer-nonlinear programming problem by a cutting plane method is given.
Feasible points, determined by Korte-Krelle-Oberhofer's lexicographic algorithm, for generating "external" or/and "internal" cutting planes are used.

Numerical results illustrates the efficiency of the given method.

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ON SOLVING MAX-MIN PROBLEMS

The solutions of various types of max-min problems are obtained by means of a recently developed cutting plane method together with an algorithm designed to find global solutions of non-convex programs. If the constraint set of the "inside optimizer" depends on the move of the "outside optimizer," the solution may be obtained by directly addressing the outside problem. Numerical examples of the different types of problems are discussed.

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ON THE POSSIBILITY OF OPTIMIZATION OF ELASTIC TRUSSES BY PSEUDOBOOLEAN PROGRAMMING

A problem of minimum volume optimal design of elastic trusses in the case when the decision variables are the cross sectional areas of bars is by its nature a problem of integer programming. Moreover if we consider indeterminate trusses for alternative loads with the constraints for allowable stresses the problem becomes nonlinear. Described problem is important from practical point of view as engineers project trusses mainly from discretely catalogued bars. In the paper general nonlinear discrete mathematical programming model for the problem is formulated and examples of practical problems are given.
To solve the problems pseudoboolean programming algorithm was applied. Computational results estimated as satisfactory are particularly discussed.

F. Fazekas, Technical University of Budapest, Hungary

MATRIX ALGORITHMICAL PROGRAMMING AT LINEAR, NON-LINEAR AND STOCHASTIC PROBLEMS /theory, methods, teaching/

As introduction, a short survey will be given on the author's papers in the Zeitsch. f. Angew. Math. u. Mech. [7], in the Internat. Series f. Num. Math. [2], in the Internat. Koll. f. Math. [3], etc., then on his book "Math. progr. by MAM", guest lecture's bulletins [2] and several postgradual notes, in which the s.c. Matrix Algoritmical Methods /MAM/ were constructed, then applied for various mathematical programming and algebraical, stochastical problems, further used by computers to different industrial tasks and in the higher teaching.

Theoretical remarks on the main variants of the MAM, e.g. DTA, STA, SMA; different basic transforms, step and spring formulas, construction and factorization of end difference; comparison with other algorithms. Methodical remarks, e.g. on the different choise of pivot elements. Computing.

Mention on the main linear algebraical tasks to solve by our DTA /STA/, e.g. on the ranking ordinary /bilateral/ and partial /unilateral/ inverting, solving of general linear unequality /spec. equation/ system, solving of symmetrical system by our decreasing SMA, complex investigation of quadratic forms by our QMA, ortogonalisation by our OMA etc.

Linear programming /LP/ by /primal/ DTA at normal max-task and by dual DTA at min-one. The DTA-s at modified and non-normal tasks. Their perturbation forms. - Special DTA forms for Chebyseff-approximations. Integer LP by mixed DTA-s. End construction of DTA; variant and parametric task. Transport LP by special DTA, STA. Advance of MAM to Boolian LP. Formation of DTA to the stochastic programming.

Quadratic programming by DTA combined with Wolfe's method. Irrational /convex/ programming for vial center; advenced variant tasks. Generalized DTA to solve non-linear unequality /spec. equality/ system, LP-analogy, comparison with NRM; utilization of DTA at nonlinear /convex/ programming, then at stochastic one.

Mention on applications of the MAM's variants in the statistics, information theory, at the stochastic systems and processes etc.

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M.Florian, University of Montreal, Montreal, Canada

A SUB-OPTIMAL ALGORITHM TO SOLVE A LARGE SCALE 0-1 PROGRAMMING PROBLEM

In this paper we develop a sub-optimal algorithm to solve a 0-1 programming problem that appears as a sub-problem during the application of Benders' decomposition. The algorithm yields a good feasible solution together with an upper bound on the distance from the optimal solution. The algorithm is very simple because it involves only a scanning process that is to be repeated with different initial solutions that are generated using the information about the relative costs of the variables.

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УМЕНЬШАЮЩИЕ АЛГОРИТМЫ МЕТОДА ОТСЕЧЕНИЯ ЦЕЛОЧИСЛЕННОГО ЛИНЕЙНОГО ПРОГРАММИРОВАНИЯ.

В докладе излагаются три алгоритма отсечения, позволяющие без ущерба для доказательства конечности отказаться в некоторых случаях от лексикографической упорядоченности при выборе строки симплексной таблицы, генерирующей отсечение. Кроме того, эти алгоритмы обладают некоторыми ценными свойствами, отмеченными ниже.

Пусть в полностью целочисленной задаче ЛП с целыми коэффициентами отброшено сперва условие целочисленности и найдено оптимальное нецелочисленное базисное решение X_0. Ему соответствует симплексная таблица и детерминант d_0. Точка n -мерного евклидова пространства X_0 является узлом некоторой решетки с определителем $|d_0|^{-n_0}$. Идея алгоритмов состоит в том, чтобы последовательно увеличивать величину определителя решетки, получая ряд промежуточных решений, являющихся узлами соответствующих решеток. Понятно, что если удастся сделать определитель равным I, то полученное промежуточное допустимое решение будет оптимальным решением исходной задачи.

В первом алгоритме применяются уменьшающие детерминант d симплексной таблицы отсечения циклического алгоритма Гомори, если симплексная таблица допустима, и отсечения полностью целочисленного алгоритма, если она недопустима.

Во втором алгоритме при каждом симплексном преобразовании таблицы вводится либо отсечение циклического алгоритма, строго уменьшающее абсолютную величину детерминанта d, либо отсечение полностью целочисленного алгоритма, строго уменьшающее величину функции цели.

Необязательно требовать, чтобы каждое вводимое отсечение не отсекало ни одну допустимую целочисленную точку. На каждом шаге третьего алгоритма строится либо отсечение полностью целочисленного алгоритма Гомори, строго уменьшающее величину функции цели, либо альтернативная пара дополнительных ограничений, каждое из которых строго уменьшает на целое число модуль детерминанта d. Выбор ветви осуществляется в соответствии с какой-либо известной оценкой. Конечность алгоритмов обеспечивается, если исходная задача разрешима.

Уменьшающие алгоритмы можно рассматривать как весьма рациональные способы получения исходной целочисленной таблицы для полностью целочисленного алгоритма Гомори, причем решение задачи может быть найдено уже на предварительной стадии.

Уменьшающие алгоритмы можно применять при реализации групповых методов дискретной оптимизации. Так как величина детерминанта оптимальной симплексной таблицы может быть произвольно велика, затруднительно применять эти методы к таким задачам непосредственно. Использование уменьшающих алгоритмов позволяет уменьшить детерминант до величин, приемлемых для групповых методов.

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ANALYSIS of APPROXIMATIONS for MAXIMIZING SUBMODULAR FUNCTIONS

Let N be a finite set and z be a real-valued function defined on the power set of N that satisfies

$$z(S) + z(T) \geq z(S \cup T) + z(S \cap T) \quad \text{for all } S, \ T \ \text{in } N.$$

Such a function is called submodular. We consider problems that involve the maximization of a submodular function.

The warehouse location problem, the maximum cut problem and several other hard combinatorial optimization problems can be posed in this framework. One more general problem of this type is to find a maximum weight independent set in a matroid, when the elements of the matroid are colored and the elements of the independent set can have no more than K colors. /The warehouse location problem is a special case of this model/. We analyze heuristics and relaxations for these problems.

Our results are worst case bounds on the quality of the heuristics and relaxations. For example, for the matroid optimization problem mentioned above, we show that a "greedy" heuristic always produces a solution whose value is at least $1 - [(K-1)/K]^K$ times the optimal value. This bound can be achieved for each K and has a limiting value of $(e - 1)/e$, where e is the base of the natural logarithm. Several other results of this type are presented.

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A MULTI-COMMODITY CONVEX COST FLOW MODEL FOR THE PLANNING OF MULTI-MODAL URBAN TRANSPORATION NETWORKS

We consider the problem of predicting origin destination demands and route flows in a network associated with an urban where several modes of travel are available. The origin-destination are modeled by means of an entropy type distribution model for each mode and route choice is assumed to be given by Wardrop's first principle of traffic equilibrium. The resulting model is a minimum convex cost multi-commodity flow problem with variable demands. The model properties are analysed and solution algorithm is given which results in a decomposition of the problem into a sequence of calculation of shortest routes, multi-commodity linear cost transportation problems and one dimensional optimizations of a convex function. Applications in the context of transportation planning are outlined.

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THE QUEST FOR A NATURAL METRIC

The interpretation of Newton's method as a steepest descent method in a natural metric will be reviwed. However it can often be difficult /for instance when the Hessian matrix is not positive definite/ to define a natural metric. Various solutions to this dilemma which have been proposed, not entirely satisfactory, will be described. Some new algorithms or possibilities will be presented.

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THE LEVEL OF SOLVING LARGE LINEAR PROGRAMMING SYSTEMS IN CSSR

This paper deals with the solving of the large problems of linear programming which arise especially in applications in economics. Some codes, based on PFI and EFI algorithms, developped in CSSR, will be discussed.

The main part of this paper will be devoted to the code which is developped by the author. This code uses the EFI algorithm with minimizing the total number of nonzero elements, the special linked-list for storage of nonzero elements, the effective usage of it and finally maitaining the sparsity during the overcome from one basis to another.

At the end of this paper some estimates of a number of operations and some computational results of the code will be presented.

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WEAKLY CONNECTED INTEGER PROGRAMMING PROBLEMS

Conventional Branch and Bound codes have had considerable success with a wide range of integer problems but there exist classes of problems where their performance is disappointing compared to alternative methods. One such class arises when a small integer problem is expanded into a multiple area model or a multiple time period model. In this case solution times grow exponentially while if a multiple time period model is amenable to dynamic programming techniques the solution time will only grow linearly. This paper will look at some possible ways of modifying branch and bound codes to take advantage of the structure of models consisting of weakly linked integer subproblems.

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A ONE PASS ALGORITHM FOR A WAREHOUSE SIZING PROBLEM

The problem we consider is one of determining warehouse size during each of T time periods, when warehouse demand for each time period is a discrete and finite valued random variable whose density function may change from one time period to the next, so as to minimize expected total cost. Expected total cost includes costs linearly proportional to warehouse size, costs incurred due either to excess capacity or inadequate capacity, and due to size changes from one time period to the next. It is known that the problem has an equivalent linear programming formulation for which the dual is a network flow problem. We present a one-pass algorithm for solving the network problem, as well as economic interpretations based on the network problem.

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ГЕОМЕТРИЧЕСКИЕ ПОДХОДЫ В ДИСКРЕТНОМ ПРОГРАММИРОВАНИИ.

Доклад состоит из трех частей и содержит результаты, навеянные геометрическими соображениями. В первой части среди задач линейного программирования выделяется класс задач, сводящихся к задачам с целочисленным многогранником планов, и имеющим эффективные методы решения. Под сводимостью задачи P к задаче Q (хорошо решаемой) понимается способ погружения множества планов M_P задачи P в множество планов M_Q задачи Q, при котором M_P оказывается проекцией M_Q и по плану (оптимальному) задачи Q удается просто восстановить план (решение) задачи P. Изучалось сведение к потоковой задаче (о циркуляции). Установлены критерии сводимости в терминах свойств структуры матрицы ограничений задачи P, которая должна быть M -матрицей. Изучаются свойства M -матриц, которые абсолютно унимодулярны и содержат в себе известные из литературы подклассы абсолютно унимодулярных матриц. Выясняется "геометрия" M -матриц, связанная с возможностью упорядочения ее строк в виде древовидной структуры. Во второй части рассматривается линейная оптимизационная задача на регулярном дискретном множестве

$$\mathfrak{M} = \bigcap_{1}^{k} \mathfrak{M}_i ,$$

где

$$\mathfrak{M}_i = \{ x \in M / (a_i, x) < b_i \text{ или } (a_i, x) > c_i \}$$

$b_i < c_i$, а M - многогранник в R^n.

Привлекая идею построения выпуклой оболочки допустимых планов путем добавления минимального числа дополнительных ограничений, удалось найти довольно общую схему получения отсечений, и доказать, что они образуют выпуклый многогранник. Исследуются свойства отсечений этого многогранника, крайних отсечений, их взаимоотношение с известными типами отсечений. Доказываются основные теоремы теории отсечений для регулярных задач, а также критерий эквивалентности неприводимых систем неравенств. В третьей части излагается экономный переход от задачи целочисленного линейного программирования с условиями $Ax = a$, $x \geq 0$, x - целое к эквивалентной задаче с условиями $A'x' \geq a'$, где число переменных меньше и они целочисленны. Это достигается путем нахождения базиса целочисленной решетки и представления общего целочисленного решения системы в виде $x = A'x' - a'$ за полиномиальное относительно параметров исходной задачи число операций, где степень полинома ≤ 5.

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THE ITERATIVE PROCESS OF LINEAR FUNCTION OPTIMIZATION ON INTEGER POINTS OF A CONE.

Integer programming problems on a cone

(1) $\max cx$ $\quad$ c, x, b - n-dimantional integer vectors
$Ax \geqslant b$ $\quad$ A - nonsingular $n \times n$ integer matrix

form the most simple but nontrivial class. The matrix $\binom{A}{c}$ may be transformed to some special form if the solution of the problem (1) is unique and bounded. In the transformed matrix all diagonal elements are positive and nondiagonal elements are nonpositive. Such matrix is called regular. Regular form of the matrix $\binom{A}{c}$ is defined uniquely if an order of rows in the matrix A is fixed and some minimality demands are satisfied. Using the polynomial algorithms for systems of linear diophantine equations we can transform problem (1) to the regular form with time complexety $O(n^5)$. Absolute values of intermediate numbers are bounded by $M^{n \log n}$ ($M = \max(|a_{ij}|, |c_i|)$).

For the problem on a cone in regular form

(2) $\max -x_n$, $(D-C)x \geqslant b$, $\quad$ x - integer

where D is diagonal matrix with positive elements on the diagonal, C is nonnegative matrix with zeros on the diagonal
iterative process

$$\xi^{(0)} = (D-C)^{-1}b \ , \ \xi^{(i+1)} = \lceil D^{-1}C\,\xi^{(i)} + D^{-1}b \rceil, \ i \geqslant 1$$

may be constructed. This process converges to the solution of the problem (2). If the equality $\xi^{(K+1)} = \xi^{(K)}$ is satisfied the process stops and $\xi^{(K)}$ is the optimum of the problem (2).

The process may be so modified that it will become monotone regarding the objective function . Other representation of matrix A as the difference A = D-C of two nonnegative matrixes generate other iterative process of the problem solving.

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ON A NEW HYPERGRAPH THEORETICAL CLUSTER TECHNIQUE

1. New cluster definition

Clusters are generally defined as maximal complete subgraphs of a graph. The main disadvantages of this definition are the following:

- clusters can partially overlap,
- no polynomial bounded algorithm is known for identifying all the clusters of a graph.

The new cluster definition, proposed in this paper, is the generalization of the concept of component, without the above mentioned disadvantages.

Let $H=(X,E)$ be a hypergraph. $X=\{x_1,\dots x_n\}$; $E=\{e_1,\dots e_m\}$

Let $w(e_j)>0\ (j=1\dots m)$ be a weighting of edges of $H=(X,E)$

If $F\subseteq E$ then $w(F)=\sum_{e_j\in F} w(e_j)$

If $S\subseteq X$ then $\mathcal{E}(S)=\{e_j \mid \exists x_i\in S : x_i\in e_j\}$

If $S\subseteq X,\ T\subseteq X$ then $\mathcal{E}(S|T)=\{e_j \mid \exists x_i\in S \cdot x_i\in e_j,\ e_j\subseteq T\}$

If $T\subseteq X$ then $\bar{w}(T)=w[\mathcal{E}(T)\cap\mathcal{E}(X-T)]=w[\mathcal{E}(T)]-w[\mathcal{E}(T|T)]$

Cluster definition: $Q\subseteq X$ is a quasi component /cluster/ of $H=(X;E)$ iff for all T proper subsets of Q $(\emptyset\neq T\subset Q)$ $\underline{\bar{w}(T)>\bar{w}(Q)}$

2. Some important properties of clusters

2.1. $Q\subseteq X$ is a cluster of $H=(X,E)$ iff for all T proper subsets of Q:
$\bar{w}[Q-T]-\bar{w}[Q]=w[\mathcal{E}(T|Q)]-w[\mathcal{E}(T|(X-(Q-T)))]>0$

2.2. All of the components of $H=(X;E)$ are also clusters /quasi components/ of it.

2.3. If $T\subseteq X$ for which $\bar{w}(T)$ is minimal, then the $Q\subseteq X$ cluster is contained either in T or in $X-T$!

2.4. If $Q\subseteq X$ and $Q'\subseteq X$ are clusters of $H=(X,E)$ then either $Q\cap Q'=\emptyset$ or $Q\subseteq Q'$ or $Q'\subseteq Q$.

2.5. Hypergraph $H=(X;E)$ has at most $2|X|-1$ clusters.

3. Identification of clusters

The polynomial bounded algorithm for identifying all the clusters of a hypergraph is based on the 2.3. propertie of clusters.

The basic routine of the algorithm is the optimal bisection of a hypergraph. The fundamental idea of the routine is the transformation of the problem into the maximal pairing problem of the supply-demand task. In this way the routine is at least as efficient as the Ford-Fulkerson algorithm.

4. Applications

4.1. Information storage and retrieval.
4.2. Research management.
4.3. Packaging of electronic circuits.

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THE GLOBAL SOLUTION OF NONLINEAR OPTIMIZATION PROBLEMS WITH NONLINEAR EQUALITY CONSTRAINTS

A new algorithm is proposed for solving optimization problems with equality constraints. The new algorithm has the desirable property of obtaining under suitable conditions a global solution, in the sense that it is an iterative procedure which yields an stationary point to the nonlinear problem regardless the initial estimate on the solution. It is therefore applicable to the solution of a system of a nonlinear system of equations.

The algorithm generates a sequence of locally convergent estimates of a stationary point of the nonlinear problem by solving optimization problems which have a quadratic objective function and linear equality constraints. Global convergence is achieved by incorporating an stepsize on the augmentation function of the associated Lagrangian for the nonlinear problem.

An asymptotic quadratic rate of convergence is obtained whenever exact second derivative of the functions are evaluated, and a superlinear rate of convergence is obtained whenever the second derivatives are approximated by some variable metric type of Hessian approximations.

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ON MODIFYING SINGULAR VALUES TO SOLVE POSSIBLY SINGULAR SYSTEMS OF NONLINEAR EQUATIONS

We show that if a certain nondegeneracy assumption holds, it is possible to guarantee the existence of a solution to a system of nonlinear equations $f(x) = 0$ whose Jacobian matrix $J(x)$ exists but may be singular. The main idea is to modify small singular values of $J(x)$ in such a way that the modified Jacobian matrix $\hat{J}(x)$ has a continuous pseudoinverse $\hat{J}^{+}(x)$ and that a solution x^{*} of $f(x) = 0$ may be found by determining an asymptote of the solution to the initial value problem $x(0) = x_0$, $x'(t) = -\hat{J}^{+}(x)f(x)$. We briefly discuss practical /algorithmic/ implications of this result. Although the nondegeneracy assumption may fail for many systems of interest/indeed, if the assumption holds an $J(x^{*})$ is nonsingular, then x^{*} is unique/, algorithms using $\hat{J}^{+}(x)$ may enjoy a larger region of convergence than those that require /an approximation to/ $J^{-1}(x)$.

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ON COMBINING THE TECHNIQUES OF REID AND SAUNDERS FOR FACTORING SPARSE LINEAR PROGRAMMING BASES

When solving linear programming problems, it is frequently necessary to compute and update a factored representation of a large, sparse basis matrix. At the recent Symposium on Sparse Matrix Computations /held at Argonne National Laboratory in September, 1975/ both John Reid and Michael Saunders described strategies for taking advantage of sparsity when updating such factored basis representations. Both schemes involve stable numerical techniques which allow greater accuracy and less growth of nonzeroes than previous schemes. Whereas Saunders's scheme is designed to allow out-of core computations, Reid's scheme runs only in-core but often produces less growth of nonzeroes. Following a suggestion made at the Sparse Matrix Symposium, we show that the two schemes may be combined to produce a hybrid scheme which allows out-of-core calculations with reduced growth of nonzeroes. We also discuss experience with an implementation of this scheme in XMP, a modular linear programming package being developed at the National Bureau of Economic Research.

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LYAPUNOV FUNCTION APPROACH IN THE DESIGN OF ALGORITHMS FOR STRUCTURED NONLINEAR OPTIMIZATION PROBLEMS

We present a method which enables us to exploit the structure of certain nonlinear optimization problems. For the sake of simplicity the idea will be shown for the case of a constrained optimization problem

$$\min f(x) \quad h(x) = 0 . \qquad /1/$$

This problem may be solved by the method of multipliers. Introduce the augmented Lagrangian function

$$Q(x,w,k) = f(x) + w'h(x) + kh'(x)h(x), \qquad /2/$$

and consider the algorithms described by the following correction formulae:

$$\delta x = - Q_x(x,w,k) \qquad /3/$$

$$\delta w = 2\, k\, h(x(w)). \qquad /4/$$

For fixed w the sequence of x values defined by /3/ will converge to the minimum of Q, say x(w). On the other hand /4/ determines the optimal multiplier vector, say w^*.

It was an open question how accurately x(w) must be computed. We give a modification in which no error bound must be given in advance, reduction of errors is automatic. This modification is:

$$\begin{aligned} \delta x &= - Q_x(x,w,k) + Kh(x) \\ \delta(w) &= 2\, k\, h(x) . \end{aligned} \qquad /5/$$

The matrix K is an appropriately chosen constant matrix. We proved that the differential equation

$$x = \delta x \qquad w = \delta w \tag{6}$$

is stable in (x^{*}, w^{*}). Therefore an algorithm of the form

$$x_{n+1} = x_n + \alpha_n \delta x_n$$
$$w_{n+1} = w_n + \alpha_n \delta w_n \tag{7}$$

converges, if the stepsizes are appropriately chosen.

One of the major advantages of the new algorithm is its stability in the presence of random errors.

The above ideas are elaboraded for the reduced gradient method, for multistage deterministic decision models and for nonlinear two stage stochastic decision models, too.

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ЛИНЕЙНАЯ УПРАВЛЯЕМАЯ СИСТЕМА С ИНТЕГРАЛЬНЫМ КВАДРАТИЧНЫМ КРИТЕРИЕМ ЭФФЕКТИВНОСТИ

Пусть поведение управляемой системы на отрезке $[t_0,T]$ описывается линейным уравнением $\dot{x}=A(t,\mu)x+B(t,\mu)u$, где A и B – непрерывные соответственно $n\times n$ и $n\times r$ матрицы-функции переменных $(t,\mu)\in Z_1=[t_0,T]\times\Omega$ (Ω – открытое подмножество R^m). $W(t,\mu)$ и $U(t,\mu)$ – квадратные симметричные соответственно n и r-мерные непрерывные на Z_1 матрицы-функции, первая из которых неотрицательно определена, а вторая – положительно определена; $g(v,\mu)$ – непрерывная скалярная функция для $(v,\mu)\in R^n\times\Omega$, которая для всех $v\in R^n$ с нормой больше некоторого числа $a>0$ удовлетворяет неравенству $g(v,\mu)\geq -b|v|^{2-\nu}$ при некоторых $b\geq 0$ и $\nu>0$. В классе управляющих функций $u(t)$ из $L_2[t_0,T]$ рассматривается задача минимизации квадратичного функционала

$$g(x(T),\mu)+\int_{t_0}^{T}(x'(t)W(t,\mu)x(t)+u'(t)U(t,\mu)u(t))\,dt,$$

где $x(t)$ – траектория с начальным условием $x(t_0)=x$, соответствующая согласно закону движения управлению $u(t)$ при значении параметра $\mu\in\Omega$, а штрих означает транспонирование. Пусть система вполне управляема. Тогда для каждого начального состояния $x\in R^n$ и $\mu\in\Omega$ существует оптимальное управление и имеет место следующая теорема.

Теорема. Для каждого $x_0\in R^n$ и $\mu_0\in\Omega$ и любого числа $\varepsilon>0$ существует такое число $\delta>0$, что если $x\in R^n$ $\mu\in\Omega$, $|x-x_0|+|\mu-\mu_0|<\delta$ и $u(t)$ – оптимальное управление для (x,μ) , то найдется оптимальное управление $u_0(t)$ для (x_0,μ_0) такое, что $\max\limits_{t_0\leq t\leq T}|u(t)-u(t_0)|<\varepsilon$.

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JUNCTION CONTROL STRATEGIES FOR AN AUTOMATED TRANSIT SYSTEM

The problem of merging two strings of high speed vehicles is approached in two ways - as a sequencing problem and a network flow problem. In the former approach vehicles occupy fixed length reference cells which move along a track at a given velocity. The vehicles on the approaching lanes are assigned to new reference positions such that non-conflict merging is achieved and an overall assignment cost, defined by the designer, is minimised. In this case the problem is formulated as a Mathematical programming problem. In the second approach vehicles follow the variable space-time headway control law and the junction acts as a vehicle flow controller which computes the appropriate velocities of the approaching vehicles while maintaining the minimum headway between them. Both these methods are implemented in a computer program and it is shown that the second method not only solves the merging problem but also provides a more satisfactory solution to the overall vehicle flow problem in the network from an operational point of view.

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NON-DIFFERENTIABLE OPTIMIZATION

The problem of minimizing a non-differentiable convex function can be solved by a technique known in the U.S. as subgradient optimization and in the Soviet Union as the generalized gradient method.

If it is assumed that there is some a priori information (bounds) on the location of the minima, and on some condition number of the function, then it can be proved that the procedure can give convergence at a "geometric rate", if for t(q), we choose a "suitable" geometric series.

Several medium-scale optimization problems have been solved using the method, and, even though convergence was not always fast enough, the theoretical results obtained were confirmed by the numerical experimentation. The results are similar to those of standard gradient methods for differentiable functions, where convergence is slow if the function is badly conditioned.

The next step is to define and implement "second-order" methods, which attempt to extend the ideas of conjugate gradients, orvariable metric algorithms defined for differentiable functions. Various proposals have been made by P. Wolfe, N.Z. Shor, C. Lemaréchal and others.

Convergence rates have been obtained for some of the methods: at this point, il seems that second order methods improve the rate of convergence in some cases, and worsen it in other cases!

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A STEEPEST-EDGE SIMPLEX METHOD FOR NETWORK FLOW PROBLEMS

We consider here the steepest edge simplex method applied to minimum cost network flow problems. As in the standard simplex method, an arc is chosen to enter the basis from among those nonbasic arcs whose associated flow augmenting paths (FAP), result in a decrease in the objective function. In the steepest edge case, however, the entering arc is determined as the one for which the cost per unit flow along its FAP divided by the square root of the number of arcs in its FAP is minimal. Increasing the flow along that arc corresponds to moving along an edge of the polytope of feasible solution on which the objective function decreases most rapidly.

In Goldfarb and Reid (1975) a practicable steepest edge algorithm was given for the general linear programming problem that was based upon suitable recurrences developed by the author. In this paper, these are specialized to networks to yield simple recurrence relations for the number of arcs in each FAP. These recurrences, together with a strategy for selecting the arc to leave the basis recently proposed by Cunningham (1974), which keeps all bases "strongly feasible", yield a powerful network simplex algorithm. A computer implementation of this algorithm is discussed and it is shown that Reid's (1975) sparsity exploiting method for updating factorized linear programming bases is particularly well suited to network flow problems.

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MODIFIED LAGRANGE FUNCTIONS IN CONVEX PROGRAMMING AND THEIR GENERALIZATIONS.

A function F(x,y) concave in x, convex in y is said to be a modified Lagrange function for a convex programming problem

$$f(x) \to \sup,\ g(x)=(g_1(x),\dots,g_m(x)) \geq 0,\ x \in G \subseteq E^n, \qquad (1)$$

if $\mathrm{Argmax}_{x\in G}(\inf_{y\in E_+^m} F(x,y))=X^*$, where X^* denotes the solution set of (1). The sufficient conditions are given for the function of the form

$$F_\lambda(x,y)= F_0(x,y)- \sum_{i=1}^{m} \lambda_i (g_i(x),\ y_i),$$

where $F_0(x,y)= f(x)+ g(x)y$, to be a modified Lagrange function for any problem (1) and to have in $G\times E_+^m$ the saddle-point set $X^*\times Y^*$ coinciding with that of the ordinary Lagrange function $F_0(x,y)$. Somewhat more strict conditions ensure that the set $X^*\times Y^*$ is stable in x with respect to $F_\lambda(x,y)$, i.e. one has $\mathrm{Argmax}_{x\in G} F_\lambda(x,y^*)=X^*$ for $y^*\in Y^*$. The connection between the stability and the convergence of the gradient method of saddle point determination is examined. If a convex function $\alpha(u)\in C^1(E^m)$ satisfies the following conditions: $\alpha(0)=0$, $\nabla\alpha(0)=0$ and $|\nabla\alpha(u')-\nabla\alpha(u'')| \geq \gamma\cdot|u'-u''|$, $\gamma>0$, then the function

$$F^\alpha(x,y)= f(x)+ \max_{t\in E_+^m} [(g(x)- t)y - \alpha(g(x) - t)]$$

is a modified Lagrange function whose saddle-point set $X^*\times Y^*$ is stable in x. The only inequality $\inf_{y\in E_+^m}\sup_{x\in G} F_0(x,y)<+\infty$ implies that $\psi^\alpha(y)= \sup_{x\in G} F^\alpha(x,y)\in C^1(E^m)$ and $|\nabla\psi^\alpha(y')- \nabla\psi^\alpha(y'')|\leq (1/\gamma)\,|y'- y''|$. The properties of $F^\alpha(x,y)$ provide the convergence of the modified dual method which consists in minimization of $\psi^\alpha(y)$ by means of the perturbed finite-step gradient method.

For a monotone point-to-set mapping $T(z): Z\to 2^{E^p}$, where $Z \subseteq E^p$ and $T(z)\neq\emptyset$ for $z\in Z$, the problem consists in determining a root, i.e. $z^*\in Z$ such that $0\in T(z^*)$. In the case of T(z) defined everywhere in E^p and satisfying the condition

$$(T(z')-T(z''),\ z'-z'')\geq\gamma\,|T(z')-T(z'')|^2,\ \gamma>0,\ \forall z',\ z''\in E^p \quad (2)$$

the following analogue of the perturbed gradient method serves the purpose:

$$z^{k+1}=z^k-\alpha l^k,\ |l^k-T(z^k)|\leq\varepsilon_k,\ \varepsilon_k\geq 0.$$

Let R(z) be any mapping satisfying both (2) and the strong monotonicity condition. Denoting by $z(w)$ the root of $T_R(z,w) = T(z) - R(w-z)$, $z \in Z$, one may define the modified mapping by formula $T_R(w) = R(w-z(w))$, $w \in E^p$. The mapping $T_R(w)$, $w \in E^p$ satisfies (2) and its roots coincide with these of $T(z)$, $z \in Z$. For practically any monotone mapping an algorithm converging to some of its roots can be obtained by applying the above gradient-like process to the corresponding modified mapping. This method of modification yields a unified approach to convex programming problems and to determination of saddle and equilibrium points as well as expands the class of the modified Lagrange functions. Each iteration of this method requires only a finite number of elementary steps. Moreover, in certain cases of continuous mappings the amount of computation required for each iteration can be substantially reduced.

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VALEURS PROPRES ET VECTEURS PROPRES DANS LES SEMI-MODULES ET LEUR INTERPRETATION EN THEORIES DES GRAPHES

On montre dans cette note l'existence de valeurs propres et de vecteurs propres pour des matrices à coefficients dans un semi-anneau.

Ces résultats correspondent à une extension du théorème de Perron-Frobenius aux semi-anneaux. On donne alors un grand nombre d'exemples liés aux problèmes de cheminement dans les graphes. L'interprétation de ces valeurs propres aux problèmes de graphe sera souvent très intéressante.

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A PARAMETRIC PRIMAL ALGORITHM FOR DISCRETE CHEBYSHEV LINEAR APPROXIMATION

In this paper a parametric primal algorithm for solving the discrete linear Chehyshev approximation problem is constructed. The algorithm imposes no restrictions on the choice of the linear approximating function, and is applied directly on the equivalent LP formulation of the Chehyshev problem. The algorithm is an adjacent extreme point type algorithm which was designed to exploit the special structure of the equivalent LP formulation, and which generates a best approximation for the Chehyshev problem in finitely many iterations.

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EFFICIENCY OF THE BRANCH AND BOUND ALGORITHMS IN A BOOLEAN PROGRAMMING

Purpouse of this report is to research the efficiency of the branch and bound method. The schem of the method that includes considerable part of the all known algorithms is described. The following three characteristics are used for classification: 1) the rule of basic problem branching to the subproblems, 2) the rule of choice of a candidate problem out of the list of the subproblems, 3) the tests allowing to reduce the enumeration of the solutions.

The branching tree is described and the search of the solutions is defined. The functions $\psi_A(p)$ and $\varphi_A(p)$ are defined where $\psi_A(p)$ is the number of the steps of an algorithm A that are needed to resolve the problem p, and $\varphi_A(p)$ is the number of problem p solutions searched by algorithm A. $\psi_A(p)$ is also equal to the number of the nodes of the branching tree. For the dichotomic branching $\psi_A(p) = 2\varphi_A(p)-1$.

The author selects classes of algorithms and boolean problems whith the efficiency estimations of the following type $\varphi_A(p) \le \binom{n+1}{k+1}$ where n is the number of variables and $k=k(p)$ is the function depending on parameters of the problem p.

The work describes some problems that for the large classes of algorithms satisfy the folloing equality $\varphi_A(p) = \binom{n+1}{k+1}$, where k is a parameter of a class containing the problem p. For large value of n a and $k \cong \frac{n}{2}$ $\varphi_A(p) \approx 2^n/\sqrt{\pi n}$

The average number of searched solutions $\overline{\varphi}(n)$ on the classes of boolean problems with nonnegative constraint matrices is found for the classes of the algorithms with simple rules of branching variable choice. For the knapsack problem exact formulae are found. Its asymptotic behaviour is as follows: $\overline{\varphi_1}(n) \approx 2^{n+3}/n^2$, $\overline{\varphi_2}(n) \approx 2^{n+1}/n$.

The first formula gives the average number of the searched solutions for algorithms taking into account the ordering of the constraint coefficients. The second formula gives the average value for algorithms with a rule of the branching variable choice ignoring the information about the constraints. Note that even in the first case the average value behaves as exponent and only as much as n times less than number of searched solutions of problems with the arbitrary number of constraints.

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FINDING OPTIMAL NUMBER, PLACE AND DISTRICT OF TRANSFORMERS IN LOW VOLTAGE ELECTRIC NETWORKS BY A SET COVERING PROBLEM

In this paper we consider as given the three-phase low voltage electric network of a territory together with its topology, technical and economic parameters and the possible settling places of the transformers.

We search for the optimal number and place of transformers, within this the placing of transformers providing minimum loss of energy and those most favourable disjunct electric circuit without loops for which it is true that at the endpoints the voltage is greater than the given value.

One of the four lines of the three-phase low voltage electric network is examined and represented by a connected graph C.

To every possible transformer-place a maximal tree C_{k_i} must be corresponded such that for the voltages at every vertex of the tree the bounding constraint concerning the endpoints should be satisfied. The possibility of constructing of this kind of a tree is proved and two procedures by which one of the maximal trees can be uniquely set up are presented.

We investigate subsets K_q consisting of the indexes of C_{k_i} /$k_i \in K$/ which satisfy:

$$\sum T_{k_i} \delta_{k_i} \longrightarrow \min$$

$$C = \bigcup_{k_i \in K} C_{k_i}$$

$$U_{k_i x} \geq \alpha \qquad x \in S_{k_i}$$

$$\delta_{k_i} \in \{0,1\} \qquad i = 1,2,\dots,m$$

where T_{k_i} - cost of investment of k_i-th transformer
$U_{k_i x}$ - voltage in x vertex of tree
α - given constant
S_{k_i} - set of vertexes belonging to tree C_{k_i}
m - number of possible transformers.

This problem can be reduced to a set covering problem in the following way: an A incidence matrix with "o-1" elements is set up, in which to every column a tree and to every row a vertex is uniquely corresponded:

$$a_{ji} = \begin{cases} 0 & \text{if} \quad x_j \notin S_{k_i} \qquad k_i \in K \quad i = 1,2,\dots,m \\ 1 & \text{if} \quad x_j \in S_{k_i} \qquad j = 1,2,\dots,n \end{cases}$$

where n is the number of vertexes of graph C.

After this the corresponding set covering problem can be written as the following:

$$\sum_{i=1}^{m} T_{K_i} \delta_{K_i} \longrightarrow \min$$

$$\sum_{i=1}^{m} a_{ji} \delta_{K_i} \geq 1 \qquad j=1,2,\dots,n$$

$$a_{ji} \in \{0,1\}, \quad \delta_{K_i} \in \{0,1\} \quad i=1,2,\dots,m.$$

The set of solutions gives the optimal number and place of transformers in the network, but the trees C_{K_i} still cover each other. For the construction of trees without covering a rule is given and its correctness is proved. On the basis of this rule from the finite set of solutions the one is selected which provides the minimal network loss.

This method is applied in practice too. According to our experiences in practical problems approximately 10% of the costs of investment can be saved.
A FORTRAN program has been developped for the SIEMENS 4004/150 computer of SZÁMGÉP at the request of ÉVITERV.

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THREE QUASI-EQUILIBRIUM ECONOMIC MODELS

As it is well known, some questions of macroeconomy can be described as a standard linear programming problem like

/*/ $\max \{cx \,/\, x \geqq 0 ,\ Ax \leqq b\}$

where A, b, c are constant arrays. The problem /*/ is equivalent to the following one:

If we find the vectors of x,v,p,w satisfying:

/1/ $Ax + v = b$
/2/ $pA - w = c$
/3/ $pv + wx = 0$
/4/ $x,v,p,w \geqq 0$,

then we have a solution for /*/. The vector x shows the activity levels, the elements of vector p are the shadow prices, the vectors v and w contain the slack variables of the primal and dual problems.

Several investigations aim at constructing models having not only satisfactory primal solutions but shadow prices useful and easily understandable for economic policymakers as well. Today this is only a routine task.

The case is a more complicated when "the economist" has some objections. "It is very good - he says - but the wages are high, the interest of capital is low, the prices of agricultural products are somewhat high, ... etc."

Satisfying his extra claims, we add to the original problem /1/ - /4/ the following constraint:

/5/ $pD \geqq f$,

where D and f are constant arrays.

We suppose that there exists a nonnegative vector p satisfying /2/ and /5/.

Generally there is no solution of /1/ - /5/ system.

We can have a solution if we weaken one of the equalities /1/ - /3/.

Satisfying the practical demands, we can solve one of the following three <u>alternative</u> problems:

I. min $\{pv + wx \;/\; /1/, /2/, /4/, /5/\}$

II. min $\{ps \;/\; Ax + v - s = b,\; s \geqq 0, /2/, /3/, /4/, /5/\}$

III. min $\{yx \;/\; pA - w + y = c,\; y \geqq 0, /1/, /3/, /4/, /5/\}$

The economic contents of s and y vectors are the quantities of shortages and the relative profit respectively.

The lecture is closed with the experiments on numerical examples.

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FURTHER RESULTS CONCERNING THE FACIAL STRUCTURE OF THE SYMMETRIC TRAVELLING SALESMAN PROBLEMS

Let $K_n = [V_n, E_n]$ be the complete undirected graph of order n, let T_n be the set of all Hamiltonian cycles of K_n and let $\tilde{T}_n$ be the set of all subsets of the Hamiltonian cycles of K_n. Let $x^F \in \mathbb{R}^{|E_n|}$ be the incidence vector of $F \subset E_n$.

We investigate properties of the symmetric travelling salesman polytopes

$$Q_T^n := \text{conv} \ \{x^T : T \in T_n\}$$

$$\tilde{Q}_T^n := \text{conv} \ \{x^T : T \in \tilde{T}_n\}$$

Extending results of Padberg and Grötschel we prove that generalizations of Chvátal's comb-inequalities, having coefficients 0,1 and 2, provide a very large class of facets of Q_T^n and $\tilde{Q}_T^n$. This result can be obtained by a couple of new and very general lifting theorems which are of the kind: if $ax \leq a_o$ is a facet of $Q_T^n(\tilde{Q}_T^n)$ then $a'x' \leq a_o'$ is a facet of $Q_T^{n'}$ $(\tilde{Q}_T^{n'})$, $n'>n$, provided that (a', a_o') is defined in an appropriate way.

Those inequalities which are facets of both the travelling salesman polytopes and the related 2-Matching polytopes are characterized.

Furthermore we give a result which indicates the degree of difficulty of the travelling salesman problem. We present a class of graphs $G=[V,E]$ containing various known infinite classes of hypohamiltonian graphs which has the property that for each graph G $\sum_{e \in E} x_e \leq a(V)$ is (a) valid for $\tilde{Q}_T^n$, $n \geq |V|$, (b) a facet of $\tilde{Q}_T^{|V|}$, and (c) not even maximal for $\tilde{Q}_T^n$, $n>|V|$.

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STABILITY IN NON-LINEAR PARAMETRIC OPTIMIZATION

Considering the parametric optimization problem

$$P(\lambda,\mu): \quad \sup \{f(x,\lambda) \mid x \in M(\mu)\},$$

we have global and local properties of the maps

$$\varphi : (\lambda,\mu) \to \varphi(\lambda,\mu) \underset{df}{=} \sup \{f(x,\lambda) \mid x \in M(\mu)\}$$

$$\Upsilon : (\lambda,\mu) \to \Upsilon(\lambda,\mu) \underset{df}{=} \{x \in M(\mu) \; f(x,\lambda = \varphi(\lambda,\mu)\}$$

and we give some applications /e.g. vedtormaximum-problems, discretization/. The main point of this paper is the local stability.

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MAINTENANCE SCHEDULING

Given a set of K machines, of capacities a(k), which service requirements r(t), t=1,..,T, the problem is to schedule downtimes of durations d(k) so as to maintain satisfactory service (in some sense). Other resource allocation problems over time admit of similar statements.

In practical problems there are certain conflict and precedence constraints among machines, and other constraints can be expected to arise in not always predictable fashion. Hence it is desirable to work with formulations which lend themselves to solution by commercial codes. These are large integer programs with multiple choice structure and K.T zero-one variables. Computational experience seems scant, but moderately sized problems can now be tackled successfully.

We have been considering, in general, the imbedding of enumerative (or heuristic) techniques in large scale branch and bound codes. For a special form of the maintenance scheduling problem we designed an enumerative algorithm which exploits the special structure and appears to solve specific problems of respectable size (say 30 to 50 machines and 52 time periods) quite readily. At the same time it lends itself well to insertion as heuristic should unforeseen constraints enforce the utilization of a more general mixed- integer programming code.

We have also formulated, in several ways, relaxations of the multiple choice sets (into S2 sets), and have conducted some numerical experiments which indicate that substantial improvements are attainable (in terms of maximal minimum coverage of the requirements), especially when "wrap-around" is permitted.

The excellent behavior of the enumerative algorithm suggests to us that enumeration (such as our method of state enumeration with use of logical implications) is unduly neglected and may be a natural tool for a wide class of scheduling problems.

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OPTIMAL SIZING OF UNDEFINED ROD STURCTURES

Static characterization of rod structures is enabled by matrix equations which form connections among rod ènd displacements loads and outside loading.

The most obvious form to start in order to define the inner forces of the structure is the basic euation synthesizing equations of equilibrium and compatibility.

$$|1| \quad \begin{bmatrix} \underline{\underline{R}} & \underline{\underline{A}} \\ \underline{\underline{A}}^{X} & \underline{\underline{0}} \end{bmatrix} \begin{bmatrix} \underline{Y} \\ -\underline{X} \end{bmatrix} = \begin{bmatrix} 0 \\ \underline{q} \end{bmatrix}$$

The contents of the matrix equation are interpreted as follows:

$\underline{\underline{A}}^{X}$ - coefficient matrix related to inner loads

$\underline{\underline{A}}$ - coefficient matrix relating to deformation

$\underline{\underline{R}}$ - elasticity matrix of the elements of the structure

$\underline{X}$ - vector of deformations

$\underline{Y}$ - vector of the unknown rod stresses

$\underline{q}$ - vector of the external loads

It is apparant from the equation system /1/, to, that for the sizing of undefined structures one has to chose among several different structures which meet the same requirements. It is therefore advisable to make the choice on the basis of an economic criterion. We are going to deal with structures where the weight of the structure will be the economic criterion.

The task is therefore the following:

The structure of minimum weight meeting the functional requirements, i.e. satisfying the equation system /1/ has to be selected from structures of a given geometry. After having written up the task and elamined the functions figuring in the same we discover that this is a non-linear programming task the permissible set of which is not convex.

In case of structures in arbitrary planes we substitute for this task a convex programming task the optimum value of which concides with the optimum value of the original task. The solution of the task requires an efficient algorithm eliminating the difficulties connected with the writing up of equation system /1/. Instead of equation system /1/ we perform a convex quadratic programming task of good quality. The algorithm chosen in accordance with the structure of the task. This algorithm was tested in case of simpler structures.

Thereafter we carry out the optimal sizing of a simple undefined rod structure.

The testing of the method is in process, for we rely on the same in sizing the quarter panels of autobusses by order of the Ikarusz factory.

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A QUASIBARRIER METHOD FOR CONVEX PROGRAMMING

It is known that a convex programming problem

$$\text{Min } \{ f(x) \mid g_i(x) \geq 0, \quad (i = 1,2,\ldots,m) \} \qquad /1/$$

can be reduced to a sequence of unconstrained minimization problems /e.g. SUMT of Fiacco and McCormick/. In the case of parametric interior point methods this is achieved by adding to the objective function $f(x)$ a barrier term $r\gamma(x)$ that favors points interrior to the feasible region over those near the boundary. As parameter $r > 0$ approaches zero, the unconstrained minimum $x(r)$ of barrier function

$$B(x,r) = f(x) + r\gamma(x) \qquad /2/$$

tends to the optimal solution x of the original problem /1/.

Fiacco and McCormick gave the convergence proof for a general barrier method with the following properties of $\gamma(x)$:

a/ $\gamma(x)$ is convex on the set
$K^o = \{ x \mid g_i(x) > 0, \quad (i = 1,2,\ldots,m) \}$,

b/ if $\{y^j\} \subset K^o$ converge to the boundary point of K^o, then $\lim_{j \to \infty} \gamma(y^j) = \infty$.

The known examples of such barrier functions are

$$P(x,r) = f(x) + r \sum_{i=1}^{m} \frac{1}{g_i(x)} \qquad /3/$$

$$L(x,r) = f(x) - r \sum_{i=1}^{m} \log g_i(x) \qquad /4/$$

The paper treats the case of another barrier function

$$Q(x,r) = f(x) - r \sum_{i=1}^{m} \sqrt{g_i\, x} \qquad /5/$$

which does not have property b/. In this case the barrier on the boundary is made only by the descent of $Q(x,r)$. This is the reason we refer to the $Q(x,r)$ as to a quasi-barrier function. The convergence proof is given under conditions analogous to those of Fiacco-McCormick and it is shown that the convergence rate is better.

For barrier functions /3/ and /4/ Fiacco- McCormick and Lootsma show that

$$f(x(r)) - f(\hat{x}) = \alpha\sqrt{r} + O(r) \qquad /3a/$$

$$f(x(r)) - f(\hat{x}) = \beta r + O(r^2) \qquad /4a/$$

In the case of the quasibarrier function /5/

$$f(x(r)) - f(\hat{x}) = r\,\omega(r)$$

where /5a/

$$\lim_{r\to 0}\omega(r) = 0$$

Moreover, if there exists a continuously differentiable trajectory $x(r)$, then

$$f(x(r)) - f(\hat{x}) = \delta r^2 + O(r^3) \qquad /5b/$$

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CLUSTER ANALYSIS BY GRAPH COLOURING

Given a set X of n objects and a distance matrix $D = (d_{ij})$ expressing the dissimilarity between pairs of these objects we want to partition X into $k < n$ clusters in order to minimize the maximum distance between objects in the same cluster. This problem is reduced to the problem of finding colorings with a minimum number of colors for a sequence of graphs with n vertices, associated with the objects, and an increasing number of edges. Bichromatic interchange as well as a new exact coloring algorithm are used to this effect.

The method has been programmed and tested on an IBM 370/125; it allows to solve exactly real problems with more than 200 objects. Moreover, for the examples solved, it takes less computing time than the well-known hierarchical complete link clustering method, which often provides only suboptimal solutions.

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ECONOMIC DISPATCH IN ELECTRIC POWER SYSTEMS

Mixed Integer Programming, based on Linear Programming, is the only one non-linear optimization method available as standard program for computers.

This paper aims for a general solution of the economic dispatch problem using Mixed Integer Programming.

The physical reality of any kind of power system operation including pumped storage plants be modeled for MIP-application. The model consists of an objective function (e.g. minimum fuel costs, minimum emission dispatch) and different constraints taking into account operational and security aspects. An approach to include power system losses is mentioned.

Based on 14 specifications defined in literature for practically usable economic dispatch models it is proved that MIP is best suited for the solution of the given problem. Furthermore, MIP allows a clearly structured mathematical model. Compared versus other similar powerful optimization methods the results obtained are more accurate with less computer running time. As a further advantage the results of the problem under discussion are available in a shorter time compared to individually programmed algorithms. There are also savings in the range of $ 25.000,- for programmers.

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A NONLINEAR PROGRAMMING MODEL IN THE PRODUCTION OF METALLIC ALLOYS

The model has been made for preparing the optimal planning of production in a company where metallic alloys are made. The aim of the company is to maximize its profit. One way to achieve this aim is to reduce the prime cost of their products by using spent basic material besides of unalloyed metal and alloying elements. The spent basic material contains alloying and contaminating materials. The use of spent basic material is bouned by the qualitative prescriptions of standard and the technological possibilities of the company. The other way to increase profit is the right choosing of the combination of the different products.

Considering the two possibilities a nonlinear programming model could be set up. With the task being solved the results mean how to choose the quantities of every single product and the quantities of the differently mixed spent basic materials used in unit quantity of every single product to achieve the maximum profit. Denote n the number of the products and m the number of the differently mixed spent basic materials. The number of the variables depends on n and m by the relation $(n+1)\cdot m$. In our special case there are 19 products and 12 spent basic materials, so we have 240 variables. There are linear constrains, and m number nonlinear ones, and also the objective function in nonlinear. The nonlinearity is caused by the expressions containing the sum of cross products of variables.

The algorithm that has been made to solve the problem is based on two linear programming tasks which are successively solved, while the value of the objective function can be increased. The optimal solution of the first task is the starting point of the second one and vica versa.

A linear programming package developed for the computer ICL System 4 was used for the practical carrying out. There are possibilities to store the basic data on magnetic disc to write the results onto magnetic tape and to read the data for revising the basic data from magnetic tape. An inserted FORTRAN IV program organizes the computer run and transforms the input data into output data according to the programming package.

The company which produces aluminium alloys employs the model in every quater of the year. It is cleared by the results what the profitable and unprofitable products are and the basis of this is the marketing strategy prepared. In this way the profit of the company has been significantly increased.

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ON THE THEORY OF SEARCH, A CASE WHERE THE MOTION OF THE TARGET IS RANDOM.

It will be assumed that the location of the target in R^n is given in terms of density function $u(x,t)$ which satisfies, if no search has been going on, an equation of parabolic type with constant coefficients /eq. (1) below with $\lambda \equiv 0$ /. The system of search will be represented by the so called search density function $\lambda(x,t)$ /see ref. [1] /. Search density function is assumed to have the following properties: 1/ $\lambda(x,t) \geq 0$ and 2/ $\int_{R^n} \lambda(x,t)dx = L_0$ - const. Furthermore $\lambda(x,t)$ will be different from zero in a region $A(t)$ with a moving boundary $S(t)$. One now wants to maximize the probability of detection of the target, given that the time available to the search, T is given. There is now the following optimal control problem for the determination of the optimal search density $\lambda(x,t)$ to solve:

$$/1/ \quad (\partial/\partial t)y(x,t) = k^2 \sum_i \sum_j a_{ij} (\partial^2/\partial x_i \partial x_j) y(x,t) - \lambda(x,t)\, y(x,t)$$

$$y(x,t) = \psi(x)$$

$$y(x,t) \to 0 \quad \text{as } |x| \to \infty$$

$$\int_{R^n} y(x,T)dx = \text{Min}$$

In ref, [2] a condition of optimality was obtained with the consequence that function $y(x,t)$ must satisfy, for $x \in A(t)$, equation

$$/2/ \quad k^2 \sum_i \sum_j a_{ij} (\partial^2/\partial x_i \partial x_j) \ln y(x,t) = \Lambda(t)$$

where $\Lambda(t)$ is an unknown function, to be determined later, and, for $x \notin A(t)$ equation

$$/3/ \quad k^2 \sum_i \sum_j a_{ij} (\partial^2/\partial x_i \partial x_j)\, y(x,t) = (\partial/\partial t)\, y(x,t)$$

The solutions of the above equations must meet on the boundary $S(t)$ in such a way that $y(x,t)$ and $n \cdot \nabla y(x,t)$, n the outward normal of $S(t)$ are continuous across $S(t)$.

In the present paper the Fourier integral approach will be used to produce a differential-integral equation for the Fourier transform of $y(x,t)$. There is no straightforward way to solve the equation.

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A RELAXATION METHOD WITH APPLICATION IN DIAGNOSTIC RADIOLOGY

Methods to obtain the internal structure of an object from its projections have been found useful in a large number of sciences and have, in particular, revolutionized diagnostica radiology over the last three years. In that field, the problem is to estimate the density distribution in a cross section of an object from x-ray data, which in practice is unavoidably noisy. The data gives rise to a large sparse system of inconsistent equations, not untypically 10^5 equations with 10^4 unknowns, with only about 1 % of the coefficients non-zero. Using the physical interpretation of the equations, each equality can in principle be replaced by a pair of inequalities, giving us the limits within which we believe the sum must lie. Previously /Mathematical Programming 8 /1975/ 1-19/, we proposed a finitely convergent relaxation method for solving this set of inequalities. That algorithm has produced a vector satisfying the constraints but not one which was optimum by any reasonable criterion. In this paper we propose an alternative relaxation method designed to converge to the minimum norm solution satisfying the constraints. We discuss the usefulness of minimizing the norm in the application area of interest. The algorithm has been implemented and is demonstrated by reconstructions of cross sections of the thorax from actual x-ray data.

Research is supported by NSF grant MCS 75-22347, NIH grant HL 4664 and NCI contract NO1-CB-53860.

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OPTIMAL DISPATCHING PROBLEM.

We consider a factory producing homogene production and supplying its consumers during a period of t days (a month, for instance). In the beginning of the period the factory has a_0 units of production which can be dispatched in the first day (or later). In the first day a_1 units of production are produced and they can be dispatched in the second day or later and so on.

The factory has three groups of consumers. There are n_1 consumers (having numbers from 1 to n_1) in the first group, the consumers of the second group have numbers from n_1+1 to n_1+n_2 and the ones of the third group have numbers from n_1+n_2+1 to $n_1+n_2+n_3$. The factory must supply the consumers of the third group three times during the period. It has to send to the ℓ-th consumer of this group c_ℓ^1 units of production to the end of the period, c_ℓ^2 units to the 20-th day and c_ℓ^3 units to the 10-th day. The consumers of the second group must be supplied in analogical conditions twice during the period and the ones of the first group once during the period.

Every consumer can be supplied only in fixed days given by a $t\times q$ -matrice $\mathcal{D}^1=(d_{ij}^1)$, a $t\times(n_2+n_3)$ -matrice $\mathcal{D}^2=(d_{ij}^2)$, and a $t\times n_3$ -matrice $\mathcal{D}^3=(d_{ij}^3)$. Here $q=n_1+n_2+n_3$ and $d_{ij}^m=1$ iff it is possible to send in i-th day the m-th consignment to the j-th consumer, otherwise $d_{ij}^m=0$. For each consumer the time of circulation of the documents (an integer $t_j>0$) is given). The factory gets money from the j-th consumer to the end of the period if the production is sent not later than in the $(t-t_j)$-th day. The price of the product is proportional to is quantity.

We consider the following problems.

1. By which conditions on the numbers a_i (the c_j^m are fixed) the factory is able to fulfil all orders of the consumers?

2. By which conditions on the numbers a_i the factory is able to fulfil all orders and can get money from all consumers before the end of the period?

3. If for given a_i it is impossible to get money from all consumers to the end of the period, how must the factory dispatch the production to get maximum money to this time?

We give mathematical formulations of these problems. The problems are reduced to the case when the factory has fictive consumers getting production only once during the month. The formulae are given which express the new variables through old ones. We get problems with more convenient formulations.

Solution of the problems 1 and 2 reduces to obtaining the conditions of the solubility for a system of linear inequalities. These conditions are formulated.

For solving the problem 3 we make a second reduction using so called special solutions. The problem 3 is reduced to solving of a multidimensional knapsack problem. Three algorithms are worked out. These are an algobithm writing out the corresponding multi-dimensional knapsacknproblem, an algorithm solving this problem and an algorithm distributing the consiguments more uniformly during the period.

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ON SOME RECENT DEVELOPMENTS IN BILINEAR PROGRAMMING

Bilinear Program is a class of nonconvex quadratic program with the following structure:

$$\max \{c_1^t x_1 + c_2^t x_2 + x_1^t C x_2 \mid A_1 x_1 \gtreqless b_1,\ x_1 \geq 0,\ A_2 x_2 \gtreqless b_2,\ x_2 \geq 0\}$$

The special structure of this program enables us to develop algorithms to obtain a global solution. Also, it has a big potential for practical applications. We will discuss such aspects of bilinear programming as:

(i) relationship to other classes of mathematical programs,

(ii) recent algorithmic developments and the results of experiments,

(iii) some applications of theoretical and practical interests.

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NETWORK IMPROVEMENTS VIA MATHEMATICAL PROGRAMMING

A class of network improvement problems is formulated in general terms as an integer nonlinear programming model. The model can be used as a decision-making aid to improve/design transport networks, packet-switching computer communication networks, etc, subject to a budget constraint. Branch and bound schemes have been suggested to solve the problem, assuming that the total user's costs (benifits) are monotonically decreasing (increasing) when projects are being implemented. Bounding and evaluating processes are carried out by solving minimum convex cost multicommodity flow problems, known as stationary traffic assignment/message routing problems. Another approach to selecting an optimal network consists in using Generalized Benders' Partitioning Algorithm to separate the continuous part of the model from the discrete one. Then, one solves minimum convex cost multicommodity flow problems in order to obtain cutting planes for selecting projects using optimal dual variables.

The objective of this study is to evaluate the computational performances of the two approaches to selecting an optimal network mentioned above. Efficient bounding processes are suggested, and special considerations for solving successive minimum convex cost multicommodity flow problems emphasized. The question of obtaining an acceptable approximate solution is also investigated.

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A programming system for choosing algorithms which solve minimizing problems with constraints

We presenting a unified implementation of iterative optimization methods which have the following structure: In each step the direction of descent is determined by a direction finding problem (linear or quadratic). The system contains for example the P1- and the P2-algorithms of ZOUTENDIJK with various anti-zickzagging rules, the methods of feasible directions of SUCHOVITSKII, POLJAK, PRIMAK, the mehtod of PSCHENITSCHNYI, the method of POLJAK, TRETJAKOV and many new algorithms which converge sublinearly or linearly. Together with some step-length algorithms (Goldstein-steplength, minimization step-length, quadratic approximation and others) the system contains more than 250 numerical algorithms. If it is given a class of practical optimization problems then we can say with our system, what is an effective algorithm (between the implementated) of solving problems of the given class relative to one of the following properties: computing time, number of iterations, behaviour near the optimum or stationary points, accuracy of the solution. We inform about some experiments with our system.

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PARAMETRIC ANALYSIS FOR INTEGER PROGRAMMING PROBLEMS.

The theory for sensitivity analysis and parametric programming in linear programming has long been developed and its value recognized. Little success has been reported in this area for the integer programming problem. A parametric integer programming procedure has been developed here for the right hand side coefficients. The method requires the use of algorithms employing Gomory cuts for the solution of the integer problem. Otherwise it is patterend along lines similar to those of the noninteger problem.

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ANALYSIS OF ECONOMICS STRUCTURE DEVELOPMENT BY MEANS OF MATHEMATICAL PROGRAMMING

By economics structure we mean grouping of economic or non-economic organizations into branches of national economy. The structure is measured by the portion of production or national income on total volume of production or of the national income.

Analysis of economics structures by means of mathematical programming is done in two ways :

1. It becomens a part of the complex models of national economy. In this case the analysis of economics structure is an independent problem or an independent model.
2. Under simplification conditions it is formulated as a problem of mathematical programming in the development of economics as a whole. By simplification conditions we mean a simplified expression of some parts of economics and also the transformation of some parts of an open model into a close one. It is firstly a question of expressing the increase of personal and social consumption, export and import, as the functions of production growth.

Methodical approach and the practical experiences obtained with the solution of the first problem as that of the goal programming of the form

$$\min d/x_t, \bar{x}_t/ \leq \rho \bar{x}_t$$

$$g_j/x_t, \bar{x}_t/ \geq 0 \qquad j = 1, \dots, n$$

$$x_t, \bar{x}_t \geq 0 \qquad t = 1, \dots, T$$

are analyzed in the report, where

$x_t, \bar{x}_t$ - vectors of the solution of the given problem and the goal vector,

$d/x_t, \bar{x}_t/$ - the quadratic criterial function /the discance function/,

$g_j/x_t, \bar{x}_t/$ - the boundary conditions the form of which depends on the type of production functions being used in the problem and,

$\rho^{\bar{x}_t}$ - the distance from the given goal stated beforehand.

Expression of the time lag between investing and introducing the investments into operation aproximates linearly under Leontief's production functions. The corresponding problem is also used for purposes of simulation. The parameters of the simulation are those of the corresponding production function as well as the resources of the system.

The report will present the results of Slovakia's economics up to the year 1980. The paradox of the structure will be mainly analyzed, which, with the reduction of the speed of production growth appears in the form of mid-term increase of personal and social consumption.

The problem 2/ has the form of Kantorovitchs problem with the previously given end structure. It differs in the functional expression of personal and social consumption, export and import, as the functions of production growth. In this way a certain simplification of the given problem solution is obtained with the simultaneous removal of negative results of the original task solution. The results of the solution and the problems connected with the solution, or the comparison with other approaches, will be presented in the report.

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ON THE SOLUTION OF TRAVELLING SALESMAN PROBLEMS

We consider the travelling salesman problem formulated as a linear programming problem in integers. Extending algorithmic ideas first proposed by Dantzig, Fulkerson and Johnson in 1954, we first derive two new classes of valid inequalities for the travelling salesman problem that are necessary to implement their approach to proving optimality of a given tour of the travelling salesman. In conjunction with the known classes of valid inequalities for this problem, i.e. subtour elimination and Chvatal's comb inequalities, an improved, thogh still partial, linear characterization of the /symmetric/ travelling salesman polytope is obtained. The algorithmic approach to proving optimality of a tour /which may e.g. have been obtained by using any of the heuristic approaches to the problem/ resembles much that one of a primal approach to integer programming and has been implemented on the computer. Computational experience with randomly generated problems as well as some problems taken from the literature is reported.

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VERALLGEMEINERUNGEN DES ZENTRUMPROBLEMS

Zu den in der Ebene gegebenen Punkten

$$P_1 , P_2 , \dots , P_n$$

kann man ein Zentrum P so bestimmen, dass seine gewichtete Entfernungssumme /Potenzsumme/ von den Punkten P_i

$$S = \sum_{i=1}^{n} s_i \cdot \overline{PP_i}$$

minimal ausfällt.

Das Zentrum P kann durch sukzessive Approximation bestimmt werden, wo für die einzelnen Schritte das gewichtete Mittel der Punkte P_i genommen wird, wobei das Gewicht der einzelnen Punkte mit s_i proportional und mit der Entfernung der vorhergehenden Approximation von P_i umgekehrt proportional ist.

Ein ähnliches Verfahren kann im Falle von gas- und oeltransportierenden Rohrleitungssystemen für die Bestimmung von Zentren angewendet werden, die auch miteinander verbunden sind.

Wir können zu einer andersartigen Verallgemeinerungen gelangen, wenn statt der Punkte P_i Kreisscheiben von gegebenem Durchmessern betrachtet werden, zu welchen Verteilungs- bzw. Sammelstationen gesucht werden. Im Falle von gleichen Kreisdurchmessern ist die Aufgabe equivalent mit der Bestimmung des Kreises, dessen /gewichtete/ Entfernungssumme von den in der Ebene gegebenen Punkten minimal ausfällt. Diese letztere Aufgabe ist auch von Interesse in Zusammenhang mit den geodetischen Ausgleichungsverfahren.

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BOUND ON THE EXPECTATION OF A CONVEX FUNCTION OF A RANDOM VARIABLE: WITH APPLICATIONS TO STOCHASTIC PROGRAMMING

This paper is concerned with the determination of tight lower and upper bounds on the expectation of a convex function of random variable. The classic bounds are those of Jensen and Edmundson-Madansky. They were recently generalized by Ben-Tal and Hochman. This paper indicates how still sharper bounds may be generated based on the simple idea of sequentially applying the classic bounds to smaller and smaller subintervals of the range of the random variable. The bounds are applicable in the multivariate case if the random variables are independent. In the dependent case, bounds based on the Edmundson-Madansky inequality are not available, however, bounds based on Jensen's inequality may be developed using the conditional form of Jensen's inequality. Some examples are given to illustrate the geometrical interpretation and the calculations involved in the numerical determination of the new bounds. Special attention is given to the problem of maximizing a nonlinear program that has a stochastic objective function.

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OPTIMAL MINING IN A DECENTRALIZED FRAMEWORK

The purpose of this paper is to investigate the rate of optimal extraction of a mineral deposit containing various grades of ore. The mine manager faces the demand of these ore and has the possibility of transforming, at some cost, one grade of ore to the higher grade. The objective of the manager is to maximize the present value of a stream of profit (social production surplus) over a given planning horizon. Either in the market setting or in a planned economy, the problem, so far described, can be neatly formulated as a multi-state variables optimal control problem.

The section 1 of the paper is devoted to a full description of our model. A novel aspect of this model is the consideration of upgrading activities which take into account the effect of the stocks of ore upon the efficiency of mining.

In section 2, we deal with the necessary conditions which allow for the determination of the optimal rate of exploitation and upgrading effort associated to each grade of ore. The main result is that under quite general conditions, the decision making of the mine manager can be decentralized completely in the sense that each grade of ore can be exploited separately as an exhaustible resource. If the cost of extraction is a linear function of the amount extracted, the mining rate is arbitrary. Under more general condition however, this rate is determined according to a linear feedback law. These findings make a sharp contrast with somes established theoritical results in the literature of resource economics.

Using the dynamic programming approach, the sufficiency of the optimality conditions in our model is proved in section 3.

Finally, in section 4, we show that our main results hold when stochastic perturbations are introduced into the model under the form of Ito differential equations with regard to the state variables changes.

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THEORETICAL CONSIDERATION ON THE COMPUTATIONAL PERFORMANCE OF BRANCH - AND - BOUND ALGORITHMS

General properties of branch-and-bound algorithms are investigated from the view point of their computational efficiency, based on a formal model which incorporates lower bound test, dominance test and upper bounding of the incumbent to prune partial problems not yielding an optimal solution, and a margin ε to facilitate the computation by being satisfied with a suboptimal solution within ε from the optimal. The parameter used to measure the efficiency is the number of partial problems decomposed before termination. Although it is commonly conjectured from intuitive understanding that the efficiency of a branch-and-bound algorithm always becomes higher if each constituent is improved (e.g., a tighter lower bound is obtained, ε is set larger and so on), many of them turn out to be false depending on search strategies (such as best-bound search, depth-first search, breadth-first search and heuristic search). In addition to giving counterexamples to the false conjectures, various cases in which such conjectures hold true are also clarified. Finally these theoretical results are compared with some simulation results.

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THEORY OF FLOWS IN CONTINUA AS APPROXIMATION TO FLOWS IN NETWORKS

A formalism is proposed to treat various problems concerning flows in compound systems consisting partly of ordinary discrete networks and partly of (two- and three-dimensional) continua. Basic theorems in network-flow theory are extended to the compound systems. Flows and tensions are adequately dealt with as distribution (in Schwartz'sense) or currents (in de Rham's sense), where it is emphasized by which kinds of "tensors" the fundamental concepts such as flows, tensions, capacities, distances, costs, etc. should most appropriately be represented. In addition to "smoothness" of the fields, the "anisotropy" of each point of the field is important. In place of "intervals" for flows in and tensions across branches of a networks, a "convex set (of the vector space of flows or tensions)" or a "convex function" is associated with each point of the continuum.

Conceptually, a continuum is an approximation to a vast, locally homogeneous and very fine network. On the other hand, from the standpoint of numerical solution, a dicrete approximation of continuum is needed. These two sorts of approximations between discreta and continua will be discussed.

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AN ALGORITHM FOR SOLVING THE TRANSPORTATION PROBLEM WITH MULTIPLE LINEAR OBJECTIVE FUNCTIONS

Consider the linear multiple objective transportation problem

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{1} x_{ij}$$

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{2} x_{ij}$$

$$\vdots$$

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{k} x_{ij} \qquad /1/$$

subject to

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad (i = 1,\ldots,m)$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad (j = 1,\ldots,n)$$

$$x_{ij} = 0 \qquad (i = 1,\ldots,m; j = 1,\ldots,n).$$

Problem /1/ can be reexpressed in vector-minimum form /2/

$$\text{"min"}\left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \,\middle|\, \sum_{j=1}^{n} x_{ij} = s_i, \sum_{i=1}^{m} x_{ij} = d_j, x_{ij} \geqq 0, \quad (i = 1,\ldots, m ; \quad j = 1,\ldots,n) \right\}, \qquad /2/$$

where c_{ij} is a k-dimensional column vector and "min" signifies that the purpose of this problem is to find all efficient solutions. Recall that a feasible solution

x^{o}_{ij} $(i = 1,\ldots,m ; j = 1,\ldots,n)$ is an efficient solution for /2/ if and only if there does not exist another feasible solution x^{1}_{ij} $(i = 1,\ldots,m; j=1,\ldots,n)$ such that $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x^{1}_{ij} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x^{o}_{ij}, \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x^{1}_{ij} \neq \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x^{o}_{ij}$

Efficient solutions for /2/ are of interest because the decisionmaker's optimal solution is efficient.

Up to now no transportation method is known by which /2/ can be solved. In this paper a multiple objective simplex transportation method is presented. This algorithm comprises three phases.

Phase I : Find an initial efficient basic solution for /2/

Phase II : Find all remaining efficient basic solutions for /2/

Phase III : Find the set of all efficient solutions for /2/.

The paper contains a complete description of the algorithm, of the underlying theory including all proofs as well as several illustrative examples.

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MULTI-COMMODITY FLOW APPROACH TO ASSIGNMENT OF CIRCUITS IN CASE OF FAILURE IN A COMMUNICATION NETWORK

In modern communication networks, switching facilities and redundant transmission channels are provided to keep communication circuits in service even in case of failure on some links in the network. Then, there arises a problem of determining the most efficient utilization of the redundant channels. This problem can be formulated as a kind of multi-commodity flow problem on a graph which represents the network consisting of the redundant channels.

In this paper, heuristic methods which consist of phase I and II are considered. In phase I, a starting solution is found by assigning each demand one by one to the minimun cost path, where the edge cost is a monotonically decreasing function of the number of unused redundant channels in the edge. The minimum cost path is determined by the following factors: number of links, number of unused channels in the path and demand assignment order. This step terminates either when all the demands are satisfied or when no more flow can be assigned because of capacity constraints. It is experimentally proved that a careful choice of cost function leads to a good starting solution which remarkably saves computation time in phase II.

When all the demands are not satisfied in phase I, we proceed to phase II improvement step. Two types of improvement methods are tried. In Method I, we take a flow demand which has not been assigned, and try to satisfy it by changing the path of a flow already assigned. The improvement step of Method II is a modification of the incremental assignment method to discrete flow.

Experiments show that both methods give solution very close to the optimum in satisfactory computation time, while branch and bound method which is implemented for comparison requires unformidable computation time for networks of practical size. Method I is found to be more effective when the network contains edges with only one or two redundant channels.

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THE NEXT GENERATION OF BRANCH AND BOUND CODES

The present generation of branch and bound codes for solving general integer linear programming problems, have concentrated on elaborate tree development heuristics based on dichotomies represented by changes to the bounds of the integer variables. This paper considers a variety of additional possibilities and examines their likely effect on the next generation of branch and bound codes.

Features considered include, other linear dichotomies, the generalization of special ordered sets, cutting plane algorithms as a special case of generalized branch and bound, user selection of dichotomies and other tree development heuristics during execution, together with post-optimal analysis and the use of information determined during the solution of one problem, to speed the solution of a closely related problem.

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A FINITELY-CONVERGENT CUTTING-PLANE ALGORITHM FOR GENERALIZATIONS OF THE LINEAR COMPLEMENTARITY PROBLEM

We use recent results of cutting-plane theory to develop interconnections between integer programming and linear complementarity, as exemplified by the following result. Theorem: If

$$\{(x,y) \mid Ax + By \geq d,\ x \geq 0, y \geq 0\}$$

is bounded and non-empty, then any valid cutting-plane for the complementarity constraints

(CMC) $$Ax + By \geq d,\ x \geq 0, y \geq 0,\ x \cdot y = 0,$$

is obtained by starting from the linear defining inequalities

$$Ax + By \geq d,\ x \geq 0, y \geq 0$$

and applying, finitely often, the following two rules (the second for $j = 1,\ldots,n$):

(i) Take non-negative combinations of given inequalities, and possibly weaken the right-hand-side.

(ii)$_j$ Having already obtained two inequalities

$$\alpha_1 x_1 + \ldots + s x_j + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + t y_j + \ldots + \beta_n y_n \geq \alpha_0$$

$$\alpha_1 x_1 + \ldots + s' x_j + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + t' y_j + \ldots + \beta_n y_n \geq \alpha_0$$

one may deduce

$$\alpha_1 x_1 + \ldots + s' x_j + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + t y_j + \ldots + \beta_n y_n \geq \alpha_0 .$$

Conversely, any inequality thus obtained is valid for the complementarity constraints.

The proof of this result is constructive, and it supplies a finitely-convergent cutting-plane algorithm for this generalization (CMC) of the linear complementarity problem.

Note that the constraints (CMC) properly include the linear complementarity problem. Indeed, the constraints of a bivalent integer program can be cast in the form (CMC), by rewriting these constraints

$$Dx \geq d$$

(BIP) $$x_j = 0 \text{ or } 1,\ j = 1,\ldots,n$$

in the form

$$
(BIP)' \quad \begin{array}{lcl} Dx & \geq & d \\ x_j + y_j & = & 1,\ j = 1,\dots,n \\ x_j y_j & = & 0,\ j = 1,\dots,n \\ x_j, y_j & \geq & 0,\ j = 1,\dots,n \end{array}
$$

and obtaining all valid cutting-planes via rules (i), $(ii)_j$.

The Theorem announced above is one corollary of a considerably more general result concerning what Balas has called "facial constraints." The general result will be given at the meeting, but is too technical to state here. Those interested may write the author for preprints.

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THE LINEAR COMPLEMENTARITY PROBLEM AND A TREE SEARCH ALGORITHM FOR ITS SOLUTION.

Consider the linear complementarity problem

$$w = q + Mz \quad , \quad w \geq 0 \ , \ z \geq 0 \text{ and } zw = 0 \ .$$

The authors have developed a tree search algorithm for the solution of this problem. Unlike any other known algorithm this algorithm makes no assumption concerning the nature of the matrix M and finds all the solutions of the problem if these exist. If no solution exists to the problem then this can also be established by this method.

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DEVELOPMENT OF A SUBADDITIVE APPROACH TO INTEGER PROGRAMMING

The subadditive approach to integer programming began with Gomory's work relating subadditive functions on group elements to facets of Gomory's corner polyhedra. One way of viewing his work is to form the dual programs:

min c t subject to:	max $\pi\ (g^o)$ subject to
$\sum_{j=1}^{n} g^j\ t_j \equiv g^o$, and	$\pi(g^j) \leq c_j$, and
$t_j \geq 0$, integer, $j=1,\ldots,n$,	π subadditive on G

where G is a finite Abelian group, g^j and g^o belong to G, and $\equiv$ means equality in the group. Subadditivity of π on G means:

$$\pi(y + h) \leq \pi\ (g) + \pi\ (h)$$

for all g, h in G (regardless of whether g and h are among g^j, g^o).

Equality of these two objectives follows from Gomory's work on facets of master group polyhedra. As usual in mathematical programming duality theorems, one direction is easy and the other difficult. Gomory's results sharpen the difficult half of the theorem by requiring that π be non-negative and satisfy

$$\pi(g) + \pi\ (g^o\text{-}g) = \pi(g^o)$$

in addition to subadditivity.

This paper will relate the further development of this theory and attempts to use it to provide a new algorithmic base for solving integer programs.

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OPTIMIZING REMOTE SWITCHING DESIGN AND ADMINISTRATION

For subscriber loop multiplexors and other digital carrier and switching systems used to concentrate rural traffic, the design and administration of the system can be optimized by means of a dynamic programming algorithm. The algorithm is presented with illustrations as to its use, and suggestions for extensions to computer timesharing.

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LOCATING STEPPING-STONE PATHS AND ASSIGNING DUAL PRICES IN MULTI-INDEX TRANSPORTATION PROBLEMS

The multi-index transportation problem (TPn) is an extension of the transportation problem (TP2) of linear programming to a problem with multiple subscripted variables $x_{i_1 \ldots i_n}$. Its constraints may be sums of these variables or double sums, triple sums etc. or combinations of them.

Several problems can be formulated as a TPn. So the well-known problem of shipping a homogeneous product from several factories to several markets at minimal transportation costs will become a three-index problem if there are different kinds of transport. An example for a four-index problem is a timetable problem where masters, classes, rooms, and periods are to be combined.

It is possible to solve a TPn by an extension of the modi-method for the solution of the TP2. HALEY resp. CORBAN have shown this for n=3, and it can be extended to any n. But they do not say anything about the computational aspect. So the determination of a stepping-stone path or the calculation of dual prices proves to be a very hard work in a n-dimensional transportation tableau or has to be done by solving a large sparse system of linear equations. Another trouble is the fact that a basis of the TPn can be non-triangular.

It will be shown in the talk how these difficulties can be overcome. It is possible to define a directed graph for any triangular basis by aid of which all the computations concerning the n-dimensional transportation tableau can be carried out. Also, a non-triangular basis can be handled in a similar way. Thus, all the remaining computational aspects of solving a TPn can be managed.

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LONG TERM OPTIMIZATION OF ELECTRICAL SYSTEM GENERATION BY CONVEX PROGRAMMING

The report describes a mathematical model for system generation long term planning. Each year new capacities of various types must be commissionned to keep the risk at the required level; at the same time, the total present-day value of investments and operation costs for the whole study period must be minimized.

By assigning a guaranteed capacity to each type of unit, the risk level requirements can be expressed with a set of yearly linear constraints linking the total capacities of each type of unit.

Furthermore, each year new capacities and total capacities of each type of units can be limited to minimum or maximum values. The resulting convex programme is solved with a feasible direction method: each iteration, the locally best direction is computed by a process similar to the gradient projection method. The choice of such a process on the special characteristics of the problem, i.e.:

- the economic function compution is time-consuming because there is no analytical expression for the operation costs, these must be an algorithmic process.
- the locally best direction is easy to find thanks to the special form of the constraints; this direction is computed by an iterative process involving a small linear system.

The mathematical model described is currently used at CPTE for the Belgian power system planning.

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AN APPLICATION OF GEOMETRIC PROGRAMMING TO THE GRAVITY MODEL OF TRIP DISTRIBUTION /Abstract/

The solution of the doubly constrained gravity model:

$$/1/ \qquad x_{ij} = r_i e^{\alpha c_{ij}} s_j$$

$$r_i, s_j > 0 \qquad i=1,\dots,m;\ j=1,\dots,n$$

$$/2/ \qquad \sum_{j=1}^{n} x_{ij} = a_i \qquad i=1,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j=1,\dots,n$$

$$/3/ \qquad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} = C$$

is an important task in many practical problems of trip distribution and spatial interaction modelling. It is known that for a given value of the gravity parameter α the solution of /1/-/2/ is unique. To fix α either the additional constraint /3/ or an other criterium should be added. The paper proves the equivalence of a natural criterium - the minimization of the information divergence between the observed (t_{ij}) and the forecast (x_{ij}) tables with the approach of entropy maximizing on the one hand, and the model /1/-/3/ on the other. The proof of the equivalence of these three approaches makes use of the duality theorem of geometric programming. In the second part of the paper an algorithm is given for the calculation of α. It is shown that the solution α of /1/-/3/ is a strictly increasing differentiable function of C, and with the calculation of the derivative of its inverse function, $\frac{dC(\alpha)}{d\alpha}$ a good estimate for α, and also an algorithm for its accurate calculation can be presented. Some numerical experiments are also reported.

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COMPACTIFICATION AND DECOMPOSITION METHODS FOR NLP PROBLEMS WITH NESTED STRUCTURES IN HYDRO POWER SYSTEM PLANNING.

In hydro system extension planning it is necessary to study the operation of alternative power systems. To this end, we must determine the optimal operation under conditions of average water yield."Optimal" means in this context that, for meeting a certain demand for power,storage and pondage stations (short-term and long-term reservoirs) are operated such that the fuel cost of thermal generating plant is at its minimum. As the Austrian system contains annual storage developments, their optimal operation must be determined over the time interval of a year. This results in a linearly constrained large-scale NLP problem. The system can be thought of as a network of interconnected storage reservoirs. The variables are the flows in the network at different discrete points of time in the considered time interval.

The system can be decomposed into local storage subsystems. We can also subdivide the time interval into "subsystems of time" (e.g. weeks, days). By subdividing the variables into subsets corresponding to the subsystems we obtain a nested-structure constraint matrix with coupling variables and coupling constraints.

The Report presents developments of optimisation algorithms which take advantage of the nested structures and reduce the amount of computational effort. These compactification and decomposition methods are based on a special gradient projection method using an orthogonal basis for the space of active constraints. The resulting program elements permit simple design of flexible software suitable to handle problems with variable nested structures (variable numbers of levels of nested structures).

Besides discussing these algorithms the Report deals with associated software problems, that is to say, questions of suitable implementation of algorithms for nested structures with a variable number of levels.

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A NEW CLASS OF PENALTY FUNCTIONS AND BOUNDS OF CONVERGENCE RATE

Let be given the following nonlinear programming problem $f(x) \to \min!$ subject to $x \in \Omega = \{z \in R^n,\ g^j(z) \le 0,\ j \in J\}$ with convex function f, g^j.

With help of axiomatix conditions a class of sequences of penalty functions being well behaved with respect to differentiability and to the growth outside the feasible set is described. As an exemple of a sequence $\{\varphi_K^\theta\}$ of penalty functions considered here we select

$$\varphi_K^\theta(x) = A_K \sum_{j \in J} \left(g^j(x) + \sqrt{g^{j^2}(x) + A_K^{-2-\theta}}\right),$$

with $\theta > 0$, $A_K \to \infty$.

In the sequel we give some theorems for the functions φ_K^θ which in fact hold for the whole class. Let denote

$x^K = \arg\min_{x \in R^n} [f(x) + \varphi_K(x)]$, $x^* = \arg\min_{x \in R^n} f(x)$;

$\tilde{x}$ - a given feasible point with $\min_{j \in J} |g^j(\tilde{x})| = \sigma > 0$.

<u>Theorem I.</u> Let Ω be compact $f \in C^1(R^n)$, $g^j \in C^1(R^n)$ $(j \in J)$

There exist integer K with $(k \ge K)$

$x^k \in \operatorname{int} \Omega$,

$$f(\tilde{x}) \geqslant f(x^*) \geqslant f(x^k) + A_K \sum_{j \in J} \left(1 + \frac{g^j(x^k)}{\sqrt{g^{j^2}(x^k) + A_K^{-2-\theta}}}\right) g^j(x^k)$$

and besides

$$A_K \sum_{j \in J} \left(1 + \frac{g^j(x^k)}{\sqrt{g^{j^2}(x^k) + A_K^{-2-\theta}}}\right) g^j(x^k) \xrightarrow[K \to \infty]{} 0.$$

<u>Theorem 2.</u> Let the conditions of theorem I hold and additionally

$$f \in C^2(R^n),\ (f''(x)\xi, \xi) \geqslant \gamma \|\xi\|^2 \quad (\gamma > 0)$$

for any pair x, ξ . There exists an integer K with $(k \geqslant K)$

$$\|x^k - x^*\|^2 < \frac{2}{\gamma}\left[\frac{A_K^{\tau-1}}{\sigma}\left(f(\tilde{x}) - f(x^*)\right) + \frac{m}{2} A_K^{-\tau-\theta}\right]$$

/ $\tau \in (0,1)$, fixed / .

Using the approach for proving theorem 2 we also get bounds of the convergence rate for some well known penalty functions, e.g.

a./ if $\varphi_K(x) = -A_K^{-1} \sum_{j \in J} \frac{1}{g^j(x)}$, then

$$\|x^k - x^*\|^2 < \frac{2}{\gamma}\left[\frac{A_K^{\tau-1}}{\sigma}\left(f(\tilde{x}) - f(x^*)\right) + m A_K^{-\tau}\right];$$

b./ if $\varphi_K(x) = \sum_{j \in J} \exp\{A_K g^j(x)\}$, then

$$\|x^k - x^*\|^2 < \frac{2}{\gamma}\left[\frac{A_K^{\tau-1}}{\sigma}\left(f(\tilde{x}) - f(x^*)\right) + m \cdot \exp\{-A_K^{\tau}\}\right].$$

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AN ALGORITHM FOR THE PARAMETRIC SOLUTION FOR NETWORK FLOW PROBLEMS WITH QUADRATIC AND CONVEX COST

The objective of this paper is to develop a method for determining the parametric minimal solution for a network flow problem when the cost is a convex function of flows along each arc. This problem could be considered as a convex programming problem and solved by one of the iterative methods available in convex programming. However, this approach does not utilize the special network structure of the flow problem with convex cost which enables the development of a more efficient algorithm. In most of the approaches in which the structure of the flow problem has been exploited, the problem has been initially solved for linear cost, then, by use of the linear approximation, the procedures have been extended to encompass convex cost.

The approach employed in this paper is the use of the special structure of the problem to determine the exact solution for a quadratic and convex cost with extension to an arbitrary convex cost by piecewise quadratic approximation. The algorithm is a combination of the Primal - Dual approach, in that the Kuhn - Tucker conditions are used to derive the optimality conditions for each arc, and the Shortest-Path approach, in that the flow along the path which is least incremental in terms of cost (shortest path) is increased. The cost of the branches are appropriately defined relative to the flow and optimality conditions throughout the calculations. It is shown in this paper how to change the solution at each iteration such that all arcs satisfy the optimality conditions, thus providing for the optimality of the solution at each iteration. Theorems are used to show that the maximum flow will be obtained, assuring the termination of the algorithm.

Problems in systems and industrial engineering, transportation and distribution systems, and electrical networks are formulated as network flow problems to show some potential areas to which the algorithm can be applied. Some extensions of the algorithm as well as applications to more general problems are given.

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PROBABILISTIC ANALYSIS OF TRAVELING-SALESMAN ALGORITHMS

Consider Euclidean traveling-salesman problems in which the locations of the n cities are uniformly distributed over a region in the plane. A simple partitioning algorithm which runs in time $O(n)$ is exhibited. It is shown that, for every $\varepsilon > 0$, the algorithm almost surely produces a tour costing not more than $1 + \varepsilon$ times the cost of an optimum tour.

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ON A SPECIAL TYPE JOB SEQUENCING PROBLEM

Let us consider a computer with $K \geq 2$ processors of equal ability. This paper deals with a sequencing problem in which n jobs with ordering restrictions have to be executed by the processors. Assume that every processor can do any of the n jobs and we are given:

- the ordering restrictions represented by a precedence network
- $d_i \geq 0$ integer numbers representing the execution time /in minutes/ of job i if one processor is used
- $k_i \geq 0$ integer numbers representing the maximum number of processors that can work on the i-th job at the same minute

We have another assumption according to the feasible sequencing: The execution of each job can be interrupted at the end of any minute. Now, the following problem may arise:

Determine the minimum number of processors that can perform all jobs at the earliest time. In the present paper this question is considered as a network flow problem of special type.

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NOTES ON SUBSTITUTE INVERSES AND THEIR RELATION TO EACH OTHER

Let A be a sparse nonsingular square matrix of order n. A is said to be reducible if there exist two permutation matrices P and Q such that PAQ is partitioned in block triangular form. If A is reducible, then the problem of inverting A may be reduced by known methods to inverting the triangular blocks. In this case PAQ and its inverse have the same block triangular structure. If A is irreducible, then all its minors of order n-1 have at least one nonzero term. Consequently the inverse of an irreducible sparse matrix is generally totally full /and so are the inverses of the irreducible triangular blocks of a reducible sparse matrix/.

Many problems can be handled by using substitute inverses instead of the explicite one. Substitute inverses may be sparse even if the explicite inverse is totally full.

A kind of substitute inverse is the pair of a lower triangle matrix B and an upper triangle matrix C such that BC = A /triangular decomposition/. Others are the factored forms of the inverse, any of them can be regarded as the set of the factor matrices: elementary column matrices for the product form of the inverse /PFI/, elementary half-row matrices /left-hand factor matrices/ and half-column matrices /right-hand factor matrices/ for the elimination form of the inverse /EFI/, alternately sequenced elementary half-row matrices and half-column matrices for the mixed factored form of the inverse /MFI/ etc.

The PFI, EFI and MFI can be described as the pairs of matrices $/B,C^{-1}/$, $/B,C/$ and $/A_L,C^{-1}/$, respectively. Here B and C are the triangular factors of A, while A_L denotes the below-diagonal-part of A. From this it follows that the PFI is generally not as sparse as either the EFI or the MFI. This assertion can be even more justified by some considerations on binary matrices.

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AN ALGORITHM FOR TRANSFORMING A SPANNING TREE INTO A STEINER TREE

The Steiner problem: Construct the minimum-lenght tree containing a given set of points $A=\{a_1,\ldots,a_n\}$ in E^r within its set of vertices. (The tree may contain other points $S=\{s_1,\ldots,s_k\}$, $0\leq k\leq n-2$, too.)

The solution is called a Steiner minimal tree and the points in the set S respectively Steiner points after J. Steiner (1796-1863), who first introduced the problem for the case n=3.

It is easy to show, that if a Steiner point exists, the tree contains three edges leading to it, and the edges meet pairwise at 120°.

Definition: A tree U with vertices $a_1,\ldots,a_n,s_1,\ldots,s_k$ is a Steiner tree on $a_1,\ldots,a_n$ if and only if it has the following properties:

1. U is not self-intersecting
2. $w(s_i) = 3$, $1\leq i\leq k$, (w(x) is the valency of a vertex x)
3. $w(a_i) \leq 3$, $1\leq i\leq n$
4. each s_i is the Steiner point of the triangle formed by the points in U with which s_i is directly connected
5. $0 \leq k \leq n-2$

An algorithm for solution of the Steiner problem was given by Z. A. Melzak (1961) and further developed by E. J. Cockayne (1970). Although a variety of geometric criteria have been devised which are useful in reducing the number of possible alternatives, the algorithm is still impractical for problems with n greater than 30. In practice even 8 points seem to take too much computing time.

This paper describes a fast recursive algorithm for transforming a given spanning tree into a Steiner tree that is shorter than the given tree. The algorithm converges in a finite number of steps and, if the initial spanning tree is minimal, gives a sufficiently close approximation for the minimal Steiner tree. Experimental results and a few new properties of Steiner trees are presented.

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MULTIOBJECTIVE DYNAMIC PROGRAMMING

In this report a generalization of Bellman's principle of dynamic programming is treated for the following class of vector optimization problems with the criterion of the Pareto-optimality (denoted by $*$):

$$F(x,u) \longrightarrow \mathrm{Max}^*$$

among all state variables $x = (x_o, \ldots, x_N)$ and control variables $u = (u_1, \ldots, u_N)$ under side-conditions of the type $x_{n-1} = g_n(x_n, u_n), u_n \in U_n(x_n)$ $(n = 1, \ldots, N)$, $x_N = a$. Here $F(x,u) \in (E^{2N+1} \to E^L)$ is a given continuous vector-valued function. Furthermore we assume (in generalization of analogous conditions of L.G. Mitten) that $F(x,u)$ fulfils the properties of separability (S) and monotony (M). This means

(S) $F(x_o, .., x_N; u_1, .., u_N) = Q_N(x_N, u_N; q_{N-1}(x_o, .., x_{N-1}; u_1, .., u_{N-1}))$

$q_n(x_o, .., x_n; u_1, .., u_n) = Q_n(x_n, u_n; q_{n-1}(x_o, .., x_{n-1}; u_1, .., u_{n-1}))$

$(n = 1, \ldots, N-1)$, $q_o = q_o(x_o)$ with continuous

$Q_n \in (E^{L+2} \to E^L)$ and $q_n \in (E^{2n+1} \to E^L)$;

(M) $Q_n(x_n, u_n; w) \succ Q_n(x_n, u_n; w')$ for all pairs w, w' of E^L with $w \succ w'$ (i.e. $w_i \geqq w'_i$ for $i = 1, \ldots, L$ and $\sum_{i=1}^{L} w_i > \sum_{i=1}^{L} w'_i$), $n = 2, \ldots, N$.

Then for the set-valued Bellman function

$f_n(x_n) := W_n^*(x_n)$ (as the optimal set of $W_n(x_n) \subset E^L$)

with $W_n(x_n) := \{ z \in E^L \mid z = q_n(x_o,..,x_n;u_1,..,u_n),$
$x_{j-1}=g_j(x_j,u_j), u_j \in U_j(x_j), j=1,..,n \}$

a multistage system of recurrence equations holds:

$$f_n(x_n) = \underset{u_n \in U_n(x_n)}{\mathrm{Max}^*} \quad Q_n(x_n,u_n;\ f_{n-1}(g_n(x_n,u_n))) \qquad (n = 2,\dots,N)$$

$$f_1(x_1) = \underset{u_1 \in U_1(x_1)}{\mathrm{Max}^*} \quad Q_1(x_1,u_1;\ q_o(g_1(x_1,u_1)))$$

if we understand the right hand side of these formulas by

$\{Q_n(x_n,u_n;w) \mid w \in f_{n-1}(g_n(x_n,u_n)),\ u_n \in U_n(x_n)\}^*$

resp. $\{Q_1(x_1,u_1;w) \mid w = q_o(g_1(x_1,u_1)),\ u_1 \in U_1(x_1)\}^*$.

Finally $f_N(a)$ represents the set of optimal values for the original introduced optimization problem.

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A GAME THEORETIC EXTENSION OF A DYNAMIC MARKETING MODEL

Considering the situation in which two firms share in a market and intend to increase their respective share by means of advertising expenditures, a differential game model is developped. Applying the min-max-principle, necessary conditions for a saddle-point are derived.

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ECONOMIZATION OF A CHEBYSHEV EXPANSION BY WAY OF TURNING IN THE TRUNCATION ERROR

An algorithm is presented to achieve the minimax(near minimax, in a strict sense) polynomial approximations by economizing p terms of the Chebyshev expansion series $\sum_{k=0}^{n} C_k T_k(x)$. To reduce the error, the economized terms are systematically turned in the remaining series as increments(or decrements) of its coefficients. The resulting approximations are given as a form:

$$f(x) \approx \sum_{k=0}^{n-p} [\, C_k + C_{2(n-p+1)-k} + c_{n-p+1-k} \,]\, T_k(x).$$

Note that $c_{n-p+1-k}$ can be explicitly determined in terms of the trigonometric identity as a quadratic fraction of the high-order coefficients of the Chebyshev series. Then, all absolute extremal deviations of the error are leveled to satisfy the same magnitude $|C_{n-p+1} + c_0|$.

Details of these fractions are given by:

$$c_0 \approx C_{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right)^2\left[\,1 + \frac{2}{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right) + \left(\frac{C_{n-p+3}}{C_{n-p+2}}\right)^2\right],$$

$$c_1 \approx C_{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right)^2\left[\,2\left(\frac{C_{n-p+3}}{C_{n-p+2}}\right) - \frac{1}{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right)\right],$$

$$c_2 \approx -C_{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right)^2\left[\,1 - \frac{4}{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right) + 2\left(\frac{C_{n-p+3}}{C_{n-p+2}}\right)\left(\frac{C_{n-p+4}}{C_{n-p+3}}\right)\right],$$

$$c_3 \approx -C_{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right)^2\left[\,2\left(\frac{C_{n-p+3}}{C_{n-p+2}}\right) - \frac{1}{n-p+1}\left(\frac{C_{n-p+2}}{C_{n-p+1}}\right)\right],$$

....

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PERFECT DUALITY IN GENERALIZED LINEAR PROGRAMMING

The following two programs are studied.

Program GLP. Let P_o, Q, and a collection of subsets $\{C_i\}$ be specified in $\mathbb{R}^n$. Find sup w from among those $w \in \mathbb{R}$ and $(z, z_{n+1}) \in \mathbb{R}^n \times \mathbb{R}$ which satisfy: for every $\epsilon > 0$, $\exists\delta$, $0 < \delta \leq \epsilon$ and $x_o \in \mathbb{R}$, $x_i \in \mathbb{R}$ such that

$$P_o x_o + \sum_i P_i x_i = Q + \delta z,$$

$$x_o \qquad - w \geq \delta z_{n+1}, \quad x_i \geq 0 \ \underline{\text{for}} \ \ i \neq o,$$

and $x_i > 0$ for at most a finite set of i, where each P_i may be freely chosen in C_i. ||

Program GLD. Find $M_{GLD} = \inf y \cdot Q$ from among those $y \in \mathbb{R}^n$ satisfying

$$y \cdot P_o = 1$$

and

$$y \cdot P_i \geq 0, \text{ for every } P_i \in C_i$$

and every i. ||

It is shown that Programs GLP and GLD are in perfect duality in the sense that (p1), if one program is consistent and has a finite value, then the other is consistent and (p2) if both programs are consistent, then they have the same finite value. Feasible perturbation vectors (z, z_{n+1}) are termed ascent vectors and rules are specified for their construction.

Perfect duality is stronger than the concept of duality equality appearing in convex analysis, which specifies: if one program is consistent and has finite value, then the other program is consistent and has the same finite value.

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COLOURING CRITERIA FOR ADJACENCY ON 0-1 POLYHEDRA

Let P be an arbitrary 0-1-polyhedron in R^E and $\mathfrak{F}$ be the system of subsets of E corresponding to the vertices of P. Let $F_1, F_2 \in \mathfrak{F}$. Then it is easy to see that F_1, F_2 are adjacent with respect to $(E,\mathfrak{F})$ (that means: the incidence vectors x^{F_1}, x^{F_2} are adjacent on P) iff $\tilde{F}_1 = F_1 \setminus F_2$ and $\tilde{F}_2 = F_2 \setminus F_1$ are adjacent with respect to a reduced system $(F_1 \Delta F_2, \tilde{\mathfrak{F}})$.

A general sufficient criterion for the adjacency with respect to $(F_1 \Delta F_2, \tilde{\mathfrak{F}})$ and thus for the adjacency with respect to $(E,\mathfrak{F})$ is given in terms of the prime implicants of $\tilde{\mathfrak{F}}$. A "colouring" of F_1 F_2, i.e. a partition into disjoint "colour classes", is called feasible if it can be obtained by a constructive colouring algorithm; this algorithm starts with the "trivial" colouring (every colour class is a singleton) and combines colour classes according to certain rules that involve the prime implicants of $\tilde{\mathfrak{F}}$. The criterion is fulfilled iff the colouring having only one colour class is feasible.

One class of polyhedra for which the criterion is necessary, too, is the class of polyhedra for which $\tilde{\mathfrak{F}}$ has only prime implicants of length 2, e.g. the polyhedra of the set packing problem and of the matching problem. In this case the criterion is fulfilled iff a certain graph is connected; the nodes of the graph are the elements of $F_1 \Delta F_2$, its arcs are the prime implicants of $\tilde{\mathfrak{F}}$.

Others polyhedra P for which the criterion is also necessary are those which are symmetrical in such a way that if a subset $\tilde{F} \subseteq F_1 \Delta F_2$ with a certain colouring property belongs to $\tilde{\mathfrak{F}}$ then also its complement $F_1 \Delta F_2 \setminus \tilde{F}$ belongs to $\tilde{\mathfrak{F}}$. In this case, we have still another criterion: F_1, F_2 are adjacent iff $\tilde{\mathfrak{F}} = \{\tilde{F}_1, \tilde{F}_2\}$. Examples of polyhedra which are symmetrical in this sense are the polyhedron of linear orderings and other ordering polyhedra.

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GLOBAL MAXIMIZATION OF CONVEX FUNCTIONS SUBJECT TO LINEAR CONSTRAINTS

In this paper we examine existing methods of convex maximization and propose a new hybrid method.

Most convex maximization methods depend on Tui cuts. However, Tui cut algorithems do not possess finite convergence and have been shown to be computationally inefficient. The other common techniques are assorted algorithems based on extreme point enumeration. Arbitrary extreme point enumeration is obviously computationally inefficient; however, it is formally finite.

Extreme point enumeration techniques can be efficient in the presence of few extreme points or few constraints. For example, an extreme point enumeration of a simplex is an adjacent extreme point search. Consequently we propose a relaxation algorithem. That is, we relax all constraints but those which constitute a simplex. We enumerate the extreme points of the simplex and determine the optimum point. We then test the optimum point against the best feasible point. If the optimum value is greater than the best feasible value, we index the most violated constraint and enumerate the extreme points generated by this constraint, etc. This procedure is formally finite and if a "good" set of "starting" constraints is used may even be computationally efficient.

However, we note that Tui cuts are constraint reduction techniques. In the absence of degeneracy, every constraint active at the extreme point at which the Tui cut is taken is made redundant. Further, after a "few" Tui cuts are generated, further cuts generally do not improve the optimal value. Consequently, a hybrid algorithem is proposed, which generates Tui cuts (and reduces constraints) until an indication of poor progress is received and then the relaxation procedure is applied.

Heuristics are also employed to obtain accelerated algorithems. Introductory computational results are presented.

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MULTILEVEL BENDERS DECOMPOSITION APPLIED TO INTEGRATED PRODUCTION / DISTRIBUTION SYSTEMS

Over the years a great many computer based techniques have been devised for, facility location, distribution and production planning. This paper develops a framework for coupling these three decision processes together. The motivation for this paper is based on the original work by Geoffrion and Graves (Management Science, Volume 20, 1974) in their paper "Multicommodity Distribution System Design by Benders Decomposition". The authors have extended the work to allow simultaneous determination of supplies levels for each product class at the various plants when raw material levels are determined. This will provide a production schedule for each product class, by production process at particular plants. These concepts have been integrated with the distribution planning system outlined be Geoffrion and Graves to allow global integrated planning for production and distribution operations.

Within the paper is the development on an efficient procedure to solve the large scale mathematical programming model via two levels of Benders Decomposition. A series of test problems provide insight to the solution of integrated production/distribution models with this methodology.

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SUPERLINEARLY GLOBALLY CONVERGENT ALGORITHM FOR NONLINEAR PROGRAMMING VIA SEQUENTIAL LINEAR PROGRAMS

The nonlinear programming problem is first cast as an unconstrained minimization of a function which is not necessarily differentiable. Any appropriate function must be able to be approximated sufficiently well at each point by a convex function. This convex function is then used to develop an optimality function, to define the algorithm, and as a basis for the convergence and rate of convergence theorems. It is shown that the rate of convergence theorem is a generalization of the iterated contraction theorem. The minimization of the convex approximation function provides descent directions for the unconstrained function. It is then shown how this minimization problem can be cast as a linear programming problem. An Armijo type step is used on the descent direction maintaining superlinear convergence. A comparison and application of the algorithms are given.

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LAYOUT PLANNING, EVALUATION AND OPTIMIZATION

Architects, engineers et cetera are frequently facing problems involving the optimal design of a structure's layout subject to certain conditions. To determine the number of buildings, the shape of each, their location and orientation on the ground(s) and the mutual placement of the individual functions within the buildings is a typical, everyday example. However, the problem is also met in other contexts such as the arrangement of electrical components in printed circuits or - to mention an entirely different field - how to move governmental institutions from the capital of a country to other cities.

The paper will outline the general structure of the problem including some special versions like "The Quadratic Assignment Problem", "The Line Ordering Problem" et cetera. Furthermore, a recently developed model by the authors is described, primarily aiming at an appraisal of a given layout according to certain criteria. The terminal-oriented model has proved to serve as a powerful tool for architects during the process of experimentation with alternative layouts. Besides of calculating the "internal transport", the model can take other measures which are usually considered to be of a "qualitative, aesthetic" nature into account such as noise, availability of daylight et cetera. A number of macro's facilitates the transition from one layout to another.

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A METHOD FOR LINEAR DYNAMIC PROGRAMS WITH THE USE OF BASIS MATRICES FACTORIZATION

The following linear dynamic program is considered.

Problem 1. Define the control $u = \{u(0), \dots, u(T-1)\}$ and the trajectory $x = \{x(0), \dots, x(T)\}$ satisfying the difference equations

$$x(t+1) = A(t)x(t) + B(t)u(t) - s(t)$$

$$x(0) = a, \quad (t = 0, 1, \dots, T-1)$$

subject to the constraints

$$G(t)x(t) + D(t)u(t) = f(t)$$

$$u(t) \geq 0, \quad (t = 0, 1, \dots, T-1)$$

and maximizing the performance index

$$J(u) = (c(T), x(T)).$$

Here $x(t) \in E^n$, $u(t) \in E^r$, $f(t) \in E^m$ while the matrices $A(t)$, $B(t), G(t), D(t)$ and the vectors a, $c(T), f(t), s(t)$ have proper dimensions and taken to be known.

In this paper a "simplex-type" algorithm for Problem 1 is proposed. The algorithm is based on the expression of the $(n+m)T \times (n+m)T$ basis of Problem 1 in terms of the set of the local $m \times m$ basises at each $t = 0, 1, \dots, T-1$. In the solution process one has to work with the local basises only, and at each step some fraction of the basises only has to be updated, the rest being the same. The updating process consists in multiplication of the local basis submatrices inverses by elementary submatrices. This allows to store the inverse in a factorized form.

The updating procedure for the change of the basis is described. The method proposed in this paper makes use of the special structure of LDP problems and provides considerable savings of processor time as well as of the core memory.

From the formal point of view the results described above may be obtained using the factorization applied to the "global" $(n+m)T \times (n+m)T$ basis. However, in fact the algorithm proposed is an extension of the simplex method for dynamic linear programs.

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MINIMIZATION OF A CONCAVE FUNCTION UNDER LINEAR CONSTRAINTS./MODIFICATION OF TUY'S METHOD./

The problem under consideration is to find a global minimum of a concave function $f: D \longrightarrow R$, where $D \subset R^n$ is a given convex /bounded/ polyhedron. We assume without loss of generality that f is defined in all space R^n and int $D \neq \emptyset$.
The first and basic for most other methods which solve this problem was the algorithm proposed in 1964 by Hoang Tuy [1] .
The algorithm described in the paper is a continuation of Tuy's idea too. Unlike the Tuy's method /see: Zwart [2] / it finds the global minimum in a finite number of iterations - for every f and D. Moreover there is no troubles with degeneracy of initial vertex of D. These advantages are a consequence of two modifications of the algorithm . The main difference between both methods lies in use of another auxiliary problem /but unfortunately it is a little more complicated/. It has the following form:

$$c^T \lambda \longrightarrow max$$
$$\lambda = (\lambda_1, \dots, \lambda_n) \geq 0$$
$$\lambda \in V(D_1)$$
$$c^T \lambda > 1$$

where $V(D_1)$ is a set of vertices of some polyhedron $D_1 \subset R^n$ closely related to D. Such a problem can be solved using the method described by Pollatschek and Avi-Itzhak [3] .
The second modification is connected with starting iteration and, in particular, with defining the first series of auxiliary problems. In addition there is proposed in the algorithm a procedure of bounding the values of solutions of auxiliary problems. It makes the idea of the method more similar to a general branch - and - bound technique known from integer programming.
It should be noted finally, that the main purpose of the paper is not to formulate an algorithm which is numerically complete and ready to code for computer. The stress is laid on a geometrial idea of the method and its formal simplicity.

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OPTIMIZING THE LONG-TERM ACTIVITY RATE OF A CLOSED POPULATION

Although population is a rather independent factor of the future, it is not without interest to speculate on the desirability of various alternative development paths. This kind of "policy studies" may serve the purpose of clarifying the objectives of population policy which often are ambiguous, especially in the more developed countries where the mere size of the population is less problematic than the structure. The translation of the ambiguous strategical goals into concrete tactical goals, things to be achieved, should be most useful even if the means of achievement must be left unspecified: The knowledge of goals is prerequisite to finding the means.

A subscriber to the normative approach exemplified by operations research is not satisfied with the methodology of most population policy studies: Only some conceived alternatives are compared, and the measuring sticks have no fixed length. In the present paper, an attempt is made to a more rigorous treatment of a specific problem of population policy.

The task is to determine the annual numbers of births required to keep the future activity rates (proportion of people in the working ages) as high as possible, either in average or continually. The population is assumed to be closed for migration, the age-specific fertility rates are allowed to vary only within specified limits, the year-to-year changes in fertility shall not be too great, and the size of the total population must also stay within predetermined limits. A solution method based on dynamic programming is outlined. Numerical results for the population of Finland are presented and discussed.

Global Stability of Optimization Problems

Consider the mappings

$$(M, \lambda) \rightsquigarrow f^{o}(M, \lambda) = \sup \{ f(x, \lambda) \ / \ x \in M \}$$

$$(M, \lambda) \rightsquigarrow \hat{f}(M, \lambda) = \{ x \in M \ / \ f(x, \lambda) = f^{o}(M, \lambda) \}$$

where x lies in a metric space X, M is a subset of X, λ lies in a set Λ "with convergence" and f is any real-valued function defined on $X \times \Lambda$.

In relation to the set-convergence, $M_t \rightarrow M$ $(t \rightarrow \infty)$ iff $M = \underline{\lim}\, M_t$ and $\forall \varepsilon > 0 \ \exists\, t(\varepsilon) \ \forall t > t(\varepsilon): \ M_t \subset U_\varepsilon M$, necessary and sufficient conditions will be given for the continuity of f^{o} and the upper-semicontinuity of $\hat{f}$. Further, connections between the upper-semicontinuity of f^{o} and $\hat{f}$ are studied, and for some problems in Banach spaces (quasi-convex optimization problems and certain problems with non-asymptotical objective functions) sufficient stability-conditions are formulated.

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A GEOMETRICAL APPROACH TO THE THEORY OF LINEAR INEQUALITIES

The topic of the paper is the theory of systems of linear inequalities. We perform proofs by geometrical arguments where it is possible. Our aim is to demonstrate the effectiveness and beauty of a proving method rather than to generate new theorems by means of this method.

First we give an almost completely elementary geometrical proof of the separation theorem. Then we deal with the problem of consequences, relying on the separation theorem. We prove the Farkas theorem and an interesting result of Chernikov which is properly the correction of an erroneous statement of Alfred Haar: a necessary and sufficient condition that all consequences of a /possibly infinite/ system of linear inequalities be consequences of finite subsystems as well.

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LINEAR PROBLEMS OF CONVEX ANALYSIS

Programming of extremal problems with convex solutions such as geometrical problems of isoperimetrical type needs unified tools of description of the whole cone of convex objects of this or that type. The general approach can be expounded on the basis of the Minkowski duality scheme, That is in the simplest cases by an identification of convex compacta with their support functions.

Because of the duality a cone in a vector space is obtained and, thus, the standart linearization technique can be applied - the technique of the Choquet ordering. Description of the ordering is given in a "computable" form bringing along a bulk of concrete results, for example, general solutions of convex programs with constraints on mixed volumes and mixed area functions. The peculiar property of the problems is that there exist at least two different vector structures in which they are convex and each of them determines complementary sets of silvable programms and respectively different Euler - Lagrange equations.

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SCHEDULING OF PARALLEL MACHINES WITH PREEMPTION

The general problem considered is that of optimally scheduling n independent jobs on m parallel machines, where there is negligible cost associated with preemption or "job splitting". Let p_{ij} be the processing time of job j on machine i, if job j is processed by machine i exclusively. If $p_{ij}=q_i p_j$, where q_i and p_j are parameters associated with machine i and job j, respectively, then the machines are said to be "uniform". If p_{ij} is arbitrary, then the machines are "unrelated". The problem of minimizing makespan /the completion time of the last job/ on unrelated machines is formulated and solved by /noninteger/ linear programming techniques. These techniques are then extended to minimize maximum lateness /with respect to given due dates/ on unrelated machines and to minimize mean completion on uniform machines. In each case, a priori bounds, polynomial in m and n, are obtained on the maximum number of preemptions required for an optimal schedule.

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THE COMPLEXITY STRUCTURE OF A CLASS OF SCHEDULING PROBLEMS

A detailed classification of machine scheduling problems can be used to generate a class M containing some 3000 specific problem types. Each of those problems allows binary and unary encodings of problem data, i.e. proportional to the sum of the logarithms of the data or of the data themselves. With respect to either type of encoding a specific problem may be solvable within polynomial time or it may be proved NP-complete. Reductions within M extend these properties to other scheduling problems as well, the current situation being that about 10 per cent of them are polynomially solvable, 70 per cent are NP-complete and the rest remains open. We examine these partitions of M and list some important open problems corresponding to various borderline cases.

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COMBINING KELLEY'S AND CONJUGATE GRADIENT METHODS

Let f/x/ be a convex function defined on some Hilbert space H. If we have on hand $x_0, x_1, \dots, x_n$ and $g_0, \dots, g_n$ $(g_i \in \partial f(x_i))$ we know that for any x in H

/1/ $$f(x) \geq \max \{ f(x_i) + (g_i, x - x_i) \ / \ i = 0, 1, \dots, n \}$$

For minimizing f, Kelley's cutting plane method [1] consists in choosing x_{n+1} so as to minimize the lower bound given by /1/: x_{n+1} solves

/2/ $$\min_x \ \max \{ f(x_i) + (g_i, x - x_i) \ / \ i = 0, 1, \dots, n \}$$

However this problem has in general no solution /the "solution" would be infinite/ and a bound is needed. We choose a bound induced by the norm of H by imposing $|x - x_n| \leq M$ a positive constant. We then set $x = x_n + M s_n$, $|s| \leq 1$. Observing that $f(x_i) + (g_i, x - x_i) = f(x_n) + (g_i, x - x_n) + f(x_i) - f(x_n) + (g_i, x_n - x_i)$ and setting

/3/ $$\alpha_i = f(x_n) - f(x_i) + (g_i, x_i - x_n) \qquad \text{/Note that } \alpha_i \geq 0, \ \alpha_n = 0 \text{ /}$$

we transform /2/ in an equivalent problem in s /the constant is dropped/

/4/ $$\min_{|s| \leq 1} \max \{ M(g_i, s) - \alpha_i \ / \ i = 0, 1, \dots, n \}$$

By linearity $\max_i \{ M(g_i, s) - \alpha_i \} = \max_\lambda \{ M(\Sigma \lambda_i g_i, s) - \Sigma \lambda_i \alpha_i \ / \ \lambda_i \geq 0, \Sigma \lambda_i = 1 \}$ and via this common trick, /4/ becomes a biconvex saddle point problem in which min and max can be inverted:

$$/4/ \Leftrightarrow \max_{\lambda \geq 0, \Sigma \lambda_i = 1} \ \min_{|s| \leq 1} \{ M(\Sigma \lambda_i g_i, s) - \Sigma \lambda_i \alpha_i \}$$

where the solution of the "min" problem is obviously $s(\lambda) = \Sigma \lambda_i g_i / |\Sigma \lambda_i g_i|$. In summary Kelley's method with Euclidean bound consists in finding the solution of

/5/ $$\begin{cases} \min | \sum_{i=0}^{n} \lambda_i g_i | + 1/M \sum_{i=0}^{n} \lambda_i \alpha_i \\ \lambda_i \geq 0 \quad \sum_{i=0}^{n} \lambda_i = 1 \end{cases}$$

where $s_n = \Sigma \lambda_i g_i$ is a <u>direction</u> along which a line-search could be performed /instead of being content with a simple step if lenght M./. Now 1/M in /5/ can be interpreted as the Lagrange multiplier associated to some constraint of the form $\Sigma \lambda_i \alpha_i \leq \varepsilon$; this means that for every $M \in [0, \infty]$ there exists $\varepsilon \in [0, \infty]$ such that the solution of /5/ solves $\min_\lambda |\Sigma \lambda_i g_i| \quad \Sigma \lambda_i = 1, \ \lambda_i \geq 0, \ \Sigma \lambda_i \alpha_i \leq \varepsilon$

which in turn is equivalent to .

/6/ $$\min |\sum_{i=0}^{n} \lambda_i g_i|^2 \quad \sum_{i=0}^{n} \lambda_i = 1, \ \lambda_i \geq 0, \ \sum_{i=0}^{n} \lambda_i = 1, \ \lambda_i \geq 0, \ \sum_{i=0}^{n} \lambda_i \alpha_i \leq \varepsilon$$

For example M=0 in /5/ corresponds to $\varepsilon = 0$ in /6/ and the method reduces then to the steepesr descent /re call that $\alpha_n = 0$ /. On the opposite hand $M = +\infty$ gives $\varepsilon = \infty$ and we get the conjugate gradient method as stated in [2] , [3] .

<u>Towards a new algorithm.</u> Let $x_0 \in H$, $g_0 \in \partial f(x_0)$. We can imagine the following algorithm: Choose $\eta > 0$ set $n = 0$.

Step 1. Compute $\alpha_0, \alpha_1, \dots \alpha_n$ defined by /3/ /a simple recurrent allows to compute them without storing the x_i's /

Choose $\varepsilon > 0$

Step 2. Solve /6/ for λ and $s_n = -\sum \lambda_i g_i$. If $|s_n| \leq \eta$ there holds $\forall x \in H \quad f(x) \geq f(x_n) - \eta |x - x_n| - \varepsilon$ therefore, in this case, stop or diminish η or ε.

Step 3. Perform a linear search along s_n as explained in [3], yielding x_{n+1} and $g_{n+1} \in \partial f(x_{n+1})$

Set $n=n+1$ and go to 1.

Convergence can be proved provided a safeguard in Step 1 prevents ε to tend to 0. Some numerical investigations have shown that this algorithm could be substantially better than [2,] [3] and variable metric methods provided

- a good method is on hand for solving /6/
- ε in Step 1 is properly chosen.

These two questions are still open.

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NOTE ON AN EXTENSION OF "DAVIDON" METHODS TO NONDIFFERENTIABLE FUNCTIONS

Some properties of "Davidon", or variable metric, methods are studied from the viewpoint of convex analysis; they depend on the convexity of the function to be minimized rather than on its being approximately quadratic. An algorithm is presented which generalizes the variable metric method, and its convergence is shown for a large class of convex functions.

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SIMPLICIAL APPROXIMATION AND SOLVING SYSTEMS OF EQUATIONS

THERE HAVE BEEN MANY RESULTS IN RECENT YEARS by various authors which relate to the use of simplicial approximation techniques for approximating fixed-points, zeroes of a vectot-valued function, and solutions to nonlinear complementarity problems. This talk is expected to include an evaluation of these results and this use of simplicial approximation from the standpoints of theory and of computational viability.

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A NEW CONSTRUCTION MANAGEMENT GAME

The new game (CMG) is a computer assisted management game for the construction industry.It is going to be played for the first time this winter and early spring 1975/76 by a nmmber of teems consisting of Scandinavian construction people.The game is followed and reported in the leading Danish construction magazine "Byggeindustrien" which in its issue for October 1975 already has brought the initial introduction to the game.

CMG is believed to be able to contribute to a better communication between theory and practice because the participants through their work with the game in a convenient and entertaining manner are trained in overlooking the network and utilizing it as a tool for management.

Besides the ordinary aspects of construction management CMG has the more specific quality that the durations of the activities are assumed to be stochastic variables following certain given probability distributions.As is well known the stochastic analysis is in fact more correct and gives the proper jobduration contrary to the deterministic analysis which tends to give too small jobdurations.The participants in CMG are very directly feeling the stochastic effect on jobduration,slackcalculations etc. In this way the participants are acquiring some background for estimating the stochastic effect in such other practical cases where a complete and proper stochastic analysis is unnecessary,too expensive or perhaps impossible to carry out.

CMG concerns a construction contract with a planned duration of 12o days,which the teams have to manage using a precedense network as a tool. There is a fine for late completion and the normal cost and expediting costs etc. are given for the various activities.There are five rounds of the game.In the beginning of each round the teams make their decisions as to which activities should be expedited etc.All the decisions are fed into the computer which to the original position adds these "man-made" data plus certain "nature-made" data namely the outcomes from the random "drawings" that the computer makes from the relevant probability distributions.The "nature-made" data give the actual durations of activities that start in the round and also which of the 25 days in the round that have such bad weather that outdoor activities cease on these days.

The sum of the original position before the round plus the "man-made" and the "nature-made" data for the round are calculated by the computer and communicated to each team.The teams make their decisions for the next round and so on till the contract is completed.

After each round the computer calculates for each team an amount in Kroner representing total now expected final cost plus fine for the whole contract.The position for selected individual teams and the average position for all teams can then be reported round for round in the magazine.

The paper for the symposium about CMG will present a more detailed description of the structure and rules of the game and report by graphs and otherwise the results of the game that is to be played this winter. Comments,experiences and possible benefits for the participants will be described and the results and experiences in connection with the stochastic conditions of the game will be dealt with.

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Methods of Conjugate Directions for Linearly Constrained Nonlinear Programming

Several numerical methods for unconstrained nonlinear optimization depend on maintaining a set of conjugate search directions; for example, Powell's non-derivative method and the accelerated conjugate direction methods of Best and Ritter. When such methods are adapted to solve linearly constrained problems, it becomes necessary to confine the search directions to a linear subspace defined by the set of contraints which are active or binding at a given point. A technique is described for maintaining conjugacy relationships among the search directions when a new constraint is added to the active set. The technique uses Householder transformations to maintain a Q-R decomposition of the matrix whose columns are the normals to the active set, and to rotate the set of search directions when a new constraint is encountered. Possible applications to quasi-Newton methods are also discussed.

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ON RESOURCES ALLOCATION PROBLEMS REDUCIBLE TO THE TRANSPORTATION PROBLEM.

The work deals with some problems of the optimal allocation of resources in networks. The problems involved differ from the resources allocation problems usually considered in the following aspects: (1) The networks have disjunctive arcs as well as arcs with negative lengths and negative costs. (2) The operation durations, due dates and resources costs are variables linearly depending on a parameter. The following two types of the problems are investigated, (1) the problems with costs linearly depending on operations durations and operations dates, and (2) the problems with costs being piecewise-linear functions.

Such problems usually appear in project planning, scheduling and transportation planning.

It is shown that the problems considered have a special structure that enables using linear programming technique, in particular, network flows methods. The finite method for solving the problems is developed, each iteratioh of the method consisting of the solution of a transportation problem of the same size as the original resource allocation problem.

The special case of the problems is the well-known Kelley problem of project cost scheduling. Using the method mentioned above for solving the Kelley problem in a network with **n** nodes and p arcs we can obtain the equivalent transportation problem in a network with n+p nodes and 3p arcs. Note that the Kelley algorithm either requires a modification of a labelling process or reduces the original network problem to the equivalent transportation problem in a network with n+2p nodes and 4p arcs.

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STABILITY REGIONS FOR OPTIMAL SOLUTIONS OF THE INTEGER PROGRAMMING PROBLEM

Some special problems of the sensitivity analysis for integer programming problems are considered.

Given the integer linear programming problem /ILP/ : max cx , $Ax \leqslant b$, $x \geqslant 0$ integer. Let $\mathcal{P}$ be a set of all possible parameters $p=(A,b,c)$; e.g. $\mathcal{P}=Z^{m\times n}\times Z^{m\times 1}\times R^{1\times n}$. Denote by $\Omega(p^{\circ})$ the set of optimal solutions of ILP cor - responding to parameter $p^{\circ}\in\mathcal{P}$. In general the problem is stated as follows : for $x^{\circ}\in\Omega(p^{\circ})$ find a set $\mathcal{P}(x^{\circ})\subset\mathcal{P}$ such that $p\in\mathcal{P}(x^{\circ})$ iff $x^{\circ}\in\Omega(p)$. In the paper special cases of the problem are considered, namely it is assu - med that some of the parameters are fixed.

One of interesting problems of such a type occurs, when A and b are constant and the set $\mathcal{P}(x^{\circ};A,b)$ of all possible vectors $c\in R^{1\times n}$, for which the ILP has an optimal solution x° , is to be found. It can be shown, that $\mathcal{P}(x^{\circ};A,b)$ is a polyhedral convex cone and that the problem of finding all the extreme rays of this cone is equivalent to the determining faces containing x° of the convex hull of feasible solutions to the ILP. Such faces are known only for some special problems / edge packing problem, some knapsack problems, integer programming over cones /. It is shown, how known results can be adapted to obtain $\mathcal{P}(x^{\circ};A,b)$ for these cases. A possibility of approximate computing of $\mathcal{P}(x^{\circ};A,b)$ as well as the problem of checking whether for a given set $\mathbb{P}$, $\mathbb{P}\subset\mathcal{P}(x^{\circ};A,b)$, are discussed.

The second class of problems occurs, when c is constant and the set $\mathcal{P}(x^{\circ};c)$ of all possible parameters (A,b) , for which x° is optimal, is to be found, It is shown, that if the ILP is a knapsack problem, than $\mathcal{P}(x^{\circ};c)$ is a polyhedral convex cone /not closed/. In the 0-1 case this cone can be completly described by the minimal covers of the inequality $cx\leqslant cx^{\circ}$. If only one parameter of the knapsack problem constraint is varied, then the stability region of the optimal solution can be calculated by solving a new knapsack problem. This new problem is much more easier to solve then the original one, because information obtained in the process of computing x^{0} can be used.

Results obtained are extended to a general ILP in which elements of one row of constraint matrix are varied.

Some other generalizations based on the transformations of the original problem are also discussed.

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A GENERALIZATION OF FENCHEL CONJUGATION LEADING TO GENERALIZED LAGRANGIANS AND NONCONVEX DUALITY.

This paper connects generalized Lagrangians to Rockafellar's duality theory by means of a generalization of Fenchel conjugation. It gives duals without duality gaps even for nonconvex problems.

In Rockafellar's duality theory one studies perturbed convex optimization problems of the form $\underset{x}{\text{minimize}}\ F(x,u)$, where u is a perturbation parameter. The dual of this problem is constructed using Fenchel conjugation. As a byproduct one gets the usual Lagrangian $L(x,u) = f(x) + \langle u,g(x)\rangle$ ($\langle .,. \rangle$ denotes inner product) to the mathematical programming problem (P): $\underset{x}{\text{minimize}}\ f(x)$ subject to $g(x) \leq 0$.

The generalized Lagrangians to problem (P) typically have the form $\underline{L}(x,u) = f(x) + G(u,g(x))$ for some function G. Using a suitable $\underline{L}$ (P) can often be solved by minimizing $\underline{L}$ for an appropriate choice of u, even in the nonconvex case.

The Fenchel conjugate f^{*} of a function f is defined by $f^{*}(x^{*}) = \sup_{x}(\langle x^{*},x\rangle - f(x))$. It can be viewed as a representation of the affine minorants of f. One of the main properties of conjugation is that a lower semicontinuous convex function generally can be recovered by double conjugation.

By allowing more general minorants than affine ones, we get a generalization of conjugation. Then, more general functions than convex can be recovered by double conjugation. For instance, for some choices of families of minorants, lower semicontinuity is almost sufficient to make a function recoverable.

Using this generalized conjugation instead of the usual, one can parallel the development of Rockafellar's duality theory. In this way we get a duality theory that gives duals without duality gaps even for some general nonconvex problems.

The Lagrangians of this theory turn out to be generalized Lagrangians. By suitable choices of minorants, most generalized Lagrangians in the literature can be obtained, e.g. those of Hestenes-Powell-Rockafellar and Arrow-Gould-Howe. The usual type of result connecting saddlepoints of the Lagrangian to solutions of the primal and the dual are also obtained.

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STEINER'S PROBLEM OF GRAPHS : APPLICATIONS, THEORY, ALGORITHMS

Given a graph with positively weighted edges, one distinguished vertex s and a vertex subset T, find a subgraph of minimal weight that contains a path from vertex s to every vertex in T.

The importance of this problem resides mainly on the fact that it can straightforwardly be interpreted in an engineering context (synthesis of networks to transport water, power, informations, etc, when the economies of scale predominate).

It is furthermore of considerable theoretical interest, as it is polynomially reducible to the class containing the traveling salesman problem and only if T is either composed of a single vertex or all vertices of the graph, are there polynomial algorithms known to solve the problem. This is true for undirected as for directed graphs.

The formulation as a 'loco' problem is discussed.

Finally various attempts to solve the problem are surveyed: (i) using optimality conditions for implicit enumeration (Dreyfus/Wagner), (ii) specializing to acyclic graphs (Nastanski), (iii) using Edmonds' branching algorithm to obtain lower and upper bounds.(Bossel),(iv) Heuristics (Keller/ Wildhaber).

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THE "TUBE-PASSING PROBLEM" AND THE TRAVELLING SALESMAN PROBLEM

Given some tubes, each with two openings, one can formulate the problem of finding the shortest circuit passing once through each tube. In graph theoretic language one can state the problem as follows:

Given an undirected graph $G=(V,E)$ with the vertex set $V=\{1,\dots,n\}$, the set of edges $E=\{\{(i,j),(j,i)\}/ i\in V, j\in V\}$ and the weight c_{ij} for each edge $\{(i,j),(j,i)\}\in E$ and a subgraph $G'=(V,E')$ with $E'\subset E$ and with degree $d(i)\leq 2$ for each vertex $i\in V$, find a Hamiltonian circuit on G using all the edges of G' with minimal total weight.

An algebraic formulation of the problem can be the following:

Given a triangular matrix $(c_{ik})_{1\leq k<i\leq n}$ and a permutation $(p(i))_{i=1,\dots,n}$ of $(1,\dots,n)$ with $p(p(i))=i$ for each i, we search a triangular matrix $x=(x_{ik})_{1\leq k<i\leq n}$ with $x_{ik}\in\{0,1\}$ minimizing the function

(1) $f(x)=\sum_{1\leq k<i\leq n} c_{ik}\,x_{ik}$ under the restrictions

(2) $\sum_{(i,k)\in Q_j} x_{ik} = \begin{cases} 2 & \text{if } p(j)=j \\ 1 & \text{if } p(j)\neq j \end{cases}, \quad j=1,\dots,n,$

(with $Q_j = \{(j,i)/\ 1\leq i<j\}\cup\{(i,j)/\ 1\leq j<i\}$),

(3) x corresponds to a connected subgraph.

A branch and bound algorithm was derived dealing with a tube-passing problem on each stage in analogy to the algorithm of LITTLE et a.. The basic reduction is very strong.

For 4o stochastically generated Euclidean problems from 10 to 60 cities the reduction was in the average 15 per cent better than the reduction of the related assignment problem. The branch and bound computer program needs only approximately $(1/2n^2 + 10n + 200)$ places for data storage. The 20-city-problem by CROES (1958) could be easily calculated by hand.

(Bibliography: D.G.Liesegang, Möglichkeiten zur wirkungsvollen Gestaltung von Branch and Bound-Verfahren, Dissertation, Köln,1974)

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SOFTWARE DESIGN FOR NON-LINEAR OPTIMIZATION

The paper is concerned with the discrepancies between the theoretical development and the computational implementations of non-linear optimization algorithms. Computational experience with readily available software and users requirements will be discussed. Finally, some guidelines for the comparison and evaluation of optimization software will be presented.

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ECONOMIC INTERPRETATION OF DUALITY IN GEOMETRIC PROGRAMMING

The purpose of this contribution is the economic interpretation of duality in geometric programming illustrated on the input-output model with substitution between primary factors.

The standard input-output models are charakterized by constant input coefficients and constant labour and capital coefficients. Suppose furthermore the constant input coefficients, but the substitution possibility between labour and capital. For this reason, we introduce the production functions Cobb-Douglas or CES in the open input-output model. Under the condition of minimization of labour costs we can now analyse the distribution of labour and capital between the particular sectors of the economy for exogenuosly given final demand. The constraints of this model are polynomials and after simple transformation we can write this model as a problem of geometric programming.

In the second part we analyse the dual model from the economic interpretation point of view. We show that the dual variables in geometric programming are elasticity coefficients in comparison to the interpretation of dual variables in the linear programming as marginal coefficients.

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AN ALGORITHM FOR SOLVING THE NONLINEAR COMPLEMENTARITY PROBLEM

The complementarity problem is the problem of finding for a given map $f: R^n_+ \to R^n$ a nonnegative vector x such that the two vectors x and f(x) are orthogonal. It is the unifying mathematical form of many problems arising in different fields such as mathematical programming, game theory, fixed point theory, ect.

Recently, two algorithms [1], [2] were published for solving this problem. The new algorithm presented in this paper is a combination of the methods in [1] and [2]. It also uses the principle of simplicial approximation. The algorithm generates a sequence of "adjacent almost-complementary" simplices terminating in a "complementary" simplex. Every point in this final simplex is an approximate solution to the complementarity problem. The possibility of using such an approximation as a new starting point of the algorithm, with a finer mesh size of the triangulation, is described. It is this feature combined with variable dimensions of the simplices on the path which results in considerable computational savings compared with other methods. Besides this numerical advantage the algorithm provides constructive proofs of f. ex. Moré's existence theorems in [3].

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ON OPTIMAL CUTTING PROBLEMS

Following a description of some examples of the application of cutting problems a discussion of the extent to which the existing solution method can handle these problems is carried out.
A case from the glass industry is then presented. The case results in a two-dimensional cutting problem where large rectangles have to be cut into smaller rectangles. At the same time a group of additional constraints has to be satisfied. The solution method is a near optimal method using knapsack functions. It is shown that the waste can be reduced by approximately 50% in comparison to the solution normally used by the company.
Finally, the cutting problem is viewed as part of a larger problem-complex and it is pointed out, that it is not always optimal only to minimize the waste.

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CHARACTERIZATION OF LINEAR COMPLEMENTARITY PROBLEMS SOLVABLE BY LINEAR PROGRAMMING

The linear complementarity problem is that of finding an n-by-1 vector x such that

$$Mx + q \geqq 0, \quad x \geqq 0, \quad x^T(Mx+q) = 0$$

where M is a given n-by-n matrix and q is a given n-by-1 vector. A necessary and sufficient condition for the solvability of the linear complementarity problem is given in terms of the solvability of a linear programming problem. This characterization leads to particularly simple linear programming methods for the solution of the linear complementarity problem for such cases as when M or its inverse is a matrix with nonpositive offdiagonal elements, or a matrix with a strictly dominant diagonal column-wise or row-wise.

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EXPERIENCES WITH THE DUAL TYPE GUB ALGORITHM OF GRIGORIADIS

For a class of decomposable linear programming problems we found that the subproblems can very well be treated by the Generalized Upper Bounding /GUB/ technique. For certain reasons we have chosen the dual type GUB algorithm of Grigoriadis /Management Science, Vol.17., No.5., January, 1971/. Grigoriadis gives a detailed description of the algorithm, however direct coding of his formulas doesn't lead to a succesfully working computer program. With a computer-minded consideration of the algorithm, paying special attention to error propagation, we have succesfully implemented it and made a series of instructive test runs. The program appeared very fast despite of the additional heavy logic and arithmetic required to control error propagation. Problem sizes varied from 8 joint constrains, 200 group constraints, 600 variables to 25 joint constaints, 200 group constraints 500 variables.

In the paper we try to give some details of the implementation of the algorithm and statistical evaluation of the test runs.

Our final conclusion is that the Grigoriadis algorithm - in our case - works efficiently.

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SOLVING A LARGE-SCALE LP PROBLEM APPLYING DECOMPOSITION

Evaluating a production plan for a screw factory having more workshops can be described by the LP-problem

$$L_i \leq \sum_j x_{ij} \leq U_i \qquad i = 1,2,\ldots$$

$$\sum_{i,j} t_{ijk\ell}\, x_{ijk\ell} \leq M_{k\ell} \qquad k = 1,2,\ldots \quad \ell = 1, 2,\ldots$$

$$\sum_{\ell} x_{ijk\ell} - x_{ij} = 0 \qquad k = 1, 2, \ldots$$

$$x_{ijk\ell}, x_{ij} \geq 0$$

$$\max / \sum_{i,j} c_{ij} x_{ij} /$$

where

x_{ij} is the quantity of the i-th product produced by the j-th technology;

$x_{ijk\ell}$ is that part of the above quantity which is produced by the ℓ-th machine /group/ of the k-th workshop;

$t_{ijk\ell}$ is the corresponding time required;

$M_{k\ell}$ is the capacity of the ℓ-th machine group of the k-th workshop;

L_i and U_i are lower and upper bounds for the production of the i-th product;

c_{ij} are the profit factors.

The number of the products is about 6000, there are more then 10 000 possible technologies and about 50 workshops. The maximal number of machine groups at a workshop is 40.

The problem was solved by a Benders type decomposition. The subproblems are transportation problems and generalized transportation problems and in each iteration the extremal problem itself was solved by a Dantzig-Wolfe decomposition.

The paper gives the details of the computational procedures and presents some results and experiences.

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PARAMETRIC INTEGER PROGRAMMING: THE RIGHT-HAND-SIDE CASE

A family of integer programs is considered whose right-hand-sides lie on a given line segment L. This family is called a parametric integer program (PIP). Solving a (PIP) means finding an optimal solution for every program in the family. It is shown how a simple generalization of the conventional branch-and-bound approach to integer programming makes it possible to solve such a (PIP). The usual bounding test is extended from a comparison of two point values to a comparison of two functions defined on the line segment L. The method is illustrated on a small example and computational results for some larger problems are reported.

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APPROXIMATIONS TO STOCHASTIC PROGRAMS

Several problems in stochastic optimization can be formulated by

$$\text{Find } \inf_{x \in U} E_\omega f(\omega,x) \text{ subject to the constraints } g_i(\omega,x) \le 0,\ i=1,\dots,m \text{ almost surely} \qquad \text{(SP)}$$

where $f(\omega,x)$ and $g_i(\omega,x)$, $i=1,\dots,m$ are real-valued functions of a stochastic parameter ω in a probability space $(\Omega,\mathbf{A},\mu)$ and a vector x in a subset U of a normed space X and E_ω denotes the expectation operator with respect to ω. Problem (SP) becomes e.g. a stochastic program with recourse if we put $x=(x_o,x_1)$, $x_o \in \mathbb{R}^n$, $x_1: \Omega \to \mathbb{R}^m$, where x_o is interpreted as a first stage decision and $x_1(\omega)$ is the decision of the second stage which can be chosen, in order to satisfy the original constraints, when the value of the stochastic variable ω has been revealed to the decision maker. Several theoretical properties of (SP), especially for stochastic programs with recourse, have been established by Rockafellar and Wets. To provide a constructive approach to optimization problems of this type is the purpose of this contribution.
Approximations to (SP), yielding minimizing sequences, are obtained by (combinations of)

a) Linearization of f and g_i, $i=1,\dots,m$ with respect to x,

b) Approximation of the probability measure μ by sequences of simpler measures μ_n,

c) (Penalty function method) Embedding of (SP) into the family of problems

$$\left\{ \text{ Find } \inf_{x \in U} \left(E_\omega f(\omega,x) + \frac{1}{\varepsilon} \sum_{i=1}^{m} E_\omega \psi(g_i(\omega,x)) \right) \right\}_{\varepsilon>0},$$

where ψ is a function such that $\psi(t)=0$ for $t \le 0$ and $\psi(t)>0$ for $t>0$. Method (c) yields a simple direct method for the Lagrangian and a constructive derivation of the (stochastic) Lagrange multipliers (considered by Rockafellar and Wets in a more theoretical framework). Special attention is paid to the question of stability of a minimizing sequence of (SP) under changes of the probability distribution μ.

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SHORT TERM WATER RESOURCES ALLOCATION

In this paper, after having recalled briefly the water resources allocation deterministic problem and the algorithm used to solve it, we discuss the obtained practical results.
In a second part we describe a practically realisible algorithm to solve the same problem with large electrical network. It is certainly quick comparing with the previous one. This algorithm mixes partitioning method and relaxation.

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DIFFERENTIAL EQUATIONS WITH TRANSFORMED ARGUMENT AND ECONOMIC APPLICATIONS

In 1969 D.Przeworska-Rolewicz gave in [1] a definition of algebraic derivation. This definition is based on two classical facts:

1° that the Voltera linear equation /of the second kind/ always has a unique solution in the space of continuous functions.

2° that the derivative of an integral with respect to its upper limit is the integrated function in the same space.

Namely:

Let X be a linear spaces over the complex scalar field. We say that a linear operator D transforming X into itself in an algebraic derivative if there is a linear operator R transforming X into itself and such that:

1° $\mathcal{D}_R = X$ and $RX \subset \mathcal{D}_D$

2° $DR = I$

3° the operator $I - \lambda R$ is invertible for every scalar λ. The operator R is called the algebraic integral.

In this paper we will consider differential equations containing an algebraic derivative. We will give algebraic and numerical solution of such. Then equations with transformed argument will be taken into consideration, among others equation with reflection.

Finally we will give an example of an economic model with investment delays taken into consideration.

As a result of taking these delays into consideration one of the constraints is a differential equation with transformed argument.

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SOLVING NONLINEAR PROGRAMS WITHOUT USING ANALYTIC DERIVATIVES. PART III: A QUASI-NEWTON METHOD FOR LINEAR CONSTRAINTS.

The first two papers in this series described the theory and implementation of a new modified Newton algorithm for the optimization of a nonlinear function subject to linear inequality and equality constraints. That algorithm is an extension of Mifflin's local variations - approximate Newton method for unconstrained minimization.

Building on the structure developed in Part I, this paper presents a new type of quasi-Newton algorithm, which, at each iteration, explicitly computes a user-determined number of rows/columns of an approximated Hessian matrix. Using a factorization of the currently active constraint matrix, a coordinate system is determined, and all derivative information is obtained relative to it by differencing function values at feasible points. A large class of matrix factorizations may be used, so that special structure and sparsity may be exploited.

Accumulation points of the algorithm satisfy the Karush-Kuhn-Tucker first order necessary optimality conditions if the objective function is continuously differentiable and appropriately bounded. Under stronger assumptions, every accumulation point of the algorithm also satisfies a second order necessary optimality condition.

The rate of convergence of the algorithm is superlinear if the function is twice continuously differentiable and there is a unique accumulation point satisfying second order sufficiency conditions.

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A NEW METHOD FOR THE SOLUTION OF A STOCHASTIC PROGRAMMING PROBLEM OF A. PREKOPA

The topic of this paper is the numerical solution of a stochastic programming problem of A.Prékopa. We consider the nonlinear programming aspects of the problem. From the nonlinear programming point of view we have a nonlinear programming problem with only one nonlinear constraint, a set of linear constraints and a nonlinear objective function. For this type of problems we developed an algorithm, which is a combination of the P1 feasible direction method of Zoutendijk, and the reduced gradient technique. The convergence of the method can be proved, and we also report our numerical experiences.

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CONSTRAINED OPTIMISATION PROBLEMS FOR SET-VALUED FUNCTIONS

First of all we present a review of known properties for unconstrained minima of convex set-valued functions, then we pass to examine the constrained problem.
The classical results known for convex constrained programs are extended for set-valued functions.
In order to do this, convexity properties are studied for families of sets and a representation theorem is proved for convex set functions in terms of "affine" functions.
It allows to introduce a subgradient relation for convex nondifferentiable set-valued functions, that can be linked to the minimum conditions.
A Lagrange multiplier theorem is then proved for an inequality--constrained convex problem and a duality theory is developed .
Finally, such a result is applied to a general equilibrium model including rent or cost from the use of space.

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ZERO-ONE MIXED INTEGER PROGRAMMING USING INTERACTIVE GRAPHICS

This paper presents experimental results of solving 0-1 mixed integer programming (MIP) problems using interactive graphics. In this application, graphics is essential to the decision making process.

There are two plots in the graphics data output, (1) the history of the value of each integer variable as a line which has an angle proportional to the variable's value for each value in the history, and (2) a tree diagram of objective function value degradation vs. sum of infeasibilities. The plots are primarily used to aid in fixing of variables to one of their bounds, and to observe the effect of fixing operations. The plots can also be used to determine the effect of changes in solution strategy.

Graphics output showing the use of this technique to decrease the iterations required to solve 0-1 MIP models will be presented. The ability to decrease the number of iterations is critically dependent upon being able to determine the controlling variables in the model and their values.

This ability, using the graphics output and for which the graphics output is essential, will be demonstrated. The fixing of key variables to a bound not only decreases the size of the model, but also forces other variables to take specific values.

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SECOND ORDER CONVERGENCE USING A MODIFIED ARMIJO STEP-SIZE RULE FOR FUNCTION MINIMIZATION

Armijo's step-size procedure for function minimization is modified to include second derivative information. Accumulation points using this prodedure are shown to be stationary points with positive semi-definite Hessian matrices.

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DUALITY IN GAUGE PROGRAMMING

Duality in quadratic, homogeneous, and ℓ^p programming models has received considerable attention over the years, reflecting the importance and widespread applicability of models involving such fundamental structural characteristics. A survey of the literature gives one the impression that these models are fairly unrelated, and in addition, the existing works are almost exclusively restricted to the treatment of just one of the models in a setting of finite dimensions and polyhedral cones. Using the advances of the past decade in the understanding of duality in extremely general convex models, we show in this paper that each of the models above is actually a specialization of a single more general, yet simple, model itself having quite specific structure. The model involves composing Young's functions with gauge functions of convex sets, and it is completely symmetric in the sense that the associated dual problem is of the same form. Working in the general non-polyhedral and infinite dimensional setting, we derive a unified theory which gives sharp extensions of the various classical duality results to our broader, more flexible model.

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TOWARD A GENERAL ALGEBRAIC MODELLING SYSTEM

Quantitative analysis is used increasingly in the social sciences, as a guide to decision-making. This is particularly true in development planning where mathematical programming represents a potentially effective tool of analysis. In spite of major advances in algorithmic development, the operational acceptance of this tool is seriously hampered by inadequate software. The major source of inadequacy is the absence of a commonly interpretable representation of the problem, model and data (by both the social scientist and the computer) at the different stages of analysis.

The paper provides a progress report on work initiated and underway at the World Bank in recognition of this problem which obstructs the routine use of highly desirable mathematical tools on a production basis. The new approach under development enables a direct communication between the mathematically skilled social scientist and the computer, given the former's ability to correctly specify a problem in algebraic form. At the same time, because the algebraic representation of the problem is computer-readable there is no loss of information regarding data or problem structure. The design criteria of the new software, and achievements to date will be presented by means of examples taken from actual applications at the World Bank. It is speculated, in conclusion, that the approach incorporates ideas that may lead to a new generation of modelling software.

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ON THE EXISTENCE AND UNIQUENESS OF SOLUTIONS IN NONLINEAR COMPLEMENTARITY THEORY

A complementarity problem is said to globally uniquely solvable if it has a unique solution, and this property does not change under translations.

A characterization of this property, which generalizes a basic theorem in linear complementarity theory, is given. Also, the conditions of Cottle, Karamardian, and Moré in the nonlinear theory, are all shown to be generalized by the new results. In particular, Cottle's condition is proved directly and several open questions concerning this condition are settled.

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PROBLEMS RELATED TO CREW PLANNING AND SCHEDULING IN A RAILROAD COMPANY. ASSIGNMENT APPROACHES AND ALGORITHMS.

The aim of the paper is first of all to describe and formulate some mathematical programming problems, which arise in a railroad company, and which are at present unsolved, at least as it regards large-scale real situation. Problems of such a kind are crew and manpower planning, optimal time-table determination.
For the crew scheduling problem a new algorithm is proposed which is based on the upper bound linear assignment algorithm. The optimal time-table problem is formulated as an optimal vertex-packing on a undireated graph, with additinal linear integer constraints; this integer linear program is structured, i.e., it has a block-angular matrix of the constraining system.
Computational experience has been made on a sample of real problems coming from the italian railroad company.

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DOUBLE-RELAXATION OPTIMALITY CONDITIONS FOR INTEGER PROGRAMMING

A new class of optimality conditions for integer and mixed-integer programming is developed, based on the use of two different relaxations of the feasible set. This approach is shown to include as a special case the ordinary (single) relaxation optimality conditions that form the basis for most integer programming algorithms. A particular choice of the relaxations is also shown to be a generalization of the Kuhn-Tucker conditions in which complementarity is replaced by "quasi-complementarity", i.e., it is shown that "near" complementarity of certain "primal-dual pairs" is sufficient to guarantee optimality. As an application of the sufficient optimality criteria, a procedure is given for the computer generation of certain classes of integer and mixed-integer problems with known optimal solutions. (Specifically, the problems constructed so far have been of the capital budgeting or facilities location types.) Extensive computational experience (which will be summarized) has shown that the test problems generated are relatively difficult integer and mixed-integer problems, and are therefore quite suitable for the testing of solution techniques. The significant difference between these groups of test problems and test problems previously described in the literature lies in the fact that knowledge of the optimal solution allows the evaluation of heuristic algorithms(and other algorithms that generate "good" rather than optimal solutions)without the initial expense of actually having to compute optimal solutions for a number of difficult problems.

In addition, these new optimality conditions suggest a number of new algorithms for integer proramming as well as a number of new optimality tests that may be added to existing techniques. Finally, certain versions of the optimality conditions permit post-optimal sensitivity analyses to be carried out in a straightforward manner. Simple examples as well as computational experience on laryer problems will be described so as to illustrate these concepts.

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AN ALGORITHM FOR NONSMOOTH OPTIMIZATION

An implementable algorithm for solving constrained optimization problems with functions that are not everywhere differentiable is presented. The method is based on combining and extending the work of Wolfe, Feuer, Poljak and Merrill. A certain specialization is the method of conjugate gradients.

The nonsmooth functions that can be dealt with are called semi-smooth, because they are defined to have directional derivatives and generalized gradient sets which are interrelated in a semicontinuous manner. The class of semismooth functions includes convex and concave functions as well as many piecewise continuously differentiable functions and extremal-value functions that have generalized gradient sets as defined by Clarke.

Accumulation points of the algorithm iterate sequence satisfy a very general necessary optimality condition depending on generalized gradients of the problem functions. This condition is also sufficient for optimality if the problem functions are semiconvex and a constraint qualification is satisfied. Under stronger convexity assumptions, bounds on the deviation from optimality of the algorithm iterates are given.

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A METHOD OF SUCCESSIVE OPTIMA FOR SOLVING THE ASSIGNMENT PROBLEM

The optimal solution of the assignment problem

(AP): minimize $\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij}$ subject to $\sum_{j=1}^{n} x_{ij} = 1$, $i = 1, 2, \ldots, n$, $\sum_{i=1}^{n} x_{ij} = 1$, $j = 1, 2, \ldots, n$,

$x_{ij} = 0$ or 1, $i, j = 1, 2, \ldots, n$

is approached through the optima of a sequence

$$(AP_1),\ (AP_2),\ \ldots,\ (AP_k)$$

of assignment problems. The first of these problems is defined in such a manner that (one of) its optimal solution(s) may be known, and any problem occurring in this sequence, as well as (one of) its optimal solution(s), is obtained from the previous problem by means of some rather simple modification. The cost matrices $C_1, C_2, \ldots, C_k$ belonging to the problems (AP_1), $(AP_2), \ldots, (AP_k)$, respectively, consist of certain rows of the cost matrix C of (AP), which is, of course, possible only if certain rows of C occur in these matrices with multiplicity. However, for any integer l such that $2 \leq l \leq k$ C_l contains at least one more different rows from C than C_{l-1}, and hence (AP_k) coincides with (AP) and $k \leq n$. (C_k can differ from C only in the order of rows and columns.)

The cost matrix C_1 of the initial problem is chosen in the following manner. If $i_1, i_2, \ldots, i_n$ are row indices such that $c_{i_k k} \leq c_{1k}, c_{2k}, \ldots, c_{nk}$ for $k = 1, 2, \ldots, n$, let C_1 be the matrix the kth row of which is equal to row i_k of C. Hence the optimal solution of (AP_1) is given by the formulae $x'_{11} = x'_{22} = \ldots = x'_{nn} = 1$, $x'_{ij} = 0$ for $i \neq j$.

The transition from (AP_{l-1}) to (AP_l) is realized by solving a <u>modified assignment problem</u> with $(n+m)\times(n+m)$ variables where $1 \leq m \leq n-1$. The feasible solutions of this modified problem are regarded as $(n+m)\times(n+m)$ permutation matrices partitioned in the following way

$$Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \\ Y_{31} & Y_{32} \end{pmatrix}$$

where Y_{11} and Y_{21} are $(n-m)\times n$ and $m\times n$ submatrices, respectively. The modified problem is formulated as follows:

(APM): minimize $\sum_{i=1}^{n+m} \sum_{j=1}^{n+m} d_{ij}y_{ij}$ subject to

(i) $\sum_{j=1}^{n+m} y_{ij} = 1$, $i = 1, 2, \ldots, n+m$, (ii) $\sum_{i=1}^{n+m} y_{ij} = 1$, $j = 1, 2, \ldots, n+m$, (iii) $y_{ij} = 0$ or 1, $i, j = 1, 2, \ldots, n+m$, (iv) $Y_{12} = 0$, (v) the value of the objective function belonging to any solution satisfying (i) - (iv) does not exceed $d_{11} + d_{22} + \ldots + d_{n+m,n+m}$, (vi) $Y_{31} \neq 0$.

Solving (APM) requires at most mn^4 arithmetic operations. The total amount of arithmetic operations needed to solve (AP) by means of the method of successive optima does not exceed $(n+1)n^5/2$.

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CIRCUITLESS GRAPHS, GENERALIZED DYNAMIC PROGRAMMING AND APPLICATIONS

Path-finding problems in /finite/ circuitless graphs are studied, and it is shown how the dynamic programming principle can be extended to a wide class of problems. A number of examples are given.

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AN OPTIMAL CUTTING ALGORITHM FOR RECTANGLE ELEMENTS

An optimization of cutting stock problem consists in solving integer linear programming problem (IPL). In such a problem each variable is connected with slitting pattern, some constraints refer to demand of ordered elements, other to supply of stock materials. The purpose of optimization is, most of all, to minimize the cost of satisfying the demand or to minimize the percentage waste. The greater number of slitting patterns is used the better effectivity of object function is attained. Making of different feasible patterns is a combinatorial problem. In practice finding all the feasible patterns is impossible.

For one-dimensional problems there exists the ability of finding an optimal solution. For solving these problems, it is necessary, in each step of simplex method, to find the solution of the "knapsack problem".

In the case of two and more dimensional cutting stock problems it is possible to find an optimal solution by the use of the guillotine cutting method.

In three-dimentional practical problems the use of guillotine cutting method is justified (for technical and physical reasons) while in many two-dimensional cutting problems this method is less effective (i.e. in the sense of object function).

Ignoring the guillotine cutting method one can get better solution but then one must solve the IPL problem with the sufficient number of variables. What number is sufficient it depends on real problems in mind. In any case it is a great one. Besides this making of new patterns, according to other methods, is undirected (contrary to guillotine method).

The Authors present an algorithm that enables making the slitting pattern for demand defined as small rectangle elements and supply as one large rectangle (it may be either a sheet or a bale in practice). The algorithm allows to obtain many patterns which are necessary to find high effective solutions of IPL problems. It also has the virtue of obtaining the patterns in short computer time. Multiple application of the algorithm is the heart of the method of solving cutting stock problems.

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UIMP: USER INTERFACE FOR MATHEMATICAL PROGRAMMING

A newly developed modelling system : UIMP is described in this paper. Using this system mathematical programming models can be generated and their solutions analysed and reported. The structure facilities embedded in the system allows the underlying structure of a user model to be at once captured and the model can be germanely defined. The use of the system in more than one context is also illustrated.

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SUFFICIENT OPTIMALITY CONDITIONS FOR COMPLEX PROGRAMMING WITH QUASI-CONCAVE CONSTRAINTS

Consider the problem, to minimize $f(x)$ subject to $g(x) \geq 0$, where $f: R^n \to R$ and $g: R^n \to R^m$ are differentiable functions. Mangasarian has given sufficient Lagrangean conditions for x_0 to be an optimal point, when f is pseudo-convex and g_i is quasi-concave whenever $g_i(x_0) = 0$. Various authors have recently extended these and similar results to the problem, to minimize $\text{Re}\, f(z, \bar{z})$ subject to $g(z, \bar{z}) \in S$, where $f: C^{2n} \to C$ and $g: C^{2n} \to C^m$ are analytic functions, and S is a convex cone. Sufficiency theorems were obtained, assuming that f has pseudoconvex real part, and g is concave with respect to S. The latter was not weakened to quasi-concavity for the tight constraints only, since $g(a, \bar{a})$ on the boundary of the cone S does not correspond to certain components of $g(a, \bar{a})$ vanishing. We show here how the assumption on g can be weakened to quasi-concavity with respect to a convex cone containing S. When S is polyhedral, this corresponds to quasiconcavity over tight constraints, as in the real case.

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STUDIES ON ANALYTICAL TRAFFIC FORECASTING AND ASSIGNMENT.

Because of the large-scale development of the motorization, the development of the national road network has an important place the long-range objectives of the transport development. The future demands in roads can be determined on the basis of the growth of traffic demands and of the objectives of land use development.

Among the aspects of the land use development the following ones seem to be most important; satisfactory connecting and exploring of the settlement network, promoting of the industrial development, approachability of the holiday resorts and excursion places and assuring the international connections.

In addition to the until now for the determination of the traffic demands used "projective method" also in Hungary emerged the necessity of the "analytical method". The latter method determines the traffic demands as a function of the alterations of the social and economical structure and can form the basis of a new method of the road network planning.

Concerning the traffic demands, the models of the weekday, weekend and international traffic separately have been studied.

In respect of the weekday traffic, more detailed studies have been accomplished and a chain of computer programmes has been constructed. The following tasks have been solved:

- selection of network models and zones,
- analysis of the structural data of the zones,
- establishment of a trip generation model, considering the accessibility of the zones,
- establishment and calibration of the trip distribution model,
- elaboration of the minimum impedance path finding algorithm and of the traffic assignment system under capacity restraint.

The paper discusses the models and the chain of computer programmes which have been tested on practical problems during the traffic studies of the variants of the motorways M 3 and M 6.

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A THEORY OF RESOURCE ALLOCATION AND A DECOMPOSITION METHOD FOR MATHEMATICAL PROGRAMMING

Resource allocation theories have probably existed since the time resource allocation problems were first recognized. Decomposition methods to solve these problems, however, have come into existence only within the last several years. One characteristic distinguishing this study from others in the field is the simultaneous presentation of a mathematically rigorous allocation theory of broad practical implications together with a simple and ready-to-implement decomposition method.

The dissertation starts with a review of the decomposition methods relevant to the study, and then proceeds from the general concepts of theoretical importance to the particularizations of practical relevance. These are represented by a method and an algorithm for solving mathematical programming problems by decomposition.

The generality of the theory ensures wide applicability of the principles developed. In fact, any systems in which the subsystem efficiencies can be quantified are candidates to be optimized by the suggested approach. Such systems may even not be represented by a model, in which case the theory becomes a practical decision making tool. In systems represented by a model (analytical, simulation, etc.) the iterative reallocation process can be easily set up emulating the steps presented in the mathematical programming decomposition method.

The coordination scheme used in the method is based on the Lagrange multipliers of the subproblems, interpreted here as the "efficiencies" with which the different subsystems utilize the shared resources. In the search for an optimal partition, resources are taken from the less efficient subsystems and given to the more efficient ones, in an iterative process that ends when the efficiencies of all subsystems are equalized and no further reallocations are necessary.

The specific implementation of the method results in an algorithm for separable geometric programs, by means of which an example is completely solved. This example is useful both for showing the steps involved in the algorithm, and also for illustrating the main advantage of the decomposition approach, namely, the simplicity with which the subproblems can be solved when they are isolated from the total problem.

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THE STRUCTURE OF LEVEL SETS OF THE DUAL FUNCTION AND THE CONVERGENCE OF SOME METHODS OF CONVEX PROGRAMMING.

Let us consider a convex programme

$$f(x) \to \sup \quad \text{under conditions} \quad g(x) \geqslant 0, \; x \in R \subset E^n \tag{I}$$

It is supposed that $\operatorname{int} R \neq \varnothing$, $f(x)$, $g_i(x)$, $i = I,\ldots m$, are differentiable functions in R. We shall denote $F(x,y) = f(x)+yg(x)$, $y \in E^m_+$, $x \in R$ and $\psi(y)=\sup\limits_{x\in R}F(x,y)$. Together with (I) we consider the dual problem

$$\psi(y) \to \inf, \; y \in E^m_+. \tag{2}$$

Lemma I. Let problems (I) and (2) be solvable, X_0, Y_0- sets of their solutions, $X_0 \cap \operatorname{int} R \neq \varnothing$. Then Y_0 is a polyhedral set.

Let $\nabla f(x)$ be a gradient of a function f at a point x, $\nabla g(x)$ be a Jacobian of a vector-function g.

Assumption A. $\operatorname{Argmax}\limits_{x\in R} F(x,y)=X(y)$, $X(y)\cap \operatorname{int} R \neq \varnothing$ for all $y \in E^m_+$.

Assumption B. There exists such a constant h that $|x^1 - x^2| \leqslant h$ for all x^1, $x^2 \in X(y)$ and an arbitrary y.

Assumption C. Function f(x) is strictly concave on R.

Assumption D. There exist such x^i that $g_i(x^i) > 0$, $i=I,\ldots,m$.

Lemma 2. Let $g(x)=b-Ax$, assumptions Aand B hold and the conditions of lemma I take place. Then the function $\psi(y)$ and the mapping X(y) are constant on the set $Y(c)= \{y \in E^m_+ : yg(x^*)=0, \; y\nabla g(x^*)=c\}$ for all vectors c and $x^* \in X_0 \cap \operatorname{int} R$.

It follows from these properties of the sets Y(c) that the subgradient method and the alternating coordinate direction method of minimization converge without the requirement of boundness of the set Y_0. We describe the latter method.

Let an arbitrary $y^0 \in E^m_+$ be given. On the t-th step we define

$$y^t = y^{t-1} + \theta_t e^{i(t)}, \quad t=I,\ldots, \tag{3}$$

where the k-th component of $e^k \in E^m$ is equal to one and the others are zeroes. The value of θ_t is determined by the condition

$$\psi(y^t) = \min_{\theta:\, y_i^{t-1}+\theta \geq 0} \psi(y^{t-1} + \theta e^{i(t)}) \tag{4}$$

The sequence i(t) can be cyclic, for example. It is shown that this process coincides with the relaxation method [I].

Theorem I. Let $g(x)=b-Ax$, assumptions A, C and D hold, the problems (I) and (2) be solvable. Then the sequence y^t defined by (3), (4) converges to Y_0. The corresponding sequence x^t converges to the solution of (I).

The subgradient method is given as a process of the following iterations

$$y^t = (y^{t-1} - d_{t-1} l(y^{t-1}))_+ \ , \ t = I,\ldots, \quad (5)$$

where $a_+ = a$ if $a \geqslant 0$ and $a_+ = 0$ if $a < 0$, d_t is a positive number, $l(y)$ is a subgradient of ψ at a point y.

Theorem 2. Let $g(x) = b - Ax$, assumptions A, B hold, the problems (I) and (2) be solvable. Then there exists such a $d > 0$ that the sequence y^t defined by (5) converges to Y_0 when $0 \leq d_t \leq d$, $d_t \to 0$ as $t \to \infty, \sum_{t=1}^{\infty} d_t = \infty$. If the function $F(x,y)$ is stable in relation to x then the sequence $x^t \in X(y^t)$ converges to X_0.

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DESIGN AND TESTING OF A NETWORK-BASED 0-1 INTEGER PROGRAMMING CODE

This paper describes the design and implementation of a modular, 0-1 integer programming package. The design is constructed according to the author's network relaxation concept in which the pure 0-1 integer program is transformed into an expanded network model. An efficient branch and bound procedure is used to resolve non-integralities of the original integer form. The advantage of this approach is the utilization of a network for fathoming subproblems. A comparison with the traditional linear programming relaxation is included.

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ON SIMPLE CONVEX POLYTOPES AND ABSTRACT POLYTOPES

Let A be an m x n matrix and let K be the set of feasible solutions of

$$Ax = b$$
$$x \geqq 0$$

We assume that $b \in R^m$ is not in the linear hull of any set of m-1 column vectors of A, and also that K is bounded. Under these assumptions K is a simple convex polytope.

It is well known how to construct an abstract polytope P, corresponding to K. The symbol j is associated with the variable x_j. The subset of symbols associated with variables which are equal to zero at an extreme point of K, define a vertex in the abstract polytope P. Conversely, every vertex of P is associated with an extreme point of K in this manner.

Axiomatically, abstract polytopes can be defined as a class of subsets of a set {1,...,n} all of the same cardinality, and satisfying certain connectivity axioms. These subsets are called the vertices of the abstract polytope. Every simple convex polytope is an abstract polytope, but the class of abstract polytopes turns out to be larger than the class of simple convex polytopes.

We discuss some recent results on the necessary and sufficient conditions for an abstract polytope to be a simple convex polytope. We also discuss some recent results on edge paths in simple convex and abstract polytopes.

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DEVELOP-IT-YOURSELF PROGRAMS IN MATHEMATICAL PROGRAMMING

Mathematical Programming techniques seem to have a strong appeal to mathematics-oriented students, but little appeal to problem-oriented students. The latter do in general not enjoy algorithmic details even if they need them for solving a problem. For this group, several develop-it-yourself programs were drafted.

In these programs the students develop the algorithms themselves step by step. For this purpose, some 10 to 20 different problems (examples) of the same mathematical structure and with increasing degree of complexity are used to motivate the student. The students start to solve the simplest examples by means of common sense. This common sense is then systematized by the following text in the sense of "What you actually did was subtracting the row minima from the single rows and assigning elements at the yielded zeros...". Such a step is a part of the algorithm and will now have to be refined when solving the next example, followed by the text: "What you actually did was...".

Such programs may require a little more student's time than the traditional teaching approach . But it has the following advantages:

(i) The student becomes familiar with many different problems of the same mathematical structure which may motivate his learning and may increase his problem sensitivity.

(ii) The student usually gets a better insight into the algorithm, e.g. the algorithm and its single steps become plausible to him; his understanding is not only mathematical and formal. His understanding is less abstract; it is more "systematic common sense".

(iii) Many students do enjoy these programs, due to the examples and due to their satisfaction and success while solving the examples.

(iv) As a consequence of (ii), the students develop a good ability to explain the problems and algorithms to others.

(v) Due to the experience with different examples and due to (ii) the students seem to develop an increase ability to program the algorithms for computers.

At the moment, the following develop-it-yourself programs are completed or under development: Ford-Fulkerson algorithm for the linear assignment problem; primal as well as primal-dual algorithms for the linear transportation problem; simplex-algorithm for linear programming; sensitivity analysis and parametric programming in LP; cutting plane techniques in integer programming; branch and bound. Others will follow.

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OPTIMAL DESIGN OF PROCESSES MODELLED IN POSYNOMIAL FUNCTIONS USING SEPARABLE PROGRAMMING

As illustrated by a large scale model of an ammonia synthesis loop, reasonably precise models of chemical processes can frequently be formulated in terms of posynomial functions. The resulting posynomial equality and inequality constrained optimization problems are only amenable to geometric programming solution techniques if the equality constraints can be reformulated as inequalities. In large process models this is generally difficult to accomplish. In this paper we use a well-known transformation to convert such problems to a non-convex and sparse but separable program which can be solved using separable programming and sparse matrix techniques. The method is applied to the optimization of the design of the synthesis loop of an ammonia process - a high dimensionality problem which involves a large number of equality constraints. Comparisons are given with conventional solution approaches.

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MANPOWER STAFFING AT AN AIRPORT: A GENERALIZED SET COVERING PROBLEM

The problem is considered of manpower staffing at an airport. Some mathematical formulations are given. The first consists in a nonlinear integer programs /NIP/, which is decomposed, by means of Bender's procedure, into a sequence of linear integer programs /ILP/; an optimal solution gives us the minimum requirement of manpower at an airport. Two algorithms are discussed: the former decomposes the NIP into ILP; the latter looks at on optimal solution of every ILP in a sequential way.
Another problem is discussed, i.e., the problem of determining the optimal levels of personel at an airport, if the average lengts of several ques are given. Such a problem leads us to a stochastic version of the preceding NIP. Bender's procedure is shown to enable us to give upper and lower bounds for the distribution function of the minimum of NIP.
Computational and software aspects are discussed in connection with experiences made on sample real problems.

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OPTIMAL CONTROLLING OF GERT NETWORKS

Decision activity networks (GERT networks) with six different types of nodes (corresponding to the project events) are investigated, where to each arc are assigned the duration (a random variable with given distribution) and the conditional performance probability of the corresponding activity. Nodes with and entrance and with inclusive-or entrance side as well as nodes with deterministic exit side are supposed to occur only within so-called basic elements that can be reduced to structures containing only "steor nodes" (nodes with stochastic exit and exclusive-or entrance side).

The time scheduling of those GERT networks consists in calculating the probabilities that the distinct terminal events occur and the distribution of project duration given that a certain terminal event has occurred. In doing this so-called activation numbers and activation functions of nodes are introduced.

As concerns cost scheduling, it is expedient to identify the possible policies (i.e. mappings of the time axis into action sets) with performance probabilities of activities depending on time. The minimization of the expected project costs, including costs depending on the total project, the events, and the activities of the project, leads to an optimal control problem. The cost functional contains simple and double integrals subject to integral equations as restrictions. That problem may be solved by means of gradient methods in Hilbert spaces.

J.L. Nicolas, University of LIMOGES, /France/

NON-LINEAR OPTIMIZATION IN INTEGERS PROBLEMS AND HIGHLY COMPOSITE NUMBERS

Soit $d(n)$ le nombre de diviseurs de n. On dit que n est hautement composé si $m < n \Rightarrow d(m) < d(n)$. Cette définition est reliée à la recherche de : $\max_{n \leq a} d(n)$. Si l'on décompose n en facteurs premiers, $n = \prod p_i^{x_i}$, on a : $d(n) = \prod (x_i + 1)$ et ce problème devient un problème de programmation mathématique :

$$\begin{cases} x_1 \log 2 + x_2 \log 3 + \ldots + x_k \log p_k + \ldots \leq A = \log a \\ \max\left(\log(x_1 + 1) + \log(x_2 + 1) + \ldots + \log(x_k + 1) + \ldots\right) \end{cases}$$

Les méthodes utilisées en théorie des nombres par S. Ramanujan P. Erdös et moi-même, peuvent s'adapter pour résoudre des problèmes d'optimisation en nombres entiers et fournissent des algorithmes de calcul très rapides.

Exemples -

1) Etant donné un entier C trouver n, et des entiers $x_1, \ldots, x_n$ tels que $x_1 + \ldots + x_n = C$ et maximisant $\prod_{i=1}^{n} x_i^{\,i}$. Ce problème était posé par T.L. Saaty dans son livre : " Optimization in integers and related extremal problems ".

2) C étant donné et α et β étant réels, trouver x et y entiers vérifiant $\alpha x + \beta y \leq C$ et maximisant $x y$.

3) Dans le livre de G. Hadley : " Non linear and dynamic programming ", p. 362, un problème de stockage des pièces de rechange d'un sous-marin, en supposant que la probabilité de rupture des pièces suit une loi de Poisson. G. Hadley traite ce problème par la programmation dynamique, mais notre méthode s'y adapte.

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SEPARATING SETS WITH RELATIVE INTERIOR IN FRÉCHET SPACES

A separating theorem, valid in finite dimensional spaces, and involving the relative interior of the sets to be separated, will be extended to Fréchet spaces.
This theorem will be elucidated by means of a few examples. The second separation theorem is a generalization of an existing separation theorem, valid in Fréchet spaces. This paper consists of two parts, part I contains the first theorem, the second part contains the second generalization.

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CONTRACTING MARKOV DECISION PROCESSES

Based on the concept of stopping time a set of successive approximations procedures for Markov decision processes with respect to the total expected reward criterion, can be generated. Every (nonzero) stopping time yields namely a monotone contraction mapping on a complete metric space (V) of real valued functions on the state space. This state space is supposed to be countably infinite or finite. The fixed point of the mappings equals the total expected reward (V^*) over an infinite time horizon if an optimal policy is used.

This fixed point is thus independent of the chosen stopping time.

The set of optimization procedures can be extended by defining for each mapping of the mentioned set a set of "value oriented" mappings. Though, a mapping A from this extended set needs not to be contractive nor monotone it remains that the sequence

$$v_n := A\, v_{n-1}\ , \quad v_0 \in V,\ n \in \mathbb{N}$$

converges to V^*. Using the concept of stopping times leads to a unifying approach; the constructed set of methods includes the existing methods for solving Markov decision processes as introduced by R. Howard, N. Hastings, D. Reetz and J. MacQueen and enables us to construct upper-lowerbounds and suboptimality criteria.

The results are achieved under less restrictive conditions than usual in literature; an unbounded reward structure is allowed and the transition probabilities are not required to be strictly defective.

If the possibility of making decisions is ignored, the problem results in solving a system of linear equations of the form

$$(I - P)x = b$$

with I the identity matrix and P a sub-Markov matrix. The constructed set of solution techniques then contains e.g. the Jacobi-, Gauss-Seidel-, and overrelaxation iterative methods for solving systems of linear equations.

In the lecture some aspects of the above theory will be treated.

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COMBINED VARIABLE METRIC - PARTIAL CONJUGATE GRADIENT ALGORITHMS FOR A CLASS OF MINIMIZATION PROBLEMS

It is generally known that variable metric methods are superior to conjugate gradient algorithms when solving unconstrained minimization problems of moderate dimension. This advantage is normally attributed to the fact that the Variable Metric methods automatically rescales the problem at every step using updated approximations to the inverse Hessian. Unfortunately, the high computational and storage requirements involved in updating and storing that approximation becomes prohibitive for large scale problems. In this paper we focus on a class of large scale unconstrained minimization problems whose special structure can be employed to exploit the advantages of variable metric algorithms at a relatively low cost. In particular we consider functions whose Hessian is of the form $H=(M+AGA^T)$ where M is a block diagonal matrix and AGA^T is a matrix whose rank ρ is considerably lower than that of H. Recent work by Bertsekas indicates that the conjugate gradient method with respect to the metric M converges in at most $\rho+1$ steps when applied to quadratic functions of the aformentioned structure. Reinitializing this algorithm every $\rho+1$ steps where M is set to its value at the beginning of each cycle, results in superlinear convergence on more general functions. The method proposed in this paper is based on the above idea however we use variable metric techniques to approximate M^{-1} and update it utilizing gradient information obtained in the conjugate gradient steps. Since the blocks of M^{-1} can be individually approximated the number of steps required for a full updating cycle equals the dimension of the largest block in M. The paper explores the properties of the proposed method and discusses its application in solving some problems having the above structure.

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ON THE COMPLEXITY OF SET PACKING POLYHEDRA

We review some of the more recent results concerning the facial structure of set packing polyhedra. Utilizing the concept of a facetproducing graph we give a method that can be used repeatedly to construct /arbitrarily/ complex facet-producing graphs. A second method, edge-division, is used to further enlarge the known classes of facet-defining graphs. By way of the theory of anti-blocking polyhedra, these results have some implications for the structure of non-integer extreme points of linear programming relaxations of set-packing polyhedra as well.

C. van de Panne, Canada

CHOICES FOR A FIRST COURSE IN MATHEMATICAL PROGRAMMING

The paper discusses, compares and comments upon the various approaches used in teaching introductory courses in mathematical programming. Specifically, the following topics are treated.

Background and interests of students, aims of the course, linear algebra requirements, geometric representation, sensitivity and near-optimality analysis, computer usage, applications, notations in transportation and network methods.

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CRITEREA FOR THE DESIGN OF AN INTERACTIVE GRAPH MANIPULATION SYSTEM (GRAMAS)

GRAMAS is an interactive system for the assistance in solving graph theoretic problems. It has been evolved with different goals. One is to provide to the research programmer an easy way of working with different data structures of graphs and sets during the design and implementation of graph algorithms. The system includes a language for interactive messages, particularly for transforming graphs in connection with various applications.

The first version of the system as well as basic algorithms are implemented in ALGOL-W on an IBM/370-158. The first experiences with the system allow to point out some general criterea for the design of interactive graph manipulation systems, in particular with regard to implementation language, interactive language, data structures for sets and large scale network test problems.

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THE ALTERNATIVE THEOREMS AND ITS RELATION WITH THE GAME THEORY AND THE ALGEBRAIC-TOPOLOGICAL PROPIERTIES OF R^n .

The importance of the Alternative Theorems in the Nonlinear Programming (Mangasarian-1969) and the difficulty of its demonstrations based strictly in matrix methods(Tuckerś existence theorems) need new systems of demonstration which allow a more natural and d irect interpretation.

In the first part we give an interpretation based in algebraic-topological propierties of the convex sets of R^n. We demonstrate three Lemmas which constitute equivalent forms of the Gordan,Gale and Stiemke theorem respectivly,

In the second part using the von Neumann´s Minimax Theorem and some well known propierties about a bipersonal 0-sum game , we give a demonstration and an interpretation fr om the most usual Alternative Theorems, as Gordon ,Stiemke, Farkas,Gale and Mangasa - rian theorems are.

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INTRODUCTORY COURSES ON NONLINEAR PROGRAMMING THEORY

Three general topics ought to be discussed and related to one another: (I) generalized geometric programming (i.e. the material described in the author's recent SIAM Review paper), (II) ordinary programming (i.e. Kuhn-Tucker multipliers, etc.), and (III) generalized parametric programming (i.e. the material developed primarily by Rockafellar). In fact, the order of presentation should be: (1) unconstrained geometric programming (2) ordinary programming, (3) constrained geometric programming, and (4) parametric programming. The main reasons are: (1) and (3) can be most easily related to linear programming (which actually need not be a prerequisite); (1) and (3) are the most natural settings in which to motivate most students (by introducing them to quadratic programming, regression analysis, both posynomial and general algebraic programming, facility location, discrete optimal control, both single-commodity and multicommodity network flows, and various types of equilibria that occur in economics and the physical sciences); (2) should come before (3) because Kuhn-Tucker multipliers play an important role in (3); and (4) is the most convenient framework in which to subsequently develop the deeper theorems of nonlinear programming (e.g. the existence theorems).

The deeper theorems and their prerequisite convexity theory (e.g. the separation theorems) should be reserved for a more advanced course. Although complementary pivot theory should be motivated relatively early in the context of (1), it too should receive a complete development only in a later course. If the introductory course is taught properly, the terminal student will be capable of model building and conversing with experts, while the ongoing student should be strongly motivated in his further studies of programming theory and algorithms.

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THE CONICAL DUALITY AND COMPLEMENTARITY OF PRICE AND QUANTITY FOR MULTICOMMODITY SPATIAL AND TEMPORAL NETWORK ALLOCATION PROBLEMS

Abstract. Consider a graph G with each node i representing the set of "producers" and/or "consumers" at a specific spatial as well as temporal location. Each link k is directed so that it represents a specific storage and/or transportation facility for transfering certain "commodities" from a given node i to another given node j. Each commodity r is produced and/or consumed by certain nodes.

Suppose that the excess quantity of commodity r produced by node i is a variable q_{ir} (which is positive when node i produces more than it consumes); and suppose that the quantity of commodity r transfered via link k (in the direction of link k) is a variable $q^{kr} \geq 0$. Conservation of each commodity r at each node i then requires that the quantity vector q (whose components are the q_{ir} and the q^{kr}) be in the cone

$$Q = \{q \mid q^{kr} \geq 0 \text{ and } q_{ir} + \sum_{[i]} q^{kr} = \sum_{(i)} q^{kr}\},$$

where [i] denotes the set of all links k directed into node i and (i) denotes the set of all links k directed out from node i.

Suppose that the unit price of commodity r for node i is a variable p_{ir}; and suppose that the unit price of transfering commodity r via link k is a variable p^{kr}. Price stability for each commodity r then requires that the price vector p (whose components are the p_{ir} and the p^{kr}) be in the cone

$$P = \{p \mid p_{ir} + p^{kr} \geq p_{jr} \text{ for each } k \in (i) \cap [j]\}.$$

The main result given here is that P and Q are a pair of dual convex polyhedral cones, whose corresponding (conical) "complementarity conditions"

$$p \in P \qquad \text{and} \qquad q \in Q,$$

$$0 = \langle p, q \rangle$$

can be used to characterize the solution sets for various important network allocation problems. The main implications of this result are that generalized geometric programming and generalized complementarity theory, along with convex analysis, monotone mapping theory, and generalized fixed point theory, can now be exploited in a much deeper study of such problems than has previously been possible.

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A NEW ALGORITHM FOR QUADRATIC PROGRAMMING

This paper describes a new projection algorithm for minimizing a quadratic form, whose matrix is symmetric and positive definite, over a convex sets intersection. This problem is equivalent to the problem of finding the projection of the quadratic form absolute minimum on to the constrained set; this projection being calculated in sense of the inner product associated with the quadratic form.

A formalization of this problem in a n-fold cartesian product space leads to a resolution algorithm based on parallel projections on to each convex set that defines the admissible region ; these projections are computed in sense of the initial inner product and are, then, easily obtained, particularly if constraints are linear. Furthermore the consideration of the product space allows us to introduce, in a natural way, an extrapolation which accelerates the algorithm convergence.

Each step of the algorithm takes into account all the insatisfied constraints and requires the resolution of a linear system.

As a rule, and owing to extrapolation, the problem solution bidding constraints are isolated after very few steps of the algorithm and the solution can be obtained in a finite number of steps by solving a Lagrange problem of minimization with equalities constraints /i.e. linear system/. For example, in the problem given by HOUTHAKKER /4 variables, 7 constraints/ the three bidding constraints are isolated after three steps of the algorithm.

Described in R^n, our method can also be applicated for solving some variational inequalities in Hilbert space. Furthermore, the use of crossing of algorithms /crossing of "dual methods"/, leads to a linearized version of the method which can be used in case of non linear constraints. This generalization will be studied elsewhere.

Among applications, some problems of statistics are investigated.

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ОПЕРАТОРЫ РЕЛАКСАЦИИ НА НЕЛИНЕЙНОМ МНОГООБРАЗИИ

Рассмотрим многообразие активных ограничений $\Omega(z^*)=\{z : f_j(z)=0, j\in J(z^*)=\{j : f_j(z^*)=0\}=\overline{1,r}\}$ в задаче выпуклого программирования

$$z^* = \arg\min\{f_0(z) \mid f_j(z)\le 0,\ j=\overline{1,p}\}\subset E^N. \qquad (1)$$

Пусть $z=(x;y)$, $x\in E^n$; $y\in E^r$, $n+r=N$. Будем считать, что градиенты $f_j'(z^*)$, $j=\overline{1,r}$ линейно независимы (2)

Тогда*) $f_0'(z^*)+\sum u_j^* f_j'(z^*)=0$, $u_j^*\ge 0$, $j=\overline{1,r}$ и в дифференциальном преобразовании $f_z'(z)=(f_j'(z))_{j=\overline{1,r}}=(f_x'(z), f_y'(z))$ окажется $\det f_y'(z^*)\neq 0$. Поэтому в окрестности $W(z^*;\delta)=\{z : \|z-z^*\|\le\delta\}$ точки z^* система $f(z)\equiv f(x;y)=0$ определяет вектор-функцию $y(x)=(y_1(x),\dots,y_r(x))$, причем $y(x^*)=y^*$, а гладкость $y_x'(x)=-(f_y'(x;y(x)))^{-1}f_x'(x;y(x))=(-f_y'(\cdot))^{-1}f_x'(\cdot)$ определяется гладкостью $f_z'(z)$. Свойства гладкости сужения $\varphi_0(x)=f_0(x;y(x))$ функции $f_0(z)$ относительно $\Omega(z^*)$ определяются соответствующими свойствами $f_j(z)$, $j=\overline{0,r}$, причем $\varphi_0'(x)=f_{0x}'(\cdot)+f_{0y}'(\cdot)y_x'(\cdot)$, $h_0(x)=\varphi_0''(x)=f_{0xx}''(\cdot)+f_{0xy}''(\cdot)+(y_x'(\cdot))^T(f_{0yx}''(\cdot)+f_{0yy}''(\cdot)y_x'(\cdot))+\sum f_{0y_\ell}'(\cdot)y_{\ell xx}''(\cdot)=\bar H(\cdot)+\sum f_{0y_\ell}'(\cdot)y_{\ell xx}''(\cdot)$, а матрицы $y_{\ell xx}''(\cdot)$ определяются из системы $\sum f_{jy_\ell}'(\cdot)y_{\ell xx}''(\cdot)=-\bar H_j(\cdot)$, $j=\overline{1,r}$, которая в силу (2) разрешима в окрестности $W(z^*;\delta)$.

Пусть $H_j(z)$ – матрица Гессе функции $f_j(z)$, $j=\overline{0,r}$, а $m_j\ge 0$ – наименьшее собственное значение (н.с.з.) $H_j(z)$, $\mu\ge 0$ – н.с.з. матрицы Грамма $y_x'(x^*)(y_x'(x^*))^T$, m – н.с.з. $h_0(x^*)$. Характер выпуклости сужения устанавливает

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*) Здесь и в дальнейшем суммирование производится от 1 до r

Теорема I (о собственных значениях). Если $f_j(z), j=\overline{0,r}$ дважды дифференцируемы в точке z^* и выполнено (2), то

$$m \geq (m_0 + \sum u_j^* m_j)(1+\mu). \qquad (3)$$

Операторы релаксации R на $\Omega(x^*)$, т.е. операторы, для которых $\|Rz - z^*\| \leq \|z - z^*\|$, $\forall z \in W(z^*; \delta)$ получим, применяя к $\varphi_0(x)$ методы безусловной оптимизации.

Чтобы избежать явного задания $y(x)$ рассмотрим оператор $P_f : W(z^*;\delta) \to W(z^*;\delta)$ псевдопроектирования на $\Omega(x^*)$: $P_f(z) = (x; y - (f_y'(x;y))^{-1} f(x;y)) = (P_f x; P_f y) = w$. Пусть $Y(z)$, $G(z)$, $H(z)$ определяются аналогично $y_x'(x)$, $\varphi_0'(x)$ и $h_0(x)$, когда в соответствующих формулах вместо $(x; y(x))$ берется z. Рассмотрим $z \in W(z^*; \delta)$ и пусть $L > 0$ – константа Липшица для $G(z)$.

Оператор градиентной релаксации определяем по формуле:

$$Rz = (x - tG(w),\ P_f y - tY(w)G(w)),\quad 0 < t \leq mL^{-2}, \qquad (4)$$

а оператор Ньютона:

$$Rz = (x - (H(w))^{-1} G(w);\ P_f y - Y(w)(H(w))^{-1} G(w)) \qquad (5)$$

Имеет место

Теорема 2. Если выполнено (2), одна из функций $f_j(z)$, $j \in \{j : u_j^* > 0\} \cup \{0\}$ сильно выпукла и $f_j(z), j=\overline{0,r}$ достаточно гладки, то существует $M > 0$ и $q < 1$ такое, что для $\forall z \in W(z^*; \delta)$:

а) в случае оператора (4) имеет место $\|R^k z - z^*\| \leq M q^k$;

б) в случае оператора (5) имеет место $\|R^k z - z^*\| \leq M \|z - z^*\|^2$.

Отыскание $z_0 \in W(z^*; \delta)$, выделение $J(z^*)$ и включение в вычислительный процесс операторов релаксации может быть произведено в рамках сходящегося с любого начального приближения общего метода [1], в котором в начальной фазе используется какой-либо метод выпуклого программирования, а в заключительной фазе последовательность приближений порождается лишь оператором релаксации, что и определяет быстроту сходимости.

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THE AUGMENTED LAGRANGIAN METHOD FOR NONLINEAR CONSTRAINTS

Because of the difficulties of following curved constraints, a common technique for allowing constraints on the variables of an unconstrained minimization calculation is to introduce a penalty for violating the constraints into the function that is minimized. Then one applies an algorithm for unconstrained minimization to the modified objective function. The penalty terms should cause the constraints to be satisfied or nearly satisfied without making the unconstrained calculation too difficult. The augmented Lagrangian method is of this type. Its penalty term depends on parameters that are adjusted in an outer iteration to the classical Lagrange parameters of the Kuhn-Tucker conditions. For each choice of the parameters an unconstrained minimization calculation is done. The results of each minimization calculation provide suitable new estimates of the parameters. This method has been studied extensively by several researchers in the last eight years. So much progress has been made that we now have an excellent algorithm for general nonlinear programming problems of up to about one hundred variables. This work is surveyed. The algorithm is described and illustrated by simple numerical examples. It is shown how the parameters are adjusted and how inequality constraints are managed. Some elegant theory is given that does not depend on any convexity conditions. Finally the augmented Lagrangian method is compared with other algorithms for nonlinear constraints.

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FENCHEL DUALITY IN BANACH SPACES

In this paper Fenchel's duality theorem, which is valid for Euclidean spaces, is generalized to Banach spaces. One of the tools is a separation theorem involving interiors of sets relative to closed linear subspaces and being valid for Banach spaces [1].

[1] J.W. Nieuwenhuis, Separating Sets with Relative Interior in Fréchet Spaces, Internal Report OR-7606, Econometric Institute, University of Groningen.

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APPLICATION OF STOCHASTIC PROGRAMMING TO ENGINEERING DESIGN

Engineering design frequently has to be carried out under a reliability constraint or in such a way that the reliability of the system has to be maximized. The corresponding mathematical problems are stochastic programming problems where special attention has to be paid to the probability distribution involved. Earlier results of the author and others show that many sophisticated stochastic programming problems involving joint probabilities of random constraints are convex nonlinear programming problems. Knowing this, several engineering design problems were formulated and solved by a research group lead by the speaker in the last few years.

The paper presents a short survey of the underlying mathematical results and describes some numerically solved problems together with computational experience. These problems belong to the following categories: reservoir system design; reservoir system operation; design of interconnected power systems; inventory control. Further possible applications will also be mentioned.

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THE PROBLEMS OF DYNAMIC LINEAR PROGRAMMING

At present the field of applications of linear programming (LP) is well known. However, both the LP theory and the basic range of LP applications are of one-stage, static nature; i.e. in this case the problem of the best allocation of limited resources is considered at some fixed stage of a system development.

However, when the system to be optimized is developing (and not only in time, but possibly in space as well) and this development is to be planned, a decision should be made for several stages in advance, and the problem of optimization becomes a dynamic, multistage one.

One can mention the general problem of dynamic linear programming (DLP), the class of dynamic transportation and distribution problems, integer DLP, etc.

Direct application of the LP methods to problems of this kind does not usually produce the required result; the LP problems thus arrived at are so large that they cannot be solved even by using the most up-to-date digital computers. Special methods therefore are required to solve these problems which take into account the specific dynamic features of the problem and employ the methods of optimal control as well as of static LP.

In this paper the basic principles of the theory and methods of DLP are presented. The pair of dual problems is formulated and the relations between them are obtained.

Two approaches to the development of DLP computational methods are discussed: one based on modern iterative methods of static LP, the other is connected with the finite methods of large-scale LP and uses heavily the ideas of factorized representation of the inverse.

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THE GENERALIZED INVERSE IN LINEAR PROGRAMMING - A GENERALIZATION OF THE SIMPLEX ALGORITHM

The results presented in this paper continue previous research using the generalized inverse of a matrix in analyzing the mathematical structure of the linear programming problem. Given impetus by a heuristic analysis of the paths selected by the simplex algorithm; motivated by statistics collected, first by G.T. McWilliams, more recently by J.P. Poon, in numerical experiments which involved solving over 900 medium to small sized, randomly generated, linear programming problems; in this paper the mechanics of the generalization of the simplex algorithm are described in detail and preliminary results from numerical experiments applying it are summarized.

Application of the generalized algorithm involves construction of paths on k-dimensional facets of the polytope of feasible solutions. Paths are obtained using the "canonical form" of the simplex algorithm to determine the orthogonal projection of the gradient, c, of the cost function, (x,c), on a k-dimensional facet. A k-variable linear programming problem must be solved in order to determine a vertex of the k-dimensional facet yielding the greatest decrease in (x,c) on that facet. When k=1, the generalized algorithm reduces to the /revised/ simplex algorithm, and then this secondary linear programming problem is trivial, being of the following form:

> Maximize α, where $V + \alpha E \geq \theta$, $V \geq \theta$ is the current vertex, and the vector E is the orthogonal projection of the vector c on a 1-dimensional facet /i.e., an edge/ of the polytope.

When $k > 1$, the generalized algorithm may be /recursively/ employed in solving the secondary problem.

Apart from solving the secondary linear programming problem, the amount of computation required to construct a path on a k-dimensional facet is essentially the same as is required by k complete "pricing out" operations. Following a path on a k-dimensional facet results in "block pivoting": the exchange of k non-basic columns for k basic columns in the current basis. Thus application of the generalized algorithm may result in the construction of a path from a vertex to a non-adjacent vertex. This is not always the case, however; in particular, it does not occur when the current vertex is adjacent to an optimal vertex.

The simplex method, a computational scheme involving repeated applications of the simplex algorithm, may be described as a procedure for obtaining a solution to a given /m equations in n unknowns/ linear programming problem by solving a sequence of simple linear programming problems, each of which involves m inequalities in one unknown. When applying the generalized simplex algorithm to obtain the solution to an m by n linear programming problem, each problem in the analogous sequence involves, essentially, (m + k-1) inequalities in k unknowns, where k may vary from problem to problem in the sequence. The generalized simplex algorithm may be viewed as providing a rational basis for a procedure which is similar in character to "multiple pricing".

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AN INTERACTIVE CODE FOR MIXED INTEGER LINEAR PROGRAMS. APPLICATIONS TO CREW AND MANPOWER SCHEDULING PROBLEMS.

The aim of this paper is to describe a recent experience, which has been made in applying some mathematical programming methods to manpower problems. Such applications have required the design of an interactive software for mixed integer linear programs. It let us develop both interactive and batch procedures; its use is compatible both with FORTRAN codes and commercial ones.

Some applications in the area of manpower planning both in an airline and railroad company are described to discuss some software demands in the applications of mathematical programming to real problems.

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MATROIDS AND n-PORTS

The existing hybrid descriptions of an n-port are identified with subsets of a set of n elements. In certain cases this system of subsets is a matroid /or, at least, has a character, similar to that of the matroids/.

In this case various problems of network theory can be formulated and partly answered in terms of matroids /applying the intersection theorem of Edmonds and the algorithm of Lawler and others/.

Hence, sufficient condition to the unique solvability and a lower bound for the maximal number of independent state variables /degree of freedom/ can be obtained.

Most of these results are weaker than those of Iri and Tomizawa /Trans. IECEJ, Japan, 1974/ and do not refer to the "nongeneral" case like the previous results of the author /Coll. Math. Soc. János Bolyai, 10., 1973 and Proc. Second European Conference on Circuit Theory and Design, 1976/. On the other hand, this approach raises some rather unusual questions in network theory, which are open /to the best knowledge of the author/ and might give deeper insight into some physical properties of the networks.

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SOLVING GENERALIZED GEOMETRIC PROGRAMS BY PRIMAL CONDENSATION.

A computationally efficient algorithm has been developped for solving generalized geometric programs. No dual formulation is used. The algorithm is based on the known condensation principle,which permits the iterative solution of the Kuhn-Tucker conditions of the primal program by solving systems of linear equations. Inequality constraints,variable bounds and vanishing variables are easily treated through the use of slack variables.

The code has been extensively tested on a set of 24 testproblems,including posynomial and signomial programs. The performance of the proposed condensation method was subsequently compared to this obtained by 16 other codes. (A collection of primal and dual based algorithms.) The main trends of this comparison will also be indicated.

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GREEDY PRICES: A SADDLE POINT CONDITION CHARACTERIZING OPTIMALITY IN GENERAL NONLINEAR PROGRAMMING

Consider a nonlinear (possibly nonconvex, nondifferentiable) optimization problem (P_u) of the form: minimize $f_0(x)$ subject to $x \in X$ and $f_i(x) \le u_i$ for $i = 1,\dots,m$, where each f_i is a real-valued function on X (some abstract space), and $u = (u_1,\dots,u_m)$ is a parameter vector in R^m. The usual theory of Lagrange multipliers seeks to characterize a locally optimal solution $\bar{x}$ to (P_u) in terms of a local saddle point $(\bar{x},\bar{y})$ of the Lagrangian function

$$L_u(x,y) = f_0(x) + \sum_{i=1}^{m} y_i[f_i(x) - u_i], \quad x \in X, y \in R^m_+ .$$

This is generally not possible without certain potentially troublesome "regularity" assumptions. But if it is, the components $\bar{y}_i$ of $\bar{y}$ can be interpreted as "equilibrium prices" associated with the "resource availabilities" u_i: the decision $\bar{x}$ remains optimal "locally" if the possibility is introduced of altering the given u by buying or selling the resources at these prices.

A broader concept of equilibrium, really more reasonable from an economic point of view, would consider distinct buying and selling prices $\bar{y}_i^+$ and $\bar{y}_i^-$, bracketing $\bar{y}_i$ but differing by an arbitrary amount. It will be shown that, in this way, one can obtain a new characterization of optimal solutions $\bar{x}$ which is always sufficient and generically necessary.

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A CASE STUDY APPROACH TO TEACHING MATHEMATICAL PROGRAMMING

A hierarchical set of five Case Studies is described, ranging from a very small introductory LP example to much larger mixed integer models. The emphasis in the paper is on experience using the Case Studies to teach various concepts of mathematical programming to graduate business students.

The introductory example concerns manufacture of one product under a capacity constraint to satisfy limited local demand and unlimited export potential. The problem is introduced, and the economics of various changes in conditions discussed, before any LP concepts are presented. Formulation as an LP model follows, and the computer solution presents a known answer and marginal economic information which has already been calculated from first principles. This generates confidence in the technique and in the computer printouts.

Later models deal with:

- optimal use of facilities in a National Park,
- changes in marginal economics associated with background changes for a multi-product system,
- facilities timing for the development of a series of shopping centres,
- production scheduling for several products with seasonal demands

All are presented as physical problems, formulated as LP or MIP models,

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A TWO-PHASE METHOD FOR NONLINEAR CONSTRAINT PROBLEMS

A method for solving nonlinear constraint programming problems based on a Newton type linearization scheme has been proposed [1] and shown to converge quadratically [2] provided the starting point is close enough to the optimum. For a general starting point this method may diverge or converge very slowly until a point close to the optimum is reached.

An excellent starting point for the above method (Phase II) is one which lies just outside the set of nonlinear constraints active at the optimum. Such a point can be obtained by a single unconstrained, minimization (Phase I) with an external squared penalty function. Both the proper choice of the penalty term coefficient and a suitable termination criterion for Phase I are important to achieve computational efficiency. These are investigated in terms of both a simple test problem (which nevertheless contains the essence of more realistic larger problems) and several larger problems.

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solved using a standard computer package and the computer printout used as the starting point for analysis of decision alternatives.

The objective is to present mathematical programming as a problem solving technique. Its broad applicability is illustrated by the range of Case Study areas covered, and further work is aimed at developing Case Studies in different fields. Eliminating computational difficulties by the use of a computer package means that models of realistic size can be considered, and that emphasis can be placed on data problems, formulation and the use of marginal economic information to evaluate relevant decision situations.

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LINEAR PROGRAMMING WITH NONLINEAR PARAMETRIZATION

In linear parametric programming the coefficients in the constraints and in the objective function depend linearly on the parameters. Here the analogous problem where the coefficients are nonlinear functions of the parameters is studied, especially for the case with only the right hand side or the objective function coefficients being nonconstant. Theorems on the properties of the set of optimal solutions and of the optimal values of the objective function are proved; these properties (continuity, function space, convexity and concavity) are related with corresponding properties of the coefficient functions. The problem of how to compute the solutions algorithmically is discussed, and the possibilities for application and a numerical example are given for illustration.

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VOOD PROCUREMENT AS A COOPERATIVE GAME

The rapidly expanding field of game theory seems to be hindered more by the lack of applications than by the inherent mathematical difficulties of the subject. The field has suffered because of the small amount of close contact between game theorists and application-oriented OR-people.

We have studied a special problem, which leads to n-person game theory with coalition structures. Our aim was to develop a method for a fair division of the profit from the cooperation and optimization of the activities of several independent companies. We developed the problem into an n-coalition game in the characteristic function form, the game having a coalition structure of two levels.

In the paper we review the best known methods and solutions of an n-person cooperative game such as the Core, Kernel and Shapley- value. We point out the necessity of developing a tailored model in our case.

We raise the question of the strength of a coalition, which in our case seems to play a major role when defining the fair division of profit. We will show that the set of feasible divisions will be heavily reduced when a coalition is transformed from a very strong to a very loose one. Difficulties in measuring the strength will be discussed.

We will also deal with the main results of the numerical part of the study, where we used data from real wood procurement process over a period of several years. Both the fighting and cooperating strategies of the companies will be discussed.

The division based on the solution of the game will be compared to the earlier heuristic division based on several complicated rules.

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CONNEXITE DANS LES MATRICES $\{-1,0,+1\}$

1. On étudie ici les matrices à éléments dan $\{-1,0,+1\}$ en utilisant le langage de la théorie des hypergraphes à partir d'une notion de connexité dont on montre qu'il s'agit d'une "bonne" généralisation de la notion de connexité dans le cas des graphes.

2. Une matrice A à éléments dans $\{-1,0,+1\}$ est considérée comme la matrice d'incidence aux arcs d'un hypergraphe "général" $H = (X,\mathcal{A})$ dont l'ensemble $X = \{x_1, x_2, \dots, x_n\}$ des sommets /resp. l'ensemble des arcs/ est en bijection avec l'ensemble des lignes /resp. des colonnes/ de A. Un arc de H est dit "pair" si cet arc est incident à un nombre pair de sommets /i.e. le nombre d'éléments non nuls de la colonne correspondante de la matrice A est pair/.

3. Les notions de chaine, cycle, cocycle, connexité, arbre définis dans le cas des graphes sont généralisées ici:

   - Un ensemble d'arcs pairs de H $\mathcal{A}' \subset \mathcal{A}$ est appelé "pseudo chaine joignant les sommets x_i et x_j" si chaque ligne de la matrice d'incidence de l'hypergraphe partiel construit sur $\mathcal{A}'$ /i.e. de la sous matrice de A obtenue en conservant les colonnes correspondante a $\mathcal{A}'$ et toutes les lignes/ a un nombre pair d'éléments non nuls sauf les lignes correspondantes à x_i et à x_j qui ont un nombre impair d'elements non nuls.

   - Un ensemble d'arcs pairs $\mathcal{A}' \subset \mathcal{A}$ est appelé "pseudo cycle" si chaque ligne de la matrice d'incidence de l'hypergraphe partiel construit sur $\mathcal{A}'$ a un nombre pair d'éléments non nuls.

   - La relation de connexité sur les sommets de l'hypergraphe H se déduit de celle de pseudo chaine:

$$x_i \, C \, x_j \iff \text{il existe une pseudo chaine joignant } x_i \text{ et } x_j \text{ ou bien } x_i = x_j .$$

4. On montre que pour un hypergraphe $H = (X,A)$ sur n sommets les propriétés suivantes sont equivalentes et caractérisent ce qu'on appelle un "h-arbre":

   /i/ H est connexe /i.e. la relation $x_i \, C \, x_j$ est satisfaite pour tout couple de sommets/ et minimal pour cette propriété.

   /ii/ H est pair /tous les arcs sont pairs/ et posséde n-1 arcs.

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CONVERGENCE QUESTIONS IN STOCHASTIC OPTIMIZATION

A promissing approach to solving stochastic optimization problems is to replace the original problem by an "approximate" problem. This usually involve discretezing the random elements or replacing the original distribution by one yielding a simpler problem. This requires the development of a theory of convergence with, whenever possible, the calculation of error bounds. We approach these problems by means of the theory of convergence of stochastic multifunctions.

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A NONLINEAR DECOMPOSITION ALGORITHM

Nonlinear programming problems quickly become unmanageable as the number of variables grows, and decomposition offers a solution to this dilemma. The problem is then broken down into a master problem and one or several subproblems, which are solved iteratively yielding solutions nearer and nearer to the optimum. Kronsjö[1] and Geoffrion[2] use a Lagrangean approach to solve the master problem, whereas the present algorithm uses a direct method, utilizing subgradient minimization, which is presently being developed by various authors. This paper reviews the Lagrangean master problem approach before the new algorithm is presented. A subdifferentiable minimization algorithm by Poljak[3] is then reviewed and modified. With the modified Poljak method the decomposition algorithm is shown to be convergent. Some suggestions for improving the numerical efficiency of the method conclude the paper.

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AN EFFICIENT IMPLEMENTATION OF THE LEMKE ALGORITHM AND ITS EXTENSION TO DEAL WITH UPPER AND LOWER BOUNDS

The Lemke algorithm is probably the most elegant method of solving the linear complementarity problem, but the classical tableau form of solution is computationally rather inefficient.

The general form of the algorithm adds an additional variable and column to the original problem, to give the form:

$$w=q+ez_o+Mz$$
$$w\geq 0,\ z\geq 0,\ w^Tz=o, z_o=o$$

The complementary pivoting procedure operates on columns of the matrix $[e,M]$ and in general transforms the whole matrix. Any symmetry in M is lost in the process, in contrast to the "principal pivoting" method of Cottle and Dantzig.

In fact the Lemke algorithm can be reorganized to use only principal pivots, thus preserving any symmetry in M. Economies can also be made in the amount of computation for each pivot, rather in the manner of the "revised simplex" method for linear programming, and it is only necessary to update a submatrix corresponding in size to the number of active constraints (non-zero z-variables). Rather than generating an inverse matrix, a stable form of triangular decomposition can be used, with all the usual advantages of numerical stability and conservation of sparseness.

The test for a "blocking variable" can also be easily generalized to deal with upper and lower bounds, so that the algorithm directly solves the more general problem:

$$\left.\begin{array}{c} w = q + Mz \\ a_i \leq w_i \leq b_i \\ z_i(a_i - w_i) \geq 0,\ z_i(b_i - w_i) \geq 0 \end{array}\right\} \text{all } i$$

This makes a further saving in computation and storage for such problems, and provides a convenient, unified form for dealing with all types of inequality and equality ($a_i = b_i$) constraints.

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DISCRETIZATION AND NUMERICAL SOLUTION OF A TIME-OPTIMAL PARABOLIC BOUNDARY-VALUE CONTROL PROBLEM

We try to solve numerically a time-optimal control problem resulting from a special one-dimensional heat-diffusion process. A thin rod shall be heated at one end-point such that a given temperature distribution $k_0 \in C(0,1)$ will be approximated in the L_∞-norm with a given accuracy ε as soon as possible. The heating process leads for every control $u \in L_\infty[0,T]$ and every control time T to a parabolic boundary-value problem, whose solution, we call it y(s,T,u), is well-known from Yegorov. So we get the problem

(SP)

$$\min_{T,u:} \; T$$
$$\|y(.,T,u) - k_0(.)\|_\infty \leq \varepsilon$$
$$\|u\|_\infty \leq 1$$
$$T > 0, \; u \in L_\infty[0,T]$$

This problem is solvable and Lempio has shown that the optimal control is bang-bang and that the jumps accumulate at most in T_0, the minimal control time. Now we consider only such feasible solutions of (SP) which possess the above bang-bang character. This leads to an equivalent problem, call it (P), whose variables are the jumps of bang-bang functions.

In order to solve (P) numerically we discretize it in the following way:

1) Consider only controls with at most k jumps, $k \in \mathbb{N}$.
2) Displace the infinite series determining y(s,T,u) by an finite series of length l_n, $l_n \to \infty$ for $n \to \infty$.
3) Approximate the L_∞-norm by computing the maximum of n function values of equidistant points in [0,1], $n \in \mathbb{N}$.

The minimal control time T_o^{kn} of this discrete problem $(P_k{}^n)$ converges against the minimal control time T_o of (P), more precisely we can show that

$$\lim_{k\to\infty} \underline{\lim}_{n\to\infty} T_o^{kn} = \lim_{k\to\infty} \overline{\lim}_{n\to\infty} T_o^{kn} = T_o$$

Further we develop an algorithm to solve $(P_k{}^n)$ numerically. This is a bisection method in the following sense: We decide by solving a special optimization problem in R^k, if for a given T a feasible control exists with respect to $(P_k{}^n)$. If yes, let T decrease, if no, let T increase.

Finally we discuss some numerical results.

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STEEPEST ASCENT DECOMPOSITION METHODS FOR MATHEMATICAL PROGRAMMING / ECONOMIC EQUILIBRIUM MODELS

A number of models have been proposed for combining econometric submodels which forecast the supply and demand for economic commodities with a linear programming submodel which optimizes the processing and transportation of these commodities. Included are models for energy planning, analyzing the world wheat market, and river basin development. We show how convex analysis can be used to decompose these planning models into their econometric and linear programming components. Steepest ascent methods are given for optimizing the decomposition, or equivalently, for computing economic equilibria for the planning models. Extensions to the dynamic core are also discussed.

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MATHEMATICAL PROGRAMMING MODELS AS INTEGRATORS: INTERDISCIPLINARY COMPUTER SYSTEMS DESIGN AND USE

Mathematical programming models often serve as integrators in interdisciplinary operations research studies. Student participation in the computer implementation of models of this type gives them a perspective from which to learn how models are created from live data in an institutional setting, how mathematical programming analyses should be performed on real problems, and how interesting and relevant mathematical research can be derived from applied problems. Illustrations from water resources planning, energy planning, machine replacement and environmental analysis will be given.

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OPTIMIERUNGSAUFGABEN IM NETZPLANMODELL

1. Terminproblem

Zunächst wird ein gegenüber der METRA-Potentialmethode erweitertes Modell der Netzplantechnik angegeben. Durch die Einführung variabler Vorgangsdauern wird die Aufgabe geringfügig komplizierter. Es werden aber Fälle lösbar, die mit der METRA-Potentialmethode nicht erfasst sind. Auch bildet das Modell günstige Bedingungen für die später zu behandelnde Ressourcenbilanzierung.

Das Terminproblem wird als lineare Optimierungsaufgabe formuliert, in welche die Vorgangsdauern D_i (MIN $D_i \leq D_i \leq$ MAX D_i) mit einbezogen werden. Für diese erweiterte Aufgabe hat Nägler einen Lösungsalgorithmus entwickelt. Auf Grund der Modellstruktur bietet sich dabei eine Verallgemeinerung des Kelleyschen Verfahrens an. Die Dualisierung der Optimierungsaufgabe führt in ihrer graphentheoretischen Realisierung zu einem Maximalstromproblem, für dessen Lösung ein Algorithmus aufgestellt wurde. Es ist ferner möglich, für jeden Vorgang eine konvexe (stückweise lineare) Dauer-Kosten-Kurve vorzugeben. Das zugehörige Programm liefert ein optimales Termin-Dauer-System, welches eine wesentliche Grundlage für die Optimierung des Ressourceneinsatzes bei komplizierter Netzstruktur bildet.

2. Ressourcenbilanzierung

Wir beziehen nun eine Menge nichtspeicherfähiger Ressourcen in die Betrachtungen ein. Das mathematische Modell geht von Kostenbewertungen für die Anzahl der zum Einsatz kommenden Ressourceneinheiten, für deren Leistungen (einschliesslich Warte- und Umsetzzeiten) sowie für Terminüberschreitungen aus.

Es ergibt sich eine nichtkonvexe bilineare Optimierungsaufgabe. Für die Lösung wird ein Verfahren vorgeschlagen, wonach eine zulässige Ausgangslösung aus dem Terminalgorithmus gewonnen und danach schrittweise verbessert werden kann.

In vier praktisch bedeutsamen Spezialfällen der Bilanz- bzw. Reihenfolgeoptimierung wurden auf der Grundlage von unabhängigen Lösungsalgorithmen Rechenprogramme aufgestellt, über deren Anwendungen bei der Ablaufplanung berichtet wird.

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OPTIMIZING THE UTILITY OF MULTI-COMMODITY NETWORK FLOWS

The paper deals with optimization of the utility of Multi-Commodity Network Flows. Utility of a commodity is a non-linear (piece-wise linear) function of its flow. In general, a commodity uses a fixed path starting from a source terminal to reach its destination. The flow of the commodities is restricted by the available capacity of the edges. A unit flow of any commodity requires integer units (≥ 1) of the edge capacities on its path. The flows of different commodities in an edge are additive though the direction may be opposite. Further, the capacity of an edge may be increased within total cost constraint by expending a known cost. The cost functions are non-linear (assumed piece-wise linear).

The problem described is very general in nature. Many network problems discussed in the recent literature form its special cases. The complexity of this general problem stems from two factors: firstly, the objective is to optimize the sum of non-linear utility functions of individual flows and secondly, the cost functions are non-linear.

An algorithm designed to solve a large real life problem is developed and under one assumption, gives the global maximum of the non-linear objective function. The problem is first transformed into a restricted basis problem having the structure of a linear programming problem. For such problems, the optimum is readily obtained. Finally, the application of the algorithm to special cases is discussed.

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THE GAME OF PURSUIT IN AN BANACH SPACE

The paper is devoted to the study of differential games in Banach space. The dynamics of the game is described by an equation of the form

$$(1) \quad x = A(t)x + u - v \quad x(t_p) = X_o$$

where $x \in X$ is a separable Banach space $A(t)$ is the linear generator of a semigroup $S(t,\tau)$, $u(t)$ denotes a measurable selector of a closed, bounded, convex subset U of a reflexive Banach space Z, $v(t) \in V$ is defined in a similar way.
If we replace u-v in (1) by an arbitrary function $k(u,v)$ then using mixed controls one can get the some result.
First we consider games with fixed time duration. The pay-off φ has the form of a continuous functional on $C[(t_p,t_k),X]$ where $[t_p,t_k]$ is a fixed interval and $C[(t_p,t_k),X]$ is a Banach space of all continuous functions on $[t_p,t_k]$ valued in X.
In the literature on differential games there is a lot of definitions of "strategy" that in turn imply a quantity of the corresponding notions of "value". A concept of strategy discussed here is those of Elliot Kalton [1] .
We also indicate its relationship to that of Friedman [2]
We are able then to prove.
Theorem
For the game described by equation (1) with the pay-off φ Lipschitz continuous on $C[(t_p,t_k),X]$ there exists a value $V(X_o,t_p,t_k)$ of the game which is Lipschitzian with respect to all the arguments.

Since $V(X_o,t_p,t_k)$ is Lipschitzian it follows from [3] that $V(X_o,t_p,t_k)$ is differentiable almost everyhere. From this we get a differential equation of Issacs-Bellman type.
In the second part of the paper we deal with pursuit - evasion games and we get.

Theorem

The game described by the equation (1) has a generalized value.
Finally we apply our results to games described by linear differential-difference equation of neutral type.
For a game of fixed duration we are able to prove the existence of optimal strategies for both players. Finaly we consider a game of pursuit described by this functional equation with a target set in the space $C[-h,0]$ where h is the maximal delay time occuring.
The conluding results constitute a generalization of the work [4] of Varaiya concerned with the finite - dimensional case.

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BOTTLENECK ASSIGNMENT PROBLEM: AN EFFICIENT ALGORITHM

An algorithm for the bottleneck assignment problem is presented, the theoretical efficiency of which is superior in comparison with the known algorithms. The problem under consideration is as follows. In a given square matrix $C = (c_{ij})_{n \times n}$ of integers we should select a sample of n elements / called a feasible solution /, one in each row and one in each column, in such a way that maximum valued element in the sample be the smallest among all feasible samples. Thus, our purpose is to find $c_0 = \min_{p \in \Pi} \{\max_{i \in \langle 1,n \rangle} c_{ip(i)}\}$, where p is a permutation of integers 1,...,n, Π is the set of all permutations. The problem was posed by Gross /Bottleneck assignment problem. RAND, P-1630, 1958/ and primal algorithm for its solution was proposed.

Our algorithm is of the threshold type. As a threshold value B for criterion serve elements of matrix C. The lower and upper bounds for the threshold may be introduced: $B_{min} \leq B = c_{ij} \leq B_{max}$. These bounds are obtained in the following way: $B_{min} = \max\{a_1,\dots,a_n,a^1,\dots,a^n\}$ and $B_{max} = \min\{b_1,\dots,b_n,b^1,\dots,b^n\}$, where a_i and b_i are respectively minimum and maximum valued elements of ith row, whereas a^j and b^j are respectively minimum and maximum elements of jth column. With the threshold being chosen, the basic step of the algorithm is performed. The matrix C^B in which: $c^B_{ij} = 1$ for $c_{ij} \leq B$ and $c^B_{ij} = 0$ for $c_{ij} > B$ is used. Now, we search for a a feasible solution in this matrix. The feasible solution exists iff the matrix C^B may be brought to the form in which there are no zeros on leading diagonal. The modified Hungarian method is applied. Selecting a column with zero in leading diagonal place, we try to permute columns in such a way that figure one appears in this place and at the same time no other column with this property loses it. When such permutation is found, the procedure is repeated with the next column and so on. It can be shown that computations of complexity $O(n^2)$ are needed to get matrix C^B in desired form or to establish that no feasible solution exists. In the latter case the new threshold is introduced and the permutation procedure starts again.

Some other formulations of the necessary and sufficient condition of feasibility as well as different methods of checking these conditions are discussed too. One of the method is based on introducing in each step some auxilliary problem,namely the maximum matching problem. We should find a maximum matching in a bipartite graph $G(V,E)$. The vertex set V consists of rows (V_1) and columns (V_2) of matrix C^B . The edge $(v_1,v_2)\in E$ iff there is one on the intersection of row v_1 and column v_2 in C^B. The matrix is feasible when an optimal solution to the matching problem is n. For the matching problem the efficient algorithm of Hopcroft and Karp is known / An $n^{5/2}$ algorithm for maximum matching in bipartite graphs. SIAM J.Comp.,No.4,1973 /.

Other examples of the auxilliary problems are: the maximum flow problem in adequatly defined network and the classical assignment problem / known algorithms for both problems are of the complexity $O(n^3)$/.

The number of the basic steps to be performed in our algorithm depends on the method of changing the threshold value. One way to do it is the progressive increase of B,starting with lower bound B_{min} . First feasible solution is optimal. The algorithm in this case will be of complexity $O(n^4)$. Using another method for changing of the threshold / alternating method / which may prove useful for large problems, there will be no more than 2lgn basic steps in algorithm.

The complexity of the algorithm in this case is $O(2n^2 \lg n)$.

R. Słowiński, J. Węglarz, Technical University of Poznań, Poznań, Poland, AN ALGORITHM FOR SOLVING A CERTAIN NONLINEAR SCHEDULING PROBLEM

In this paper, the following problem is considered. A set of n heterogeneous machines process in parallel an amount of raw material. Let $c_i(v_i)$ be the known cost of processing v_i units of raw material on machine $i=1,2,\ldots,n$. The amount of raw material processed at any moment on machine i lies within bounds: $v_i \in \langle a_i, b_i \rangle$, where $0 \leq a_i \leq b_i$. The objective is to find the number of periods of work for every machine and amounts of raw material processed in these periods which minimize the cost of processing the total amount of raw material, that is not less than certain required amount.

The above problem forms an atypical Single Product Production Problem. The point of the matter of this atypicallity is that the requirement for the product only refers to production as a whole; the inventory holding cost is not considered and furthermore we consider n facilities with different cost functions, producing the same product in parallel. Finally, the number of periods for particular facility is here a variable of the optimization problem.

Of course, the considered problem also has an economic aspect to it, but we would rather stress its applications in technological processes like grinding or mixing which occur in the building materials, metallurgical or glass industries.

In this paper, we develop and discuss an algorithm for solving optimally the above problem, for $c_i(v_i)$ nondecreasing convex or concave, $i=1,2,\ldots,n$. In the algorithm some special properties of optimal solutions are used, and due to this, it may be useful in practice.

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COMBINATORIAL PROPERTIES OF BIMATRIX GAMES

In this paper we reveal some combinatorial properties of the bimatrix games; these properties are obtained by studying a certain associated graph.

More precisely, if a bimatrix game is given by the payoff matrices A,B, each of them with m rows and n columns, one attaches to this game a graph with mn vertices (i,j); (i=1,...,m; j=1,...,n)-two vertices (i,j),(k,l) being joined

-either by a directed edge ((i,j),(k,l)) when i=k and $b_{ij} > b_{kl}$, or j=l and $a_{ij} > a_{kl}$,

-or by an undirected edge [(ij),(k,l)] when i=k and $b_{ij}=b_{kl}$, or j=l and $a_{ij}=a_{kl}$.

One studies the properties of the associated graph for a bimatrix game and one examines the characteristic properties in the case when it corresponds to a matrix game.

One uses some of these results in order to establish a method for finding an equilibrium point in a bimatrix game with one player having two pure strategies. Some other results for any bimatrix game are given.

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IMPLEMENTATION OF THE HEURISTIC MIXED INTEGER PROGRAM TO THE MATHEMATICAL PROGRAMMING SYSTEM

We have recently developed a general purpose mathematical programming system for a domestic large scale computer. This system has four functions as its sub-systems, which are the linear programming system, the data management system, the mixed integer programming system and the special structured large scale linear programming system. What we are to report here is about the mixed integer programming system, especially the heuristic method that is implemented in it.

Our basic principles in designing the mixed integer programming system were as follows.

(i) consideration of elementary usage;
Users with relatively poor experience in MIP can use this system easily and can get reasonable computational efficiency.

(ii) consideration of sophisticated usage;
Users can select proper strategies taking advantage of the structure of their problems.

(iii) stability of computation time;
Users can obtain reasonable solutions that correspond to the computational cost, even if their available computation time is limited.

Considering these points we adopted two methods in our system, one is the branch-and-bound method, the other the R-optimal method.

The branch-and-bound method ([1],[2],[5],[7],[9]) is adopted in not a few mathematical programming systems such as MPSX,OPHELIE,UMPIRE etc. One of the most important characteristics of this method is that one can obtain a globally optimal solution. But it seems to have some disadvantages such that in many cases computation time varies considerably according to the computational strategy one adopted and that computation time increases exponentially depending on the number of integer variables.

In the R-optimal method, given a feasible integer solution, one tries to improve the solution by means of varying R integer variables at a time and repeat such trials so long as the solution is improving. This kind of method was applied to the pure integer programming problem by Hillier [4], to the travelling salesman problem by Lin [6] and the plant location problem by Sā [8]. One defect of this method is that solutions obtained by this method are generally locally optimal. But its advantage is that computation time is proportional to l^R (where l is the number of integer variables) and is stable irrelevant to the structure of problems. Compared with the branch-and bound method, it seems to work with remarkably high efficiency for problems which do not have strong structural relations among integer variables. In our program we set R=1 and 2, but even by such a simple algorithm, numerical experiments showed fairly good results.

In our mixed integer programming system, we adopted these two methods to make up for each other's defects and by combining them, high computational efficiency can be expected.

Finally, computational results comparing our R-optimal method and the branch-and-bound method of MPSX are given in Table 1 and 2. The program of Table 1 is coded in FORTRAN and it is processed totally in core. The program of Table 2 is coded in the assembler language and it uses external memory devices. These two programs were made for the test of the R-optimal method.

After these experiments, we were convinced that the R-optimal method is effective for mixed integer programming problems of practical scales, so we proceeded to implement this method in our general purpose mathematical programming system. Table 3 shows the result of the tests of our system. BBMIX is the procedure that applies the branch-and-bound method and HEUMIX is the procedure that applies the R-optimal method.

problem		m×n	l_{0-1}	Computational time(CPU minutes)			functional value
				Cont. LP	Search	Total	
I	MPSX	111×507	7	0.18	0.42	0.60	850430.5
	R-Opt.			0.26	0.06	0.32	850430.1
II	MPSX	165×426	12	0.11	0.97	1.08	1001287.0
	R-Opt.			0.20	0.60	0.80	1001286.8
III	MPSX	126×179	25	0.04	0.12	0.16	885.150
	R-Opt.			0.02	0.03	0.05	885.149
IV	MPSX	131×184	20	0.04	2.10	2.14	736.370
	R-Opt.			0.22	0.10	0.32	763.549
V	MPSX	18×78	36	0.02	0.21	0.23	1702.696
	R-Opt.			0.01	0.14	0.15	1715.589
VI	MPSX	67×114	28	0.02	0.05	0.07	224.126
	R-Opt.			0.01	0.10	0.11	224.125

Table 1

machine: IBM 360/75J
problem: minimize
m = no. of constraints
n = no. of variables
l_{0-1} = no. of 0-1 integer variables
Cont. LP: time to solve continuous LP

Prob.		m×n (density)	1	10-1	Comp. time (CPU seconds) Cibt.LP	Best int.	total	Is Optimality proved?	Functional value
VII	MPSX	72×1092		20	2.4	291.0	595.0	no(time cut)	3244.0
	R-Opt.	(3.97%)			2.4	53.4	95.4	yes	3329.0
VIII	MPSX	92×1512		20	3.0	30.6	292.8	no(time cut)	3149.0
	R-Opt.	(3.11%)			3.0	58.2	66.3	no(Comp.err.)	3052.0
IX	MPSX	72×1092	20		1.8	45.6	138.0	yes	2868.0
	R-Opt.	(3.97%)			1.8	34.8	64.7	yes	2868.0
X	MPSX	92×1512	20		2.4	77.4	294.0	no(time cut)	2682.0
	R-Opt.	(3.11%)			2.4	69.0	110.4	yes	2666.0
XI	MPSX	197×478		85	4.2	32.4	294.0	no(time cut)	458947.0
	R-Opt.	(1.43%)			4.2	32.4	105.0	yes	458947.5

Prob.		m×n (density)	1	min or max	Comp. time (CPU seconds) Cont.LP	Best int.	Total	Is optimality proved?	Functional value
VII	BBMIX	72×1082	20	min	0.62	5.53	30.0	no	3244
	HEUMIX	(3.97%)			0.62	9.96	20.71	yes	3285
IX	BBMIX	72×1092	20	min	0.47	7.27	11.02	yes	2868
	HEUMIX	(3.87%)			0.47	7.56	15.02	yes	2868
XII	BBMIX	561×376	112	max	3.80	51.33	59.70	no	2236
	HEUMIX	(0.40%)			0.0	10.15	12.14	yes	2463

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NEW TOOLS FOR 0-1 KNAPSACK PROBLEM - O(n) REDUCTION ALGORITHM AND DOMINANCE CONSTRAINTS.

The paper deals with 0-1 knapsack problem (KP)

$$\max (c_1x_1+\ldots+c_nx_n)$$

subject to $a_1x_1+\ldots+a_nx_n \leqslant b,$

$x_i=0$ or 1, $i=1,\ldots,n.$

The reduction of KP consists in assigning values to as many as possible variables before applying the main algorithm. This approach was introduced by Ingargiola and Korsh [3] using standard continues relaxation of KP with one additional constraint of the form $x_j=0$ ($x_j=1$) in attempt to fathom corresponding subproblem. The best possible estimation of number of operations as a function of n is $O(n^2)$ for their method. Recently Dembo and Hammer have proposed O(n) reduction scheme applying weaker fathoming test. In the first part of this paper O(n) reduction algorithm is described with fathoming criterion equivalent to the original one of [3]. Simple modification of this algorithm can be used for reduction of general (non-binary) KP. The possibility of use of the algorithm within enumeration method is also discussed introducing subset of so-called test variables with the property that some test variable must be reducible, if any free variable is reducible.

In the second part of the paper a partial order (so-called dominance relation) is introduced in the set of variables. Assumming $c_1/a_1 \geqslant \ldots \geqslant c_n/a_n$, we say that x_i dominates x_j iff $i \leqslant j$, $a_i \leqslant a_j$ and $c_i \geqslant c_j$. Examining in enumeration process subproblem with constraint $x_i=1$ ($x_i=0$) we can also put

$x_j=1$ $(x_j=0)$ for all x_j dominating (dominated by) x_i. This technique usually reduces the size of the search tree and gives bounds tighter than obtained by pure linear programming. Possible generalization of this approach is also discussed, using dynamic programming. Finally, two-phase reduction algorithm is proposed adapting dominance constraints in the second phase.

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ON THE MULTIPLE MINIMUM RISK PROBLEM

This paper presents a method to solve the multiple minimum risk problem of stochastic programming:

$$Ax \leqslant b$$

$$x \geqslant 0$$

$$\max P\{\omega \mid c'_1(\omega)x \geqslant u_1\} = Z_1,$$

$$\max P\{\omega \mid c'_2(\omega)x \geqslant u_2\} = Z_2,$$

$$\cdots\cdots\cdots\cdots$$

$$\max P\{\omega \mid c'_r(\omega)x \geqslant u_r\} = Z_r$$

where A is a m x n matrix, x is a n- dimensional vector, b is a m - dimensional vector, $c_k(\omega) = (c_{kl}(\omega))$, $1 \leqslant k \leqslant r$, $1 \leqslant l \leqslant n$ are vectors whose elements are real - valued random variables defined on a probability space $\{\Omega, K, P\}$, $u_1, \dots u_r$ are scalar numbers.

We assume that $c_k (k=1,\dots,r)$ has a nonsingular multinormal joint distribution.
The concept of the multiple minimum risk solution is introduced.
The results are part of a doctoral thesis.

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ON STOCHASTIC ZERO DEGREE GEOMETRIC PROGRAMMING

A methodology has been devised for zero degree of difficulty geometric programs in which constraint and objective coefficients are random variables. A limiting distribution, under rather general conditions, is derived for this case.

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ALGEBRAIC PROPERTIES OF THE MATROIDS AND THEIR APPLICATIONS TO SCHEDULING

Considering the connection between the matroid theory and the theory of binary relations, we give a new definition of a matroid on an infinite set, and we prove the equivalence with the other definitions /R.Rado, L.Mirsky, etc/ Many properties of the finite matroids can be translated for the infinite matroids, the proofs being simplified. This language allows us to formulate the scheduling problem and suggests us a manner for solving it in some particular cases.

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LEAST-SQUARES WITHOUT NORMAL EQUATIONS

The problem of fitting a noon-linear function to data is often approached using a "least-squares" criterion. However, the method whereby normal equations are formed has often been shown to be unstable.

It will be shown that it is unnecessarily ill-conditioned, and that the required solution may be obtained more accurately by using direct transformation of the linearized observational equations.

J.J.Strodiot - N.V.Hien, University of Namur, Namur, Belgium

AN EXPONENTIAL PENALTY METHOD FOR MINIMAX PROBLEM WITH CONSTRAINTS

In this paper an exponential penalty method is applied to a nondifferentiable minimax problem with constraints given by inequalities. The strategy is to transform the constrained problem into a sequence of unconstrained problems which are easier to solve. The penalty functions have two parameters and are constructed so that all convergence subsequences of solutions to the easier problems converge to a solution of the constrained problem. It is a generalisation to our problem of Murphy's penalty method for nonlinear programming. As in nonlinear programming the exponential penalty method has the advantages of the exterior penalty method i.e. the starting point may be infeasible.

The main result of this paper is that our method has also the advantages of the interior penalty method : the trial solutions become feasible after a finite number of iterations. The proof reduces the unconstrained problem into a convex programming problem and uses extensively the concepts of convex analysis i.e. subgradients of convex functions and directional derivatives.

Since the trial solutions belong to the feasible domain after a finite number of iterations we need only establish the rate of convergence for feasible trial solutions. We show that if the problem has an unique constrained saddle point and if uniformly convexity and concavity assumptions are satisfied, then the rate of convergence depends of the choice of the parameter in the penalty function. This proposition allows us to deduce a good choice of these parameters. All the proofs in this paper are given without differentiability assumptions.

D. Stumpe, TH Otto von Guericke Magdeburg, Magdeburg
Boleslav-Bierut-Platz

EINIGE BEMERKUNGEN ZUR LÖSUNG VON KONKAV-KONVEXEN SPIELEN MIT HILFE DER OPTIMIERUNGSTHEORIE

Betrachtet man die entwickelten Lösungsmethoden in der Spieltheorie, so stellt man fest, daß sie mehr oder weniger mit der Optimierungstheorie verbunden sind. Die in [2] durchgeführten Untersuchungen führen zu der Vermutung, daß der aussichtsreichste Weg, für weitere Spielkomplexe Lösungsverfahren zu finden, ebenfalls über die Optimierungstheorie führen wird.

Wir betrachten ein antagonistisches Spiel

$$G = \langle X, Y\ H \rangle$$

mit den Strategiebereichen

$$X = \{ x \in R^n :\ f_i(x) \geq 0\ ,\quad i = 1 \dots k\ ,\ x \geq 0 \}$$

$$Y = \{ y \in R^m :\ g_j(y) \leq 0\ ,\quad j = 1 \dots l\ ,\ y \geq 0 \}$$

und der stetigen Gewinnfunktion

$$H = H(x,y).$$

Entwickelt man eine verallgemeinerte Lagrangefunktion der Art

$$F(x,y;\, u,v) = H(x,y) + \sum_{i=1}^{k} v_i\, f_i(x) + \sum_{i=1}^{l} u_i\, g_i\,(y)\ ,$$

so wurde in [2] der folgende Satz bewiesen:

Satz: Seien F und G wie oben definiert, so folgt aus der Gleichung

$$\sup_{(x,u) \geq 0}\ \inf_{(y,v) \geq 0} F(x,y;\, u,v) = \inf_{(y,v) \geq 0}\ \sup_{(x,u) \geq 0} F(x,y;\, u,v)$$

die Gleichung

$$\sup_{x \in X}\ \inf_{y \in Y}\ H(x,y) = \inf_{y \in Y}\ \sup_{x \in X}\ H(x,y)$$

Ist dabei (x^*, y^*, v^*, u^*) ein Sattelpunkt von F für $(x,y,u,v) \geq 0$, so ist (x^*, y^*) ein Sattelpunkt von H und es gilt

$$F(x^*, y^*, u^*, v^*) = H(x^*, y^*)$$

Sei H(x,y) in x eine konkave und in y eine konvexe Funktion, seien weiter $f_i(x)$ (i=1...k) konkave und $g_i(y)$ (i=1...l)

konvexe Funktionen, erfüllt das Spiel G die Slaterbedingung (in $X \times Y$ existiert ein innerer Punkt) und seien X und Y kompakt, so gilt auch die Umkehrung des Satzes.
Auf Grund dieser Tatsache kann man unter gewissen Voraussetzungen mit Hilfe der Kuhn-Tucker-Theorie folgendes äquivalente Optimierungsproblem finden:

$$Z(x,y,u,v) = x'(-F_x) + v' F_v + y' F_y + u' (-F_u) \rightarrow \min$$

NB:

$$-F_x (x,y,u,v) \geqq 0$$
$$F_v (x,y,u,v) \geqq 0$$
$$F_y (x,y,u,v) \geqq 0$$
$$-F_u (x,y,u,v) \geqq 0$$
$$x \geqq 0, \; y \geqq 0, \; u \geqq 0, \; v \geqq 0,$$

wobei die Menge der Optimallösungen identisch ist mit der Menge der Sattelpunkte von G.
Interessant sind die numerischen Lösungsverfahren. Es wurden (bzw. werden noch) Untersuchungen durchgeführt, inwiefern spezielle Schnittebenen- bzw. Gradientenverfahren, wie sie in [1] und [3] angegeben wurden, auf diese Problematik anwendbar sind.

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GENERALIZATIONS OF THE STANDARD TRAVELING SALESMAN PROBLEM.

The standard traveling salesman problem (STSP) is to find a minimal Hamiltonian circuit of a weighted graph.

A number of procedures have been developed for solving the STSP but it is not known that which grows less than exponentially with the number of vertices of a graph.

The STSP has many practical applications in scheduling theory, vehicle routing, the implementation of algorithms in complex computing machines, the construction of information systems, etc , and to meet additional requirements of some of these applications the STSP has been generalized in many ways.

Most of problems which can be considered as generalizations of the STSP are to find the optimal routes for a group of salesmen provided the routes satisfy a set of requirements. Then a given generalization is obtained by specifying meanings of terms "optimal", "group of salesmen" and " set of requirements imposed on routes".

One of the aims of this talk is to present a set of generalizations of the STSP which correspond to all introduced so far and known to the author meanings of the above mentioned terms. Some problems are reducible to others, in particular to the STSP but there are only a few simple generalizations which are not NP - complete problems.

The STSP is closely related to the assignment problem (AP) which has been also generalized in many ways, for instance as the multidimentional assignment problem or as the independent assignment problem. Thus the following question arises : how the generalizations of the STSP are related to those of the AP.

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A. Prékopa, Technical University of Budapest and Hungarian Academy of Sciences Computer and Automation Institute, Budapest
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PLANNING OF OPTIMAL EXTENSION OF INTERCONNECTED POWER SYSTEMS USING STOCHASTIC PROGRAMMING

The interconnection of power systems can be described by a system of inhomogeneous linear inequalities. In order to satisfy the increasing load demand, new generating and transfer capacities have to be installed. The most economic project evaluation leads to the introduction of an objective function and thus we are given a /in general nonlinear/ mathematical programming problem. Since such systems contain random elements e.g. generation outages, peak load deviation and transfer capacity deviation, it is reasonable to prescribe a /high/ reliability level and thus we obtain a stochastic programming problem of probabilistic constrained type. This problem is solved by a joint application of simulation and nonlinear programming procedures.

Jacek Szymanowski, Andrzej Ruszczyński, Institute of Automatic Control Technical University of Warsaw

CONVERGENCE ANALYSIS FOR TWO-LEVEL ALGORITHMS OF MATHEMATICAL PROGRAMMING

For large scale problems of mathematical programming decomposition and hierarchical methods are often used. The upper level algorithm (coordinator) invokes several subalgorithms which perform lower-level minimizations. In various theoretical works in the field it is assumed that the lower-level solutions are unique and are computed precisely. The last assumption can hardly be satisfied in practice. Usually computation of lower-level solutions requires infinite number of function evaluations and therefore truncation has to be used. The basic problem which has to be dealt with now is the question of the effect of inaccuracy in lower-level minimizations on the properties of the upper-level algorithm. If we let local minimizations run for too long we would consume more computer time than needed; on the other hand too early truncation would disarrange the upper level algorithm. In the work the way of truncating lower-level minimizations in two-level algorithms has been suggested. It has been shown that there is no need to compute local solutions always with utmost accuracy. As a base for the considerations the approach of Polak to minimization algorithms with approximate linear search has been taken. The convergence theorem for the suggested two-level algorithm with truncation has been proved. The general definition of two-level methods which has been used covers primal methods and balance methods as well. Finally some detailed questions concerning application of various algorithms in the general framework have been discussed.

Arie Tamir, Tel-Aviv University, Tel-Aviv, Israel

ERGODICITY AND SYMMETRIC MATHEMATICAL PROGRAMS

The paper provides new conditions ensuring the optimality of a symmetric feasible point of certain mathematical programs. It is shown that these conditions generalize and unify most of the known results dealing with optimality of symmetric policies. The generalization is based on certain ergodic properties of nonnegative matrices.

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DIE ANWENDUNG VON MODIFIZIERTEN LAGRANGE-FUNKTIONEN IN DER BLOCKOPTIMIERUNG

Im Vortrag wird eine Lösungsmethode der folgenden konvexen Blockaufgabe vorgeschlagen:

$$\max\left\{\sum_{i=1}^{n} f_i(x_i) \,\middle|\, \sum_{i=1}^{n} g_i(x_i) \geqq b,\ x_i \in X_i,\ i=1,\dots,n\right\}. \quad (1)$$

Zu diesem Zweck werden die Menge der Verteilungen $U=\{u=(u_1,\dots,u_n) \mid u_i \in R^m, \sum_{i=1}^{n} u_i = b\}$ eingeführt und modifizierte Lagrange-Funktionen allgemeiner Art $M_i(x_i,u_i,y)=f_i(x_i)+\sum_{k=1}^{m}\mu_k(g_{ki}(x_i)-u_{ki},y_k)$ konstruiert (s.[1]), wobei mit $y=(y_1,\dots,y_m)$ der nichtnegative Vektor der dualen Variablen der Restriktionen der Aufgabe (1) bezeichnet wird. Es sei $\Psi_i(u_i,y)=\sup_{x_i\in X_i} M_i(x_i,u_i,y)$, $\Phi(u,y)=\sum_{i=1}^{n}\Psi_i(u_i,y)$, Y^* - die Menge der optimalen Dualvariablen und U^* - die Menge der optimalen Verteilungen. Zuerst wird bewiesen, dass $U^* \times Y^*$ mit der Menge der Sattelpunkte der Funktion $\Phi(u,y)$ auf $U\times R^m_+$ zusammenfällt, wenn die Funktionen $\mu_k(g_k, y_k)$ bestimmte Eigenschaften besitzen. Bei fast den gleichen Bedingungen ist $\Phi(u,y)$ eine stetig differenzierbare und konkave Funktion der Veränderlichen u. Wenn $\frac{\partial \mu_k(g_k,y_k)}{\partial g_k}=[y_k - K g_k]_+$, $K>0$, so kann man für das Auffinden von Sattelpunkten der Funktion $\Phi(u,y)$ auf $U\times R^m_+$ die Methode der Gradientenprojektion anwenden. Dabei erhalten wir folgenden Prozess:

$$u_i^{t+1} = u_i^t + \alpha^t\Big(\nabla_{u_i} M_i(x_i^t,u_i^t,y^t) - \frac{1}{n}\sum_{j=1}^{n}\nabla_{u_j} M_j(x_j^t,u_j^t,y^t)\Big),\ i=1,\dots,n,$$

$$y^{t+1} = \Big[y^t - \alpha^t \sum_{i=1}^{n} \nabla_y M_i(x_i^t,u_i^t,y^t)\Big]_+,$$

wobei $x_i^t \in \operatorname{Arg}\max_{x_i\in X_i} M_i(x_i,u_i^t,y^t),\ i=1,\dots,n,\ u^0\in U,\ y^0 \geqq 0.$

Wenn α^t den Bedingungen $0<\underline{\alpha}\leq\alpha^t\leq\bar{\alpha}<\min\left\{\frac{2K}{n};\frac{2}{K}\right\}$ genügt, so konvergiert dieser Prozess zu einem Element der Menge $U^*\times Y^*$ und stellt deshalb eine Lösungsmethode der Aufgabe (1) dar. Bei der Spaltenzerlegung einer gewöhnlichen linearen Aufgabe kann man mit Hilfe dieses iterativen Prozesses eine interessante Methode der linearen Optimierung erhalten.

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OPTIMALITY CRITERIA AS MATHEMATICAL DUALS

The paper treats the problem of designing for minimum weight structures which may be idealised as assemblages of plane-strain finite elements. Typical constraints consists of limitations upon the nodal displacements of the structure, limiting stress levels in each element and minimum size requirements for each element. The straightforward approach of using standard mathematical programming techniques to solve these problems has fallen into disfavour in the aerospace industry, the largest practical user, because of the large numbers of variables and nonlinear constraints, typically of the order of 10^3 variables and 10^3 constraints. Instead elements are iteratively resized using imprecise heuristic formulae whose convergence properties are sometimes very good but sometimes very bad. These resizing formulae and the methods developed around them are known as optimality criteria methods.

It is demonstrated that under certain conditions the minimum weight design problem has a nonlinear dual problem of a concise form which can be established via the Lagrangian saddle-point conditions. Upon examination there are very close similarities between the forms of the dual and the optimality criteria resizing formulae. Comparisons demonstrate why such variable convergence porperties have been obtained from some optimality criteria methods. The rigorous dual problem adds a hitherto absent degree of logic to the heuristic optimality criteria formulae. It is shown how better optimality criteria methods can be developed using the insight gained from examining the dual problem. It also raises the question whether all optimality criteria methods are nothing but poor, inaccurate representations of the general mathematical dual problem.

Fianlly it is demonstrated that the form of the dual problem is such as to effect considerable reductions in overall problem size and that special-purpose algorithms can be devised to permit its rapid solution. This suggests that mathematical programming may yet return to favour for this class of problems by virtue of duality and strengthens the importance which duality has for the solution of large, structured, optimization problems.

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BRANCH AND BOUND AND SHORTEST PATHS

In the discrete optimization, a new problem $g^* = \min_{x \in E \cap S} g(x)$ (Q) is considered instead of the problem to be solved $f^* = \min_{x \in S} f(x)$ (P) through a socalled embedment of the essential demand

$$\tilde{x} \in E \cap S \wedge g(\tilde{x}) = g^* \Rightarrow f(\tilde{x}) = f^* .$$

Based on the description by Mitten (Op.Res.18,24-34,1970) a branch and bound algorithm for the solution of the problem (Q) is characterized as an iteration in the second order power set $P^2(E)$ where a new collection of subsets from E is generated by a branching and a rejection operator from a given collection of subsets. The branching operator is also substantially characterized by a strategy (choice function) that decides which subsets are subdivided for every step when the optimality criterion is not fulfilled. A strategy where a node having the smallest lower bound is subdivided is termed a minimal strategy. A strategy where all the active nodes to be branched are subdivided is called parallel strategy.

In the tree T generated by a branch and bound algorithm a non-negativ edge length is assigned to an edge that symbolizes the generation of a new problem in the branching of a node by the difference of lower bounds of the generated node and the subdivided node. Due to the requirement that the function value $g(\hat{x})$ is assigned as the lower bound to a terminal node of one element $\{\hat{x}\}$ the problem is equivalent to the determination of a shortest path from the root of the tree T to a node from set of the one-element terminal nodes of T.

Thus, a branch and bound algorithm simultaneously determines a graph, particularly a tree, stating also a shortest path in this tree. The tree construction proceeds until the optimality criterion is fulfilled. Then a strategy means a method of determination of a shortest path wanted. A minimal strategy corresponds to the well-known algorithm by Dijkstra while the parallel strategy corresponds to the Moore algorithm. The latter strategy is of particular significance for the interpretation of discrete dynamic optimization algorithms as branch and bound method.

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LINEAR PROGRAMS IN TOPOLOGICAL VECTOR LATTICES

Usually infinite linear programming problems are studied in ordered topological vector spaces. In this spaces closure of the cone of nonnegative elements is the only relation between order and topological properties. However, in many practical cases there are additional relations between order and topology, e.g. the spaces are order complete topological vector lattices with the order topology (TVL).

Based upon structural properties of TVL the following duality theorem is obtained:

Theorem 1: Let be given the dual consistent linear programs

$$cx = \sup!$$
$$Ax \leqq b,\ 0 \leqq x \leqq k, \tag{1}$$

and

$$y'b + x'k = \inf!$$
$$A'y' + x' \geqq c,\ y' \geqq 0,\ x' \geqq 0, \tag{2}$$

where A is a continuous linear operator from TVL X into the ordered topological vector space Y, X' (Y') is the adjoint space to X (Y), A' is the adjoint to A and $c \in X'$.

Each of the following conditions is sufficient for the equality of the optimal values of the objectives from (1) and (2):

(1^0) $\{x \mid 0 \leqq x \leqq k\}$ is $\sigma(X,X')$-compact.

(2^0) There is a $x_0 \in X$ such that $b - Ax_0 \in \tau(X,X')$ int P_Y (P_Y is the cone of nonnegative elements in Y).

(3°) There is a total set X^0 of order-continuous linear functionals on X, $c \in X^0$, and A is $\sigma(X,X^0)$– $\sigma(Y,Y')$-continuous.

Theorem 2 gives some information about the structure of the optimal solution of (1).

<u>Theorem 2:</u> Let X be a TVL without atoms. If under condition (1°) for each band $B \subset X$, $B \neq \{0\}$, we have $A^{-1}(0) \cap B \cap \{x \mid -k \leq x \leq k\} \neq \{0\}$, then (1) has an optimal solution which is a projection of k in a band from X.

A special case of this theorem is the Ljapunov-theorem on the range of a vector measure. It gives an unified and easy approach to some generalizations of the Ljapunov-theorem, proved by several authors in the last time and related to bang-bang-principle and the existence of nonrandomized statistical tests. A further application of TVL is the following reformulation and extension to infinite spaces of the Tucker complementary slackness theorem. We have

<u>Theorem 3:</u> Let Y be a TVL of minimal type, and AX and $A'^{-1}(0) \cap P_Y^+$ weakly closed sets. Then the band in Y, generated by $AX \cap P_Y$ is the polar of the band in Y', generated by $A'^{-1}(0) \cap P_Y^+$ and vice versa.

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MODELLING AND COMPUTING SOLUTIONS TO LARGE OPERATIONS RESEARCH PROBLEMS BY USING NETWORK FORMULATIONS

The talk will give a survey of recent work by T. H. C. Smith, V. Srinivasan, V. Balachandran and the author on the formulation and computation as network problems of a number of operations research problems such as, ordinary and bottleneck travelling salesman problems, the decision critical path problem, work load smoothing, sources to uses, machine scheduling and sequencing, etc.. In several cases (such as for travelling salesman problems) computational experience will be presented. The usefulness of operator parametric programming techniques for the ordinary and generalized transportation problem will be discussed.

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CERTAIN KINDS OF POLAR SETS AND THEIR RELATION TO MATHEMATICAL PROGRAMMING

In this paper is discussed some special polar correspondences, which are related to the Minkowski polarity, known from convex analysis.

They represent a natural generalization of the concepts of blocking and anti-blocking polyhedra, developed by D.R. Fulkerson and used in the study of certain problems in mathematical programming and combinatorics.

Here emphasis will be placed on the study of necessary and sufficient conditions to ensure the validity of the polar correspondences in question.

At the same time an economic interpretation will be given within the framework of activity analysis.

ABSTRACT

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OPTIMAL DISSECTION OF SIMPLICES

The bisection algorithm is a minimax procedure for locating a fixed point of a continuous function taking an interval into itself. The algorithm can also be viewed as dissecting a 1-simplex (interval) into smaller 1-simplices by the insertion of a point. We generalize the latter problem to n dimensions and ask for a sequential dissection scheme to minimize the maximum diameter of a subsimplex after k dissections. We obtain bounds on the asymptotic rate at which the minimax diameter can be reduced and dissection schemes that approach these bounds. Based on these schemes we give near-optimal triangulations for computing fixed points.

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ON THE ACCELERATING PROPERTY OF AN ALGORITHM FOR FUNCTION MINIMIZATION WITHOUT CALCULATING DERIVATIVES

This paper reports some recent results on the speed of convergence of a general algorithm for function minimization without calculating derivatives. This algorithm contains Powell's 1964-algorithm as well as Zangwill's second modification of this procedure. The main results are Theorem 3.1 and 4.1 which show that if the algorithm behaves well then asymptotically almost conjugate directions are build and therefore the algorithm has an every-iteration superlinear speed of convergence. The results hinge on ideas of McCormick and Ritter (1972) and Powell (1964).

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A SURVEY OF RECENT ADVANCES IN MATHEMATICAL PROGRAMMING SYSTEMS

In the last few years the power of sophistication of mathematical programming systems, both commercial and experimental, has continued to expand. Some of this progress is due to the full scale implementation and exploitation of ideas which have already been current for some time and some if it is due to new ideas still not widely recognized. In the former category we have the full flowering of the large-scale in-core concept based on "supersparsity", due to J. E. Kalan. In the latter category there is the powerful approach of D. C. Rarick for obtaining feasible solutions to linear programs, of M. A. Saunders for problems with non-linear objective functions and new or improved techniques for updating the basis inverse and for pivot selection in the simplex method. In addition there has been much promising new work for problems with special structure and in general non-linear programming, based on techniques in current use.

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ON OPTIMAL PATTERN FLOWS

In this paper, we shall present some algorithms for finding the optimum solution of the flow problems, in which several constraints of non-network flow type are imposed on special arc flows. These constraints may be linear, nonlinear, or combinatorial. We call them *pattern constraints,* because in many cases they are associated with certain patterns of flows on special arcs. Also, we call such flows *pattern flows.* To find out maximal pattern flow and minimal cost pattern flow, and to show an extension of the Critical Path Method are the main objectives of this paper. In general, we can't solve them by usual network flow algorithms. They are concerned both with network flow problems and with more general mathematical programming problems. In this connection, we shall use Benders decomposition to find the optimal pattern flows and shall exhibit a parametrization of Benders decomposition to solve the extended Critical Path Method.

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and
D.B. Weinberger, Yale University, New Haven, Connecticut, USA

SYMMETRIC BLOCKING AND ANTI-BLOCKING RELATIONS FOR GENERALIZED CIRCULATIONS

Furkerson and Weinberger have determined blocking and anti-blocking relations for integral, feasible flows in a capacitated supply-demand network. In this manuscript we derive similar relations in the context of circulation networks. This approach provides immediate generalization to arbitrary linear subspaces using the notion of generalized circulations. In this general framework a striking symmetry is displayed between the blocking and anti-blocking relations. Implications of this material are discussed, including simple proofs of the earlier results of Fulkerson and Weinberger and certain new integral packing and covering results.

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IMPROVED ALGORITHMS FOR CONCAVE PROGRAMMING UNDER LINEAR CONSTRAINTS

This paper is concerned with some improvements to a cutting method the author has proposed in an earlier paper (1964) for solving a concave programming problem with linear constraints. A new finite algorithm is given, which is essentially a modified version of the first algorithm proposed in the mentioned paper. Also a proof is provided for the convergence of the second algorithm proposed in the same paper, the convergence being understood in the following sense: given $\varepsilon > 0$, the algorithm reaches an ε-optimal solution after a finite number of steps. (an ε-optimal solution is a solution whose value differs from the optimal value by less than ε). Some problems, closedly related to concave programming, are pointed out.

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ON FIXED POINT METHODS IN MATHEMATICAL PROGRAMMING AND RELATED QUESTIONS

The developement of Mathematical Programming in the last decade has demonstrated the increasing importance, both from a theoretical and a computational practical point of view, of fixed point methods in a wide area of subjects from nonlinear programming, control theory, variational calculus, game theory, general economic equilibrium theory. This paper deals with some general problems concerning the theory and applications of these methods and contains the following items:

I. Description of an abstract fixpoint scheme, which is in a sense an axiomatization of Scarf's theory and hopefully can provide an unified basis for treating a large variety of theoretical and computational problems in different fields.

II. Applications to combinatorial properties of convex sets, variational inequalities, minimax theorems, and related questions. In particular, a new proposition is obtained which is similar to Kuratowski-Knaster-Mazurkiewicz's theorem in the classical approach and has interesting implications.

III. Applications to stability properties of systems of inequalities and extremal problems. It turned out that a convex consistent system of the form: $x \in D$, $G(x) \in M$ is stable (in a natural sense) if and only if it is not critical , i.e. if $0 \in \text{int}\,(G(D) - M)$ (here D is a convex subset of a locally convex topological space, M a convex closed cone in R^n and G an M-convex mapping from D into R^n). Since extremality implies critical-ness, these results provide necessary extremality conditions for a wide class of convex and "convexifiable" problems.

Aydin Ulkucu, California Polytechnic State University, San Louis Opispo, California, USA

A PROBABILISTIC EFFICIENCY STUDIES OF MATHEMATICAL PROGRAMMING ALGORITHMS

Although the most important characteristic of a mathematical programming algorithm is its efficiency, the concept of efficiency has not been established. However, it has already been demonstrated that the deterministic methods are inadequate and the literature contains scattered results of some attempts in probabilistic studies.

In this paper a unified approach to algorithmic efficiency concept is introduced and previous related work is briefly reviewed. Several classes of probability measures for generating random problems and their properties are presented. A simulation procedure for experimental studies to estimate the average behavior of mathematical programming algorithms is proposed and the results of some recent work on probabilistic efficiency evaluations are reported.

S.Vajda, The University of Sussex, Brighton, England

CONTROL OF MANPOWER SYSTEMS BY LINEAR PROGRAMMING

We consider a graded population with flows between the grades, and we are interested in a situation when the structure of the population is controlled by some "management".

In principle, the control can be performed by adjusting the transfer rates, the numbers of new entrants, possibly the numbers of those leaving, or by a combination of such influences.

The rates and numbers mentioned are non-negative, and it is therefore natural to use techniques of linear programming to solve constructively some of the following problems:

When the transfer rates are given, then we can determine which structures can be attained starting from a given structure, and from which starting structures a desired structure can be attained in one, two, ... steps. It is also of interest to know whether a structure attainable in n steps can then be maintained. Such questions can be answered by the Simplex Method.

If the population is expanding (or contracting), parametric programming can be used.

When the transfer rates are not given, but can be chosen, then problems analogous to those stated can be solved by a method similar to that used to solve the Transportation Problem.

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SOME RESULTS ON THE MATHEMATICAL THEORY OF BINARY MIXTURES SEPARATION

1. The characteristic of separating element. A two-component flow (x,y) is separated into two flows (parts) $(kx, \ell y)$ and $((1-k)x, (1-\ell)y)$. It is supposed that a convex monotonic function $\ell(k)$ independent of x and y and such that $\ell(0)=0$, $\ell(1)=1$ is known. If $k=\ell$ then there is no separation. If k=1 and $\ell=0$ then the separation is referred to as ideal. An element with the characteristik $\ell_1(k)$ is considered to be better than an element with the characteristic $\ell_2(k)$ for $k \in D$ if $\ell_1(k) < \ell_2(k)$, $k \in D$.

2. The bichromatic digraph such that its nodes correspond to the separating elements is associated with a separating scheme; there are also two absorbing nodes. Each non-absorbing node originates two ares of both colours. The colour attributed to an are is defined by the meaning of inequality relating ℓ and k.

3. The ratio of the maximum input flow of one element to the flow on the scheme output is referred to as the circulation coefficent.

4. A scheme where the substitution of any element by an element with the characteristic $\ell=k$ does not impruve its characteristic is called feasible.

5. The synthesis of a separating scheme reduces to the enumeration of feasible schemes, the calculation of their characteristics and the ordering graphs accordingly to the rule in 1. subject to the circulation constraints.

This paper gives the scheme feasibility principle. The synthesis problem is reduced to a convex programm which a set of specialized computer codes is proposed.

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GENERALIZED LAGRANGE MULTIPLIERS IN INTEGER PROGRAMMING

In this paper we use the following form of the linear integer programming problem:

$$\max \ \underline{c}'\underline{x}$$
$$A\underline{x} \leq \underline{b}$$
$$x_j = 0 \text{ or } 1 \qquad j = 1,\dots,n$$

where $\underline{c}, \underline{x} \in R^n$, $\underline{b} \in R^m$ and A is a matrix, sized mxn. The GLM method, as it was suggested by H Everett, is many times referred in operation research papers. The aim of the present paper is to show how the GLM method, can be used in a complex algorithm. First of all we give a new optimality test. Though the test is a general one, its use in integer programming is detailed. It is a well-known property of the enumeration algorithms that it is much easier to find the optimal solution than to prove its optimality. The new test can shorten the proof. Therefore a new question arises here: to find Lagrange multipliers producing a given feasible /optimal/ solution. We show not only the existence of such multipliers but we choose the best one in a certain sense. The proof of the existence shows how the gaps can be avoided. This is important because the gaps are the greatest difficulty of the GLM method. In any case, if we have a new feasible solution, we can get an upper estimation of the optimal value of the objective function.

Other utilizations of Lagrange multipliers are also studied. The relation between the Lagrange multipliers and surrogate constraints is examined at the end of the paper.

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DESIGN OF A PIPELINE ROUTE. AN OR APPROACH TO AN ENGINEERING DESIGN PROBLEM

A pipeline is to be built such that total yearly costs are minimized. The design variables are the route, the diameter and thickness of the pipe, and the number and size of the pumps. An heuristic procedure is proposed. This is composed of: a onevariable optimization problem, an algorithm to find the shortest route of a graph and a dynamic programming solution to the optimal design of a pump-pipe system in series.

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FINITE GROUPS AND INTEGER LINEAR PROGRAMS. AN UNIFYING SURVEY AND SOME NEW PROPERTIES

The aim of this paper is first of all to review the group approach for an integer linear program /ILP/. More precisely, for an ILP the related group minimization problem is considered over a finite groupG. Overview is made of recent results about G and its use to solve ILP. Succesively the paper is devoted to the study of all homomorphisms of G. This enables one to state that if

/1/ $\sum_{g \in E} t(g) \cdot g = g_0$

$t(g) \geq 0$ integers $\forall g \in E \subset G$

is the group equation related to the ILP, then a family $\mathcal{F}$ of valid cuts may be found by considering a new group equation:

/2/ $\sum_{h \in Q} t(h)h = h_0$

$t(h) \geq 0$ integers $\forall h \in Q \subset H$

where $h = \gamma(g)$ for some $g \in E$ and γ is a homomorphism of G on to H.
The problem is studied of existence of extremal minimal cuts in $\mathcal{F}$. Let $P(G, E, g_0)$ denote the convex hull of solutions of /1/ and let $P(H, Q, h_0)$ denote the convex hull of solutions of /2/. At last, the correspondence is investigated between the irriducible points or the vertices of $P(H, Q, h_0)$ and those ones of $P(G, E, g_0)$.

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TIGHTER EQUIVALENT FORMULATIONS OF INTEGER PROGRAMMING PROBLEMS

Computational experiences /e.g.H.P. Williams, Math. Programming Study, 2, pp. 180-197/ suggest that an integer programming problem /IP/ is much easier to solve if the feasible region to the corresponding linear programming problem /LP/ is smaller, i.e., if the continous formulation of a given IP is tighter. In this paper we show how to obtain such a formulation using the dynamic programming procedure for the rotation of a given constraint in any direction without adding or eliminating any integer feasible solution, described by F.Kianfar /Operation Research, 19 pp. 1373-1392/.

We introduce so called the best direction of such a rotation and define a strongest cut as a constraint that cannot be rotated. A IP, in which every constraint is a strongest cut, is called an almost linear /integer programming/ problem. We proved that every IP can be transformed to an equivalent almost linear problem with the same number of variables and the number of constraints increased at most by one. A linear optimum to the corresponding LP is often integer for such problems. If such a case does not happen we construct a cut that cuts off linear optimum but does not cut off any feasible solution and rotate this cut in its best direction. We repeat adding new cuts until a linear optimum is integer.

We show that all Gomory cuts and many other ones can be rotated. In computational experience we study the method of integer form, since the rotation decreases the effect of roundoff errors. We study an influance of the rotation on the number of cuts needed and on the total solution time. Different directions of rotation are investigated.

There are many special IPs which are almost linear problems /e.g. set covering, partitioning and packing problems/, while e.g. Miller's et al. formulation of the traveling salesman problem is not an almost linear problem. We solve some problems, taken from the literature, in the way similar to described in above mentioned Williams's paper using the IBM MIP/370 System.

So far dynamic programming and linear programming have been used as the separate method for solvin IPs. In this paper it is shown how both dynamic programming /rotation/ and linear programming /cutting plane method, the branch - and - bound method/ may be used simultanously in solving IP problems.

A B S T R A C T

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A NECESSARY AND SUFFICIENT CONDITION FOR THE AGGREGATION OF LINEAR DIOPHANTINE EQUATIONS

Several authors have given sufficient conditions for the weights with which two Diophantine equations are to be multiplied before being aggregated to a new single equation with exactly the same set of integer solutions. Here some both sufficient and necessary conditions will be established in the finite case for an even more general form of linear constraints standardized to $\sum_{j=1}^{N} a_{ij}x_j = 0$, $i=1,2$.

The principal idea is based on the fact that the set of values a linear form can take - called its spectrum - is often very poor as compared to the Cartesian set of the generating integers x_j, $j=1,\ldots N$. A method for easily calculating the spectrum of a linear form is given. Aggregating two equations means establishing a linear combination of the two corresponding spectra the coefficients of which are the aggregation weights. If and only if this linear combination does not contain any spectral correlation point other than the origin, the equation found by aggregation is equivalent to the two given ones.

Determining the above linear combination requires the knowledge of the spectral correlation set. A method to find this set is given.

Aggregating $m > 2$ equations according to the foregoing principle is theoretically possible en bloc and gives the best aggregation weights. But the computational requirements for the determination of the correlation set $S_{12\ldots m}$ grow rapidly, the cardinality of this set tending towards that of the Cartesian set of the generating integers x_j, $j=1,\ldots N$. Sequential aggregation combined with some new "shake out" algorithms will here perhaps be a practicable way. These algorithms have certainly not yet found their final shape, but the main ideas are developed by what might be called "spectral programming".

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ON OPTIMIZATION IN EDGE-CHROMATIC SCHEDULING

Some types of multi-period assignment problems may be reduced to finding an edge coloring $H = (H_1, \dots, H_k)$ of a multigraph G ; usually several criteria are to be considered for characterizing a good assignment. First the number $f_1(H) = k$ of periods should be kept reasonably small and next one may wish to optimize a function $f_2(H) = c_1 h_1 + \dots + c_k h_k$ where h_i is the cardinality of H_i . It is known that whenever G is not bipartite these objectives may be contradictory.

In this paper we try to characterize the so called efficient points of this multiobjective optimization problem. It is shown that if G is a simple graph with maximum degree d , then all edge colorings H which maximize $f_2(H)$ satisfy $f_1(H) \leq d + 1$. By using an extension of the coloring technique previously used one may derive a similar result for multigraphs.

Finally we describe a class of multigraphs G for which $q(G) = d$ and all colorings H maximizing $f_2(H)$ satisfy $f_1(H) \leq d + 1$ (where d is the maximum degree).

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THE OPTIMAL RECOURSE PROBLEM

An optimal recourse problem is a sequential stochastic decision process where at each instant of time a decision must be selected on the basis of prior observations (possibly the decision maker might only have access to partial or distorted information about the past of the process). The objective is to minimize the total expected cost. We seek necessary and sufficient conditions. The existence of "usable" characterizations of the optimal solution rest on certain regularity conditions that are standard for classical (deterministic) optimization problems but also on a specific property of stochastic problems. The results are applied to some problems arising as multistage stochastic programs, stochastic control problems and to economic growth problems.

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AN APPLICATION OF DYNAMIC PROGRAMMING TO A BUILDING GAME WITH DOMINOES

The following game with dominoes is probably well known: One builds a row of upright dominoes with interdistances which are just smaller than the height of the dominoes. Then one gives a push to the first domino. The fun of the game is to see how subsequently all dominoes fall down. But before one can enjoy this spectacle one has to build the row. If one builds the row from left to right there is a great risk that the whole row will fall before it is completed, the hand is not so firm as it should be. One can make this risk somewhat smaller by leaving first some positions free. The problem is how to build the row in order to minimize the (expected) building time.

We assume that the only risk is that a just placed domino falls (to the left with probability p_l and to the right with probability p_r). The time to place a domino is assumed to be one unit of time.

It is clear that the problem can be formulated as a total costs dynamic programming problem. If there are N dominoes to be placed on prescribed positions, then there are 2^N possible states. Each state can be represented by a subset of $V := \{1, 2, ..., N\}$, indicating the occupied positions. In state $A \subset V$ one can place a domino on each of the free positions. The transition probabilities are given by p_l and p_r and the structure of A.

It is well known that in such problems the optimal strategy is a stationary one. But it is not necessary to consider all stationary strategies. It is possible to prove that the optimal strategy is a sequence. Assume that the N positions are numbered in some way, the strategy which choses in each state A the free position with the smallest number is called a sequence.

The optimal sequence can be found by induction. If $p_l = p_r$ an optimal sequence can be constructed in the following way: Give the number N to the middle position (or one of the middle positions), the number N-1 to the middle position of the left or right part, and so on.

For N = 8, for instance, one can get

x	x	x	x	x	x	x	x
4	5	7	1	8	2	6	1

But of course there are a lot of other optimal sequences.

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THE ECONOMIC INTERPRETATION OF DUALITY FOR PRACTICAL MIXED INTEGER PROGRAMMING PROBLEMS

The dual of a Linear Programming Model provides a means of attaching valuations to the constraints. When the model represents a practical situation these valuations have an economic significance. Moreover if the original problem possesses a unique optimal solution these valuations on the constraints provide a means of obtaining it. Such a procedure has the attraction that, for many practical problems, it provides a means of deducing an optimal operating pattern through a system of prices on the scarce resources.

Many attempts have been made to define the dual of an integer programming model. Most of these attempts suffer from serious shortcomings in that many of the attractive mathematical properties of duality in linear programming are lost. It is well known, for example, that a system of prices alone will not generally suffice to determine the optimal solution.

In this paper we examine some of the ways of attaching valuations to the constraints of an integer programming model to see if these valuations have a useful economic interpretation. Constraints arising in the following contexts in practical mixed integer programming models are considered:

/1/ Depot Location Problems

/2/ The Fixed Charge Problem

/3/ Problems with Logical Conditions

/4/ An Electricity Generation Problem.

In some of these problems a meaningful economic valuation on the constraints is a practical necessity. For example it is sometimes stipulated that the price of electricity should be determined according to the marginal cost of its production. In a linear programming model this could be obtained from the shadow prices /dual values/ on the demand constraints. Where generators can only work at zero, or above a minimum threshhold level the model contains integer variables. In order to determine a suitable price for electricity it is still necessary to place a valuation on the demand constraints.

Finally the reformulation of a mixed integer programming model /particularly if this leads to a unimodular model/ can sometimes lead to meaningful valuations on the original constraints.

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A PARAMETRIC APPROACH TO THE SOLUTION OF LINEAR AND QUADRATIC PROGRAMMING PROBLEMS.

The following singular parametric quadratic program:

$$\text{(I)} \qquad \varphi(x,t) = \tfrac{1}{2}(x, Qx) + (d_0 + t d_1, x) \longrightarrow min$$
$$x \in \Delta = \{x \mid a^1 \leq x \leq a^2\}$$

is considered.Here t is a scalar, Q - a positive semi-definite $n \times n$ matrix, d_0 , d_1 , $x \in E^n$,and the components of a^1 , a^2 may take values $\pm \infty$ .Denote

$$M(t) = \underset{x \in \Delta}{Arg\, min}\, \varphi(x,t)$$

The main idea of the parametric method consists in constructing of the piecewise linear trajectory of minimizers $x(t) \in M(t)$, $t \geq \bar{t}$ .It may be shown that such a trajectory for the problem(I) generally has jumps.

Definition.The point $x \in M(t)$ is said to be locally extendable if there exists a vector v such that

$$x + \alpha v \in M(t+\alpha)$$

for all sufficiently small $\alpha \geq 0$.

Two criteria of the local extendability are proposed. The first one contains a construction for evaluation of the direction vector of the $x(t)$ trajectory while the second allows us to find jumps or discontinuities of $x(t)$.

A class of parametrizations for which there exists a continuous poligonal path $x(t) \in M(t)$, $t \geq \bar{t}$ for the problem(I) is discussed.Namely,the d_0 and d_1 have to satisfy the following relation

$$\text{(2)} \qquad \beta d_0 + \gamma d_1 \in R(Q) = \{x \mid \exists y : x = Qy\}, \; |\beta| + |\gamma| > 0$$

The direction vector v is defined as a solution of the following minimization problem:

$$\text{(3)} \qquad \tfrac{1}{2}(v, Qv) + (d_1, v) \longrightarrow min$$
$$v_i = 0 \quad , \; i \in I(x(t_k))$$

where the index set $I(x(t_k))$ is obtained from $I(x(t_{k-1}))$ by means of some recurrence relation. Note that the two sets differ in one element only.

The advantages of the proposed parametric method have to be evaluated in terms of computational efforts for the problem(3) solution. There are available a number of numerical methods which may be used to this end, the LDL^T factorization technique, perhaps, being most efficient. It is important to emphasize that this technique is applied here to the singular matrix Q.
The parametric approach described above may be successfully applied to different classes of structurized linear and quadratic programs.

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A DIFFERENTIAL EQUATION APPROACH TO NONLINEAR PROGRAMMING AND ITS APPLICATION TO GLOBAL OPTIMIZATION

A method is presented for finding a local optimum of the equality constrained nonlinear programming problem. A nonlinear autonomous system is introduced as the base of the theory instead of the usual approaches. The relation between critical points and local optima of the original nonlinear programming problem is proved. Asymptotic stability of the critical points is also proved. A numerical algorithm which is capable of finding local optima systematically at the quadratic convergence rate is developed by using an idea similar to Branin's method for solving simultaneous nonlinear equations. Some numerical results are given for global optimizatiom.

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POWER, PRICES AND INCOMES IN VOTING GAMES

A priori measures of the power of different voters in a decision-making body have been proposed by Shapley and Shubik and by Banzhaf. These measures are based on the number of times a voter could change the outcome by changing his vote. However, if we postulate that voters require an *incentive* to change their votes (for example, in the form of payments by special interest groups), then it may be shown that there exist equilibrium "prices" for the various voters, and an optimal payment strategy for the interest groups. The prices and optimal strategies are computed as primal and dual optimal solutions to a special new class of linear programs. The above results lead to a new way of measuring relative power of voters in a voting body.

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ON ONE-DIMENSIONAL MULTIMODAL OPTIMIZATION

Algorithm for multimodal optimization combines two strategies: the first is global for search of intervals of mein local optima and the second is local for precise evaluating of this local optima. Global description of objective function $f(x)$ is statistical, i.e. global model is Wiener process. For global search is used the method (the OBM) that is optimal in Bayes sense in accordance with this statistical model. After each evaluation of $f(x)$ it is tested whether the interval of the local optimum is found. In the case it is not found the OBM continues the optimization. Otherwise the model of $f(x)$ in this interval is unimodal function and local method with parabolic interpolation of $f(x)$ is used for evaluation the local optimum with the given accuracy. After evaluation of the local optimum the OBM continues the optimization. The optimization ends if the probability of finding of the absolute optimum with the given accuracy exceeds level 0.99; this probability is evaluated in accordence with the statistical model of $f(x)$. The results of optimization of some multimodal functions show high efficiency of this algorithm.

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CHOOSING AMONG ALTERNATIVE DECISIONS INVOLVING MULTIPLE CRITERIA

In this paper an interactive method is presented for helping a decision maker to choose the best decision from a finite set of decisions. The method is in the spirit of my work with Wallenius, but differs from the earlier method in that a convex set is not assumed, and an alternative that may be dominated by a convex combination of decisions may be optimal and may be identified by the procedure. The same kind of questions used in the earlier work is used and it is believed that the method will be easily used by managers.

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MATROID INTERSECTION PROBLEMS WITH GENERALIZED OBJECTIVES

For $\mathbb{B} := \{0,1\}$ and ordered sets $(H,\leq)$ the objective $f : \mathbb{B}^n \to H$ shall be maximized under the restriction $x \in S \subseteq \mathbb{B}^n$. The GREEDY algorithm as well as Matroid Intersection algorithms can be formulated for this problem. The question is for which objectives f and which restrictions S do the algorithms work and solve the above defined Boolean optimization problem [BOP].

Sufficient conditions for the applicability of the most simple algorithm, the GREEDY one, characterize the structure of S in close relation to matroids.

For the objective f a certain monotonicity condition is assumed. An important special case is the restriction of the form of the objective to

$$f(x) = \mathop{*}_{x_i=1} f(e_i) \qquad [RBOP]$$

where e_i denotes the i-th unit vector and * is the binary operation in a semigroup $(H,*,\leq)$. In this case the monotonicity is equivalent to the algebraic assumption that $(H,*,\leq)$ is an ordered semigroup.

Two different Matroid Intersection algorithms are discussed. The first one is a generalization of the Hungarian method; by successive transformations the original problem is changed to a simple problem with the same set of solutions. The second one is a generalization of the max-flow-min-cost algorithm of FORD and FULKERSON due to LAWLER; at each step an element $x \in S$ with increasing $|x| := \Sigma x_i$ is found which has maximal value f(x) among all $y \in S$ with $|x| = \Sigma y_i$. For convenience only the problem RBOP will be considered. The combinatorial structure of the restriction S is closely related to the intersection of two matroids. The algebraic structure of the system $(H,*,\leq)$ is more special than in the previous case of an ordered semigroup; for both algorithms different additional axioms (e. g. strong divisibility) are assumed which guarantee the validity of the algorithms.

M A T H E M A T I C A L   P R O G R A M M I N G   S O C I E T Y

A B S T R A C T S

/SUPPLEMENTARY VOLUME/

IX. INTERNATIONAL SYMPOSIUM ON MATHEMATICAL PROGRAMMING

BUDAPEST - HUNGARY

August 23-27, 1976

HUNGARIAN ACADEMY OF SCIENCES

BOLYAI JÁNOS MATHEMATICAL SOCIETY

C O N T E N T S

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THE TEACHING OF MATHEMATICAL PROGRAMMING TO COMPUTER SCIENTISTS

The primary motivation for teaching mathematical programming to computer science students has the same origin as a number of other application-oriented topics, although there is some justification for introducing the techniques so that they can be used to solve problems arising within the computer science field. /The use of an integer l.p. algorithm for certain threshold problems in logical design is a case in point./

Unfortunately, pressures to include competing material is high. In our own case /at Sydney University/, the result has been that, in a two year course for pass students /representing about 40 p.c. of a student's time over this period/, lectures on mathematical programming are about 1 p.c. of the total course material. However, students come to the lectures with a reasonable grounding in the computational aspects of linear algebra.

The aim of the lectures is to bring the student to a point that he could usefully become a member of a group constructing or maintaining a mathematical programming package. Any "black-box" approach to packages is avoided, and model-building is de-emphasised as being primarily the concern of the operations researcher, not the computer scientist. Some attention is given to the specification and use of commercial packages: however, this aspect of mathematical programming is considered secondary to ensuring that students have a grasp of the computational techniques employed.

Research topics found to be suitable for a computer science department raise problems. The difficulty of using commercial packages for experimenting with new techniques is considerable and is alleviated to a considerable extent by using modular suites such as that constructed by Land and Powell.

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A BEHAVIOURAL APPROACH TO INFORMATION DEPRIVED MARKETS THROUGH COMPUTER ASSISTED COMBINATORICS

Considering the problem of atomistic markets, where potential buyers and sellers pair up randomly and negotiate deals based on their respective reserve prices, one finds that there is more than one price in the market for the same service or commodity. Examples of such markets are the real-estate markets, the used-car markets, or the employment market for executives.

This paper attempts to analyse, in these atomistic markets, the formation of transaction prices and transaction volumes through computer simulation. Seven different models are used: the markets are differentiated according to the amount of price information and bargaining skill made available to buyers and sellers.

The major conclusions drawn from the computer simulation are the following:

1. Average transaction prices will tend towards equilibrium prices of perfect markets. An increment of equally shared information and/or bargaining skill will accelerate this process. If this increment is unequally shared, however, the process will be retarded.
2. Transaction volumes will be higher than in perfect markets. This tendency will be little affected by a change in bargaining skills, but will be significantly and inversely affected by incremental price information.

Collectively, these two conclusions have tremendous welfare implications: in atomistic markets, ignorance increases transaction volumes, while average transaction prices remain unaffected.

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SOME RESULTS IN CONVEX ANALYSIS WITH APPLICATIONS TO A GENERALIZED TRAVELLING SALESMAN PROBLEM.

A generalized travelling salesman problem is considered by which the salesman is not obliged to visit all the points of a region, but only the subset of them, where the "utility" is not lower than the cost.

The problem consists in finding an optimal vertex in a subset of vertices of an assignment polytipe.

A stepping stone procedure is proposed to find an optimal solution.

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SCHEDULING UNIT-TIME TASKS WITH TREELIKE PRECEDENCE ON M PROCESSORS TO MINIMIZE MAXIMUM LATENESS

The problem treated is one of sequencing tasks on m identical processors, where there is a precedence ordering between certain tasks, as given by a directed graph which is a tree. The processing times of all tasks are assumed to be equal. Associated with each task there is a due date. An efficient algorithm is given for finding a schedule which minimizes maximum lateness. The algorithm may also be used to solve the problem of finding the shortest schedule without late tasks.

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POLYNOMIAL BOUNDING FOR NP-COMPLETE PROBLEMS

A large class of NP-complete combinatorial optimization problems may be reduced efficiently, i.e. by a polynomial algorithm , to k-parity matroid problems. Branch-and-bound or heuristically-guided-search [1] methods are currently used to solve NP-complete problems [2] and are particularly interesting for this class of matroid problems, since it is possible to use subgradient techniques in order to obtain possibly tight bounds to the optimum which are quite suitable to guide the search [3]. However subgradient methods are not polynomial bounded. In this work a polynomial bounded method is proposed for estimating the value of the optimum of a broader class of problems: k-parity weighted clutter problems. At the i-th iteration the method adds to the lower bound, initially set equal to zero, the value of a minimum weight assignment I of a digraph representing the parity constraints. The circuits of the clutter contained in I are then contracted, producing a new clutter over a smaller set of elements. This procedure generalizes the method presented in [4] for computing lower bounds to the length of a shortest hamiltonian cycle of a network.

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NECESSARY CONDITIONS WITHOUT DIFFERENTIABILITY OR CONVEXITY

We consider a mathematical programming problem in which the functions involved satisfy a Lipschitz condition, but need be neither differentiable nor convex. Using the concept of "generalized gradients", it is shown how one may prove an analogue of the usual first-order necessary conditions. The well-known results for smooth or convex data are seen to be special cases of the theorem. Finally, we discuss a new constraint qualification called "calmness", with the help of which we prove that for "most" problems, the necessary conditions hold in the stronger Kuhn-Tucker form.

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DISSEMINATION AND MAINTENANCE OF MATHEMATICAL PROGRAMMING SOFTWARE: EXPERIENCE WITH M.P.O.S.

In the past two years, Northwestern University's Computing Center has distributed to a large number of universities and research institutions a mathematical programming package designed for CDC 6000/CYBER computers. MPOS (Multi-Purpose Optimization System) is a simple-to-use integrated Fortran system for solving small to medium size problems in linear, quadratic, and integer programming, by choosing among eleven algorithms. The purpose of this paper is to describe our procedures for distributing and maintaining MPOS. Our distribution policy is aimed mainly, but not exclusively, to University Computer Centers. Because of this orientation, a great deal of effort is spent on producing good user's documentation. The maintenance activities of MPOS involve several individuals who spent a fraction of their time on that project. The modular design of MPOS allows each programmer to be responsible for his/her algorithm. An extensive archive of test problems (supplied by users or machine generated) is used to certify improved or new algorithms. Surveys of the user community at Northwestern have been useful in evaluating MPOS, determining user needs, and planning the development of future versions.

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NECESSARY CONDITIONS FOR THE SOLUTION OF CERTAIN PROBLEM OF STOCHASTIC LINEAR PROGRAMMING.

The stochastic linear programming problem (in the form: min $c^T X$, subject to $A x \geq b$, $x \geq 0$) is examined in which the parameters c, A and b depend on the set of random variables: $t_1 \dots t_r$, with $t \in R^r$.

The aim of the present paper is to solve the minimization problem utilizing the Kuhn - Tucker saddle - point method in the particular case of linear constraints. More precisely the conditions on the LP minimizati on problem are determined under which the solutions found with the saddle - point method be all and only the solutions of the starting LP problem.

To do so the following "feasible" sets are introduced:

$$X_f = \left\{ x \in X \;\middle|\; T(x) \text{ not empty set} \right\}$$

$$T_f = \left\{ t \in T \;\middle|\; X(t) \text{ not empty set} \right\}$$

Where sets X and T are:

$$X = \bigcap_{t \in R^r} X(t)$$

$$T = \bigcap_{x \in R^n} T(x)$$

and with:

$$X(t) = \left\{ x \in R^n \;\middle|\; x \geq 0, \quad A x \geq b \right\}$$

$$T(x) = \left\{ t \in R^r \;\middle|\; \text{for fixed } x, \text{ A and b are obtained such that the inequality } A x \geq b \text{ is satisfied} \right\}$$

Then, the convexity of these feasible sets is studied, because it plays a fundamental rôle in establishing necessary conditions for the solution of the stochastic LP via Kuhn Tucker saddle - point method.

In particular a necessary condition is given for the existence of solutions - via Kuhn - Tucher method - in the case in which A be deterministic and b stochastic.

Further, a necessary condition is given for the existence of solutions when A stochastic, b deterministic and under the hypothesis that T_f be convex.

Lastly, the general case in which both A and b are stochastic is examined.

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FULKERSON'S CONTRIBUTION TO LARGE SCALE PROGRAMMING

This paper will review joint research with Ray Fulkerson on the travelling salesman problem and network flows. Also his contribution to multicommodity network problems and its influence on the column generation approach and the decomposition principle.

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ON REGULARITY CONDITIONS IN CONSTRAINED OPTIMIZATION II : VARIATIONAL THEORY, FRITZ JOHN AND KARUSH-KUHN-TUCKER CONDITIONS

In the first paper of this series the authors introduced a minimal regularity condition for a nonlinear programming problem without regard to its dual variables or multipliers. The new condition involves both the constraint function *and the objective function* of the problem. In this paper, the implications of this regularity condition are traced for variational theory, Fritz John, and Karush-Kuhn-Tucker type results. The setting of the problem studied is in locally convex Hausdorff topological vector spaces. Best possible Kuhn-Tucker theorems are obtained for both the convex and the (linearly) Gateaux differentiable case. On the way to the latter the best possible Fritz John theorem is obtained. The relations to other recent work in first order optimality are traced and applications in control, stochastic programming, etc., indicated.

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SOME PROBLEMS AND RESULTS IN UNCONSTRAINED COMPUTER OPTIMIZATION TECHNICS

The subjects related to effective solving of nonlinear programming problems on the computer constitute an actual direction of research. The paper presents the author's researches regarding the efficiency, stop tests, etc. of some unconstrained algorithms in test problems solving. A special interest in shown mixed type algorithms.

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ON TWO PROBLEMS CONCERNING PATHS IN A GRAPH

The first problem concerning the optimal solution between two fixed vertices of a connex graph when, compared to a precedent situation, there appear modifications of the values associated to the arcs, performing the least number of computations.

The following cases of modifications have been considered:

1. The value of the arc is reduced by a given number;
2. The value of the arc is increased by a given number.

In the two cases, based upon the teorethical results, it is shown wich are the elements, in the matrix of optimal values M, that remain unchanged and which must be calculated, using Dantzig's algorithms.

The second problem concerning a method to determine all the "critical bonds" in an graph.

A critical bond is understood to be the only elementary path with a length equal one to the path between two fixed vertices of the graph a path of type $\prec (t\cdot\alpha)$.

Are introduced the new notion of restricted graph for all the external (internal) incidental arcs of a vertex and method for the construction of the introduced graph, after which , the necessary and suficient conditions for existence of the bonds are given.

Based on the above notions, an algorithm with three steps is given, considering the external incidental arcs.

The study is completed by appling two practice problems classes:

1, the determining of the elementary path with a minimal number of bonds

2. the determining of the path or with a minimal number of bonds (t,α), from a set of paths having the same minimal value.

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GEOMETRIC APPROACH TO DISCRETE PROGRAMMING

The report consists of three parts and contains the results induced by geometric ideas. The first part deals with a class of linear programs reducible to the problems with the integer polyhedra of feasible solutions. These problems have efficient algorithms. The problem P is said to be reducible to the problem Q(solvable efficiently) if there exists a method for mapping the set M_p of the feasible solutions of the problem P in the set M_q of the feasible solutions of the problem Q such that M_p is a projection of M_q and it is possible to obtain efficiently a solution (an optimal solution) of the problem P from a solution (an optimal one) of the problem Q. The reducibility of some problems to the flow network problem (the circulation problem) is studied.

The criteria of reducibility are established in terms of properties of constraint matrix that should be a matrix of a special type called an M-matrix. The authors investigate the properties of M-matrices that are totally unimodular and contain most of the known classes of the totally unimodular matrices. The geometry of M-matrices is found out to be connected with a possibility of putting their rows in a tree struture.

The second part deals with a linear optimization problem on a regular discrete set $\mathfrak{M} = \cap \mathfrak{M}_i$
where $\mathfrak{M}_i = \{x \in M / (a_i, x) < b_i$ или $(a_i, x) > c_i\}$
b_i, c_i and M is a polyhedron in R^n.
Using an idea of constructing the convex hull of feasible solutions by means of adding the minimal number of additional constrains authors work out a general scheme of generating the cuts, that being proved to be the convex polyhedron. The propeties of the polyhedron and of the extreme cuts are studied and their relations with other known cuts are found. The principal theorems of the cut theory for regular problems are proved as well as a criterion of equivalence for irreducible inequality system is established.

The third part of the work describes the efficient method of reducing the linear integer problem with constrains Ax = a, x is integer nonnegative vector to an equivalent problem with constraints $A'x' \geq a'$, x is integer, where the number of variables x′ is less than the number of variables x. The method is based on founding a basis of integer lattice together with a representation of a general integer solution of a system in a form of $x = A'x' - a'$. The method requires the polynomial number of operations with respect to the parameters of original problem, the power of the polynom being no more than 5.

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QUADRATIC KNAPSACK PROBLEMS

Binary (0, 1) optimization problems having a quadratic objective function, and one linear constraint are considered. A variety of problems leading to this model are presented. An algorithm for their solution is described; this algorithm is based on producing a sequence of linear knapsack problems which provide upper and lower bounds to the optimum of the original problem. Numerous experiments with variants of this algorithm have been carried out in order to select the most efficient one. The metaORic question "If quick then dirty?" has been examined.

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WHAT EVERY MODELER SHOULD KNOW ABOUT OBJECTIVE FUNCTION APPROXIMATIONS

Most applications of mathematical programming require the modeler to exercise some discretion in estimating or approximating the objective function to be optimized. We give a simple à priori bound relating the amount of objective function error to the amount of error thereby induced in the solution of the corresponding optimization problem. This furnishes a natural criterion to guide the choice of an estimated or approximate objective function. The criterion can often be applied via simple graphical constructions in the case of linear separability, and we show that it is generally equivalent to the familiar Chebyshev criterion -- which thereby provides direct access to a powerful array of established results and techniques for the general case. Additional results in a similar vein will be presented.

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ON THE METHOD OF SUCCESIVE PROJECTIONS

Let K be a polyhedral cone

$$K = \{x \mid A^T x \leq 0, \quad x \in R^n, \quad A \in R^{n\times m}\},$$

and for a given vector $c \in R^n$ let the function $\varphi(x)$ be defined as

$$\varphi(x) = \begin{cases} c^T x/\|x\|, \text{ if } x \neq 0, \ \|x\| = (x^T x)^{1/2} \\ 0, \text{ if } x = 0. \end{cases}$$

Here we investigate the method of succesive projections for solving the problem

$$\text{Max} \ \{ \varphi(x) \mid x \in K\}, \tag{1}$$

which is closely connected to the problem of finding optimal feasible directions in convex programming.

It is known that there exists a submatrix consisting of the column vectors of A

$$A_o = [a_{i_1}, a_{i_2}, \dots, a_{i_p}], \quad p \leq n$$

such that A_o is a matrix of maximal rank and the solution of (1) can be expressed as

$$x_o = (I - A_o A_o^+) c, \quad A_o^+ = (A_o^T A_o)^{-1} A_o^T$$

which is unique up to a multiplication with a positive scalar /see e.g. [1],[2]/. Here A_o^+ denotes the pseudo-inverse of A_o.

Let X denote the set of column vectors of a matrix X and let them correspond to each other by convention.

According to the method of succesive projections, column vectors of A are selected for or dropped from the set $\{A_i\}$ in the $(i+1)$ st step by some reasonable criteria in such a manner that the sequence

$$\{A_1\}, \quad \{A_2\}, \ldots$$

converges to an optimal set $\{A_o\}$. Let

$$\{\overline{A}_i\} = \{A\} \setminus \{A_i\}$$

and let e_k, e_j be unit vectors of suitable order.

We propose to modify Rosen's selection rules [2] as follows:

a) if $A_i^+ c > 0$, select the vector $\overline{A}_i e_k \in \{\overline{A}_i\}$ for which

$$c^T (I - A_i A_i^+) \overline{A}_i e_k / \| (I - A_i A_i^+) \overline{A}_i e_k \| > 0$$

is the largest and adjoin it to $\{A_i\}$,

b) if $A_i^+ c > 0$ does not hold, then drop the vector $A_i e_j \in \{A_i\}$ for which

$$e_j^T A_i^+ c / \| e_j^T A_i^+ \| \leq 0$$

is the smallest, i.e. omit it from $\{A_i\}$.

It has been mentioned by Zoutendijk [1] that rule a) seems to be promising. In fact, it is optimal in the sense that if $\{A_i\} \cup \{a_k\} = \{A_o\}$, then rule a) selects the vector a_k. Similarly, if $\{A_i\} \setminus \{a_j\} = \{A_o\}$ then rule b) drops the superfluous vector a_j. Moreover there exists at least one sequence

$$\{A_1\}, \{A_2\}, \ldots, \{A_p\} = \{A_o\}$$

with the property

$$A_i^+ c > 0, \ \text{rank}\ (A_i) = i, \ i = 1, 2, \ldots, p.$$

This assures that such type of algorithm may converge. However convergence is not proved yet.

Additionally, we have developed a computer code for solving (1) by using Householder's QR-factorization [3]. The values $n = 12$, $m = 10,15,20,25,30$ were chosen and matrix A and vector c were generated 100-times by a random number generator uniform in $(-1,1)$. For the ratio

$$\frac{N_-}{N} \quad \frac{\text{no. of all dropped vectors}}{\text{no. of all selected vectors}}$$

and for N/100, the average number of selections the following values have been observed:

m	10	15	20	25	30
N_-/N	0.004	0.035	0.088	0.109	0.081
N/100	5.060	7.680	10.730	12.870	12.750

According to experiences, slower convergence occurs if there is an active subset of column vectors of A that is close to being linearly dependent.

The results may be of use in selecting active constraints in solving linearly constrained problems.

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(i) Take non-negative combinations of given inequalities, and possibly weaken the right-hand-side.

$(ii)_j$ Having already obtained two inequalities

$$\alpha_1 x_1 + \ldots + s x_j + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + t y_j + \ldots + \beta_n y_n \geq \alpha_0$$

$$\alpha_1 x_1 + \ldots + s' x_j + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + t' y_j + \ldots + \beta_n y_n \geq \alpha_0$$

one may deduce

$$\alpha_1 x_1 + \ldots + s' x_j + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + t y_j + \ldots + \beta_n y_n \geq \alpha_0 .$$

Conversely, any inequality thus obtained is valid for the complementarity constraints.

The proof of this result is constructive, and it supplies a finitely-convergent cutting-plane algorithm for this generalization (CMC) of the linear complementarity problem.

Note that the constraints (CMC) properly include the linear complementarity problem. Indeed, the constraints of a bivalent integer program can be cast in the form (CMC), by rewriting these constraints

$$\text{(BIP)} \qquad \begin{aligned} Dx &\geq d \\ x_j &= 0 \text{ or } 1,\ j = 1,\ldots,n \end{aligned}$$

in the form

$$\text{(BIP)'} \qquad \begin{aligned} Dx &\geq d \\ x_j + y_j &= 1,\ j = 1,\ldots,n \\ x_j y_j &= 0,\ j = 1,\ldots,n \\ x_j, y_j &\geq 0,\ j = 1,\ldots,n \end{aligned}$$

and obtaining all valid cutting-planes via rules (i), $(ii)_j$.

The Theorem announced above is one corollary of a considerably more general result concerning what Balas has called "facial constraints." The general result will be given at the meeting, but is too technical to state here. Those interested may write the author for preprints.

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THE ASSIGNMENT PROBLEM WITH COMPATIBLE JOBS

In this paper a generalization is presented of the classical job-machine assignment problem. It is assumed that some but not all jobs may be assigned to the same machine. The problem defined in this way generalises both the assignment problem and the problem of determining the chromatic number of a graph. A branch and bound algorithm is proposed using a combination of the hungarian method and Lagrangian relaxation.
Computational experience on a CDC 6500 is discussed.

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APPLICABILITY OF NETWORK TECHNOLOGY TO INTERACTIVE DECISION MAKING

Lack of satisfactory treatment of the human engineering and multi-criteria aspects of problem-solving constitute two of the major obstacles to more widespread use of mathematical programming today. The purpose of this paper is to discuss how the dramatic recent improvements in the computer implementation technology of network flow algorithms and the major advances in mini-computer design can be used to partially overcome these obstacles. Specifically, the paper focuses on how network flow technology can be combined with interactive mini-computer capabilities in order to produce substantial advances in the design of interactive man/machine mathematical programming systems, information systems, and current procedures for coping with multi-criteria decision making.

H. W. Kuhn, Princeton University, Princeton, N.J., U.S.A.

NONLINEAR PROGRAMMING DUALITY AND ECONOMICS

Historically, the economic interpretation of dual linear programs has played a central role in applications and algorithms. The purpose of this paper is the development of a theory of duality in nonlinear programming starting from a new and natural economic motivation. This interpretation extends in a straightforward manner the duality of linear programming. It produces a pair of dual programs that include, as special cases, a wide variety of previous results.

P.J. Laurent, University of Grenoble, France, and
C. Carasso, University of Saint Etienne, France.

AN ALGORITHM OF SUCCESSIVE MINIMIZATION IN CONVEX PROGRAMMING.

$$(P)\quad \begin{cases} \text{Consider the problem of minimizing :} \\ \qquad f(x) = \sup_{t\in T} (< x,b(t) > - c(t)) \\ \text{with the constraints} \\ \qquad < x,b(u) > - c(u) \leq 0 \text{ , for all } u \in U \\ \qquad x \in W \subset \mathbb{R}^n \end{cases}$$

where T and U are arbitrary sets, b and c are bounded mappings from $T \cup U$ into $\mathbb{R}^n$ and $\mathbb{R}$ respectively and W is an affine variety of $\mathbb{R}^n$.

At each iteration we introduce, as in the Cheney-Goldstein algorithm, a polyhedral approximation of the problem :

$$(P^\nu)\quad \begin{cases} \text{Minimize :} \\ \qquad f(x) = \max_{t\in T^\nu} (< x,b(t) > - c(t)) \\ \text{with the constraints :} \\ \qquad < x,b(u) > - c(u) \leq 0 \text{ , for all } u \in U^\nu \\ \qquad x \in W \end{cases}$$

where T^ν and U^ν are suitable finite subsets of T and U. Let α^ν be the (finite) amount of (P^ν) and W^ν the set of solutions (which is an affine variety).

A general exchange theorem (a generalization of the Stiefel theorem) is given in order to exchange a set of new elements $\hat{A}^\nu \subset T \cup U$ with a subset $\bar{A}^\nu$ of $A^\nu = T^\nu \cup U^\nu$:

$$A^{\nu+1} = (A^\nu \setminus \bar{A}^\nu) \cup \hat{A}^\nu$$
$$T^{\nu+1} = A^{\nu+1} \cap T$$
$$U^{\nu+1} = A^{\nu+1} \cap U$$

The set $\hat{A}^\nu$ is determined by considering the sub-problem :

$$(SP^\nu)\quad \begin{cases} \text{Minimize :} \\ \qquad h^\nu(x) = \text{Max } (f(x) - \alpha^\nu \text{ ; } g(x)) \\ \text{with the constraint} \\ \qquad x \in W^\nu \end{cases}$$

where $g(x) = \sup_{u\in U} (< x,b(u) > - c(u))$.

An approximated solution of (SP^ν) is obtained by considering a string of nested minimization problems :

Setting $W_o^\nu = W^\nu$,

$$(SP_i^\nu)\quad \begin{cases} \text{Minimize} \\ \qquad h_i^\nu(x) = \text{Max}\,(f_i^\nu(x) - \alpha^\nu\ ;\ g_i^\nu(x)) \\ \text{with the constraint} \\ \qquad x \in W_{i-1}^\nu \end{cases} \qquad i=1,\dots,m^\nu,$$

where $f_i^\nu(x) = \underset{t \in A_i^\nu \cap T}{\text{Max}} \;(< x,b(t) > - c(t))$

$g_i^\nu(x) = \underset{u \in A_i^\nu \cap U}{\text{Max}} \;(< x,b(u) > - c(u))$

A_i^ν is a finite subset of $T \cup U$, $i=1,\dots,m^\nu$,

W_i^ν is the set of solutions of (SP_i^ν) , $i=1,\dots,m^\nu$.

All operations (specially the computation of the functions f and g) can be done approximatively.

The convergence is proved under very general conditions (but without any assumption of the Haar type).

A notion of ε-solution is introduced and it is proved that such an ε-solution is obtained after a finite number of iterations.

The algorithm includes all the algorithms of the Rémès or Töpfer type for solving best approximation problems without Haar conditions.

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A GAME PLAYED BY LINEAR OPTIMIZERS

Considering the problem

(1) $\max\limits_{x \in M(y)} \{xA^1y + p^1x + q^1y\}$

and

(2) $\max\limits_{y \in N(x)} \{xA^2y + p^2x + q^2y\}$,

where (1) and (2) are the pay-off functions of player I and player II respectively and the sets of polyhedrons M(y) and N(x) are the sets of strategies. Equilibrium points of such games are characterized and computed by linear parametric programming results.

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SOME PROBLEMS OF THE APPLICATION OF MATHEMATICAL PROGRAMMING IN AGRICULTURAL PLANTS

The Statistical Department of our university has prepared medium-term development plans for several agricultural plants in the last years. For this purpose we have elaborated linear programming models which are practically applicable in any larger Hungarian agricultural plant. For instance for the simultaneous optimizing of the structure of production and the resources we have used the following model:

$$\begin{array}{lllllll}
A_{11}\underline{x}_1 & + A_{12}\underline{x}_2 & & & & \leqq & \underline{b}_1 \\
 & A_{22}\underline{x}_2 & - A_{23}\underline{x}_3 & & & \leqq & \underline{0} \\
A_{31}\underline{x}_1 & + A_{32}\underline{x}_2 & & - A_{34}\underline{x}_4 & & \leqq & \underline{0} \\
A_{41}\underline{x}_1 & + A_{42}\underline{x}_2 & + A_{43}\underline{x}_3 & - A_{44}\underline{x}_4 & & \leqq & \underline{0} \\
A_{51}\underline{x}_1 & + A_{52}\underline{x}_2 & + A_{53}\underline{x}_3 & + A_{54}\underline{x}_4 & + A_{55}\underline{x}_5 & = & \underline{b}_5 \\
\hline
\underline{c}_1^* \underline{x}_1 & - \underline{c}_2^* \underline{x}_2 & + \underline{c}_3^* \underline{x}_3 & - \underline{c}_4^* \underline{x}_4 & + \underline{c}_5^* \underline{x}_5 & = & \text{maximum,}
\end{array}$$

where:

$\underline{x}_1$ = vector of commodity plant production,
$\underline{x}_2$ = vector of foodder production for home consumption,
$\underline{x}_3$ = vector of livstock production,
$\underline{x}_4$ = vector of resources of production /manpower, machinery/,
$\underline{x}_5$ = vector of other variables,
1st line = conditions of area and area-ratio,
2nd line = balance of production and demand of fodder,
3rd line = resources needed for plant production,
4th line = need for other resources,
5th line = other conditions,
$\underline{c}_1$ and $\underline{c}_3$ = difference of monetary income and changing costs,
$\underline{c}_2$ = changing costs of fodder production,
$\underline{c}_4$ = fixed costs,
$\underline{c}_5$ = specific earnings belonging to the other variables.

The improved version of the above model allows the simultaneous determination of the optimal structure and technology of production.
Our lecture explains these models and our experimences gained in the course of their application.

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MATHEMATICAL THEORY OF VON NEUMANN EQUILIBRIA

An extended von Neumann model consists of four matrices $(A_1, A_2; B_1, B_2)$ considered as linear transformations of ordered spaces $(R^n, S) \longrightarrow (R^m, T)$ or /duality/ $(\tilde{R}^m, T^*) \rightarrow (\tilde{R}^n, S^*)$. The ordering cones are supposed to be closed, solid and pointed. The equilibrium of such model is a pair of vectors $x \underset{S}{\geqslant} 0$, $p \underset{T^*}{\geqslant} \theta$ such that there exist $\lambda > 0$ and $\mu > 0$ satisfying

$$\lambda x A_1 \underset{T}{\leqslant} x B_1 \quad ; \qquad B_2 p \underset{S^*}{\leqslant} \mu A_2 p \;;$$

$$\lambda x A_1 p = x B_1 p > 0 \;; \qquad x B_2 p = \mu x A_2 p > 0 \,.$$

If $A_1 = A_2 = A$ and $B_1 = B_2 = B$, then the model is called simple. For a simple model at equilibrium we have $\lambda = \mu$ and the conditions of equilibrium may be written in a simplified form as follows

$$\lambda xA \underset{T}{\leqslant} xB \;, \quad Bp \underset{S^*}{\leqslant} \lambda Ap \;, \quad xBp > 0 \,.$$

If the model is simple, the ordering is coordinate-wise and the matrices are non-negative, then the model is the usual von Neumann model as considered in the classical paper of von Neumann. The economic applications to consumtion, savings and wages problems, international trade, economic regulation, open economies, capital and investment forced many authors to generalize the classical model. For those reasons, the non-negativity of matrices has been postponed, the coordinate-wise ordering has been replaced by arbitrary orderings and each of the matrices was split into two with different economic meaning.

The lecture gives a survey of the results about the existence of equilibria and the structure of possible equilibria levels, i.e. the λ's and μ's at which for a given model an equilibrium exists. It is based mainly on recently published papers and some unpublished results.

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THE STATE OF DEVELOPMENT AND APPLICATION OF NETWORK ANALYSIS IN ČSSR

Considering the problem of application of Network Analysis in various areas of national economy in Czechoslovakia.

In Czechoslovakia there is a group for Network Analysis in the Central Expert Section for Operation Research. Our group joins together experts from all branches of the national economy; it secures their education by with the aid of lectures, courses, seminars, etc.; it secures also the neccessary information for them by the means the Bulletin. Every year in October it organizes an expert meeting at Gottwaldov in which even the participants from abroad take part.

The Network Analysis in Czechoslovakia makes itself steadily more and more useful in the management of our national economy at various levels. The applications are carried out not only in investment arragements, building industry, machinery, maintenance, transport, but also in untraditional branches such as medicine, persueing of transpassers, the state management and the management of the development of science and technology.

The greatest progress has been done in the investment arragement. In this year a new regulation has been worked out for to secure the fulfilling of the tasks of the state plan under the obligation.

This regulation sets a due usage of networks for the preparing of investment arrangements organisation, for the evidence of these constructions, for the contracting with higher suppliers and for the directing of the realization process itself.

Another important field of the network application is the management of the development of science and technology. In the year 1975 a hand-book of methodology was worked out for the complex realisation management of wide and important research tasks. This mentioned handbook of methodology was acknowledged by the COMECON.

But newertheless there is still quite a great gap between the theoretical part on one side and the applied one on the other.

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RATE OF CONVERGENCE OF A CLASS OF METHODS OF FEASIBLE DIRECTIONS IN NONCONVEX PROGRAMMING

Rate of convergence of the Zoutendijk procedure 1 [1, p. 73] and the Zukhovitskii-Polyak-Primak procedure [2] for solving problems of the form

$$\min\{ f_0(x) \mid f_i(x) \le 0,\ i=1,2,\dots,q\}$$

was investigated by Pironneau and Polak [3] under the assumptions that functions $f_0, f_1, \dots, f_q$ are convex and the solution of the problem is a vertex of the constraint set. We are considering these methods without the assumptions mentioned above. It is supposed that the set of cluster points of the process contains at least one such point which satisfies the standard second order sufficiency conditions and strong regularity condition (it is obvious that under such conditions the process would converge to this point).

An example is given which shows that the methods considered do not converge linearly if controlling sequence construction follows the rule proposed in [1], [2]:

$$\delta_{k+1} = \begin{cases} \delta_k, & \text{if } \eta_k > \delta_k \\ \frac{1}{2}\delta_k, & \text{if } \eta_k \le \delta_k, \end{cases}$$

where

$$\eta_k = \max_{\|y\|\le 1} \min\{(-f_i'(x_k), y) \mid i \in I_k\},\quad I_k = \{i=1,2,\dots,q \mid -\delta_k \le f_i(x_k)\} \cup \{0\}.$$

The main result is that the following modification of the controlling sequence construction rule

$$\delta_{k+1} = \begin{cases} \delta_k , & \text{if } \eta_k^2 > \delta_k \\ \frac{1}{2}\delta_k , & \text{if } \eta_k^2 \le \delta_k \end{cases}$$

leads to linear convergence of the process.

The proof of this result is given in [4].

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CONTRIBUTIONS TO PERTURBATION THEORY OF INFINITE NONLINEAR PROGRAMMING PROBLEMS

Let X, Y, Z, A be Banach-spaces and let $f: X \times A \to \mathbb{R}$, $g: X \times A \to Y$, $h: X \times A \to Z$ be Fréchet-differentiable functions. Let M be a subset of X and let K be a convex cone in Z with its vertex at the origin and nonempty interior. We consider the family of nonlinear programming problems indexed by $a \in A$:

$$(P_a) \quad \operatorname*{Inf}_x \{ f(x,a) \mid x \in M,\ g(x,a) = 0,\ h(x,a) \in K \}.$$

The optimal response function $P: A \to \mathbb{R} \cup \{-\infty, \infty\}$ is defined by $P(a) = \operatorname{Inf} \{f(x,a) \mid x \in M,\ g(x,a)=0,\ h(x,a) \in K\}$ for $a \in A$. The problem (P_0) corresponding to $a=0$ is considered as the unperturbed problem. We give estimates of the directional derivatives (resp. Fréchet-derivative) of P at $a=0$ in terms of the Kuhn-Tucker-functionals appearing in the Maximum-Principle for the unperturbed problem (P_0). Let x_0 be a solution point of (P_0). Then, if $l_1 \in Y'$ and $l_2 \in Z'$ are the Kuhn-Tucker-functionals corresponding to equality constraints and inequality constraints, the 'shadow-price-formula'

$$DP(0) = D_a f(x_0,0) + l_1 D_a g(x_0,0) + l_2 D_a h(x_0,0)$$

is shown to hold under suitable assumptions.

Some applications to optimal control problems with perturbations in the dynamics and control or state constraints are given.

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EXPERIENCES IN SOLVING MIXED LINEAR - NONLINEAR PROGRAMS

Solving mixed linear-nonlinear programs can be approached in various ways. This paper presents some computational experience in solving an actual application program and programs generated by a test problem generator. The method used to solve the application program is similar to the method of approximate programming (MAP). The method used to solve the generated programs is the Dantzig-Wolfe generalized programming algorithm.

W. Oettli, Universität Mannheim, Mannheim, W. Germany

THE PRINCIPLE OF FEASIBLE DIRECTIONS FOR CONTINUOUS MINIMAX-PROBLEMS

This talk discusses various aspects of the application of the principle of feasible directions for the solution of continuous minimax-problems

$$\min_{x} (\max_{t} f(x,t))$$

in the case, when the functions $f(\cdot,t)$ involved can be locally approximated by convex functions. Particular attention is given to the possibility of a fixed (pre-assigned) step-length. A modification is described in which only binding constraints (but not ε-binding constraints) are taken into account.

U. Passy, Technion, Haifa, Israel

PSEUDO DUALITY IN MATHEMATICAL PROGRAMMING

Duality plays an important role in both the theory and applications of Mathematical programming. Most of the results orginâted from Fenchel's conjugate correspondence. However, the conjugate correspondence is intimately connected with convex (concave) functions. In the present paper a relation between stationary points of non convex program and of its Legendre transform is investigated.

C. E. Pfefferkorn, Institute for Advanced Computation, Sunnyvale, California, (USA) and J. A. Tomlin, Stanford University, Stanford, California (USA)

DESIGN OF A LINEAR PROGRAMMING SYSTEM FOR THE ILLIAC IV

This paper outlines a design for implementing a linear programming system on the ILLIAC IV computer. The central concern is to take advantage of the special features of the ILLIAC IV (64 parallel processing elements, large fast disk memory and relatively small fast core memory) and at the same time to take advantage of the sparsity of real large-scale linear programs and the (mostly serial) methodology which has been developed to exploit this sparsity. This requires both the adaption of existing techniques to a parallel environment and the development of new parallel techniques for efficient sparse matrix processing. It appears that this can be done successfully and that ILLIAC IV should be able to solve problems considerably larger than those which can be attempted on serial computers.

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NONLINEAR PROGRAMMING METHODS IN THE PRESENCE OF NOISE

The problem under investigation is how one must modify ordinary nonlinear programming methods if the values of all functions and their gradients are noise-corrupted. The modified methods are of some interest for deterministic problems also.

Three methods for solving nonconvex problems with equality constraints are studied: Lagrange multipliers, penalty functions and augmented Lagrangian methods. In the deterministic case the methods include unconstrained minimization at each iteration. In the proposed versions of the methods these auxilary problems are not solved exactly but only one gradient step of minimization is made. It is proved that the algorithms coverge with probability $1-\delta$, δ depending on the initial approximation, on the noise level and on the process parameters.

One of the best known methods for solving convex unsmooth minimization problems over convex sets is the subgradient projection method. It converges if the minimum point is unique. We modify the method by adding a regularizing term for problems with nonunique minimum points. The regularization parameter is changed at each iteration. The mean square convergence of the algorithm to the solution with the minimal norm is proved.

For general stochastic unsmooth convex programming problems the verison of Uzawa's iterative procedure is proposed. It converges without the usual assumptions about strict convexity of the function to be minimized. In particular we presents a simple and stable algorithm for solving linear programming problems and matrix games in the presence of noise.

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REMARKS CONCERNING TEACHING OF LINEAR PROGRAMMING

Giving introductory courses and writing a book on linear programming for mathematician students, the author met many problems the character of which were how to make things mathematically clear, elegant, exact and elementary. The author will shortly describe his own way to present the lexicographic primal, dual methods, their connection, Gomory's algorithm for the all integer problem, the criss-cross method etc. All these are based on already published ideas of primarily Dantzig, Orden, Wolfe, Gale, Tucker, Gomory etc. The author thinks, however, that he may present some new observations.

Stephen M. Robinson, University of Wisconsin-Madison, U.S.A.

REGULARITY AND STABILITY IN OPTIMIZATION AND EQUILIBRIUM PROBLEMS

We consider the problem of nonlinear programming, and certain more general equilibrium problems which can be expressed as variational inequalities. For these problems, we examine the behavior of the solution sets when the functions entering into the problems are perturbed in various ways. This approach yields information about the stability of the solutions which extends the applicability of the current methods (primarily via the implicit-function theorem) to cases in which the solution sets may not be single-valued and the classical implicit-function theorem may not be applicable.

To establish a background for these results, we review some work on regularity of systems of linear and nonlinear inequalities which has been carried out in the last three years. Some simple characterizations of regular systems are now available, and these turn out to have implications, not only for theoretical analysis, but also for computation, particularly with respect to the ways in which constraints are represented in routines for solving optimization problems. We show, for example, that mathematically equivalent ways of writing constraint sets may have very different stability properties, so that they are not at all equivalent for computational purposes.

Finally, we review some practical applications of these results to the problem of roundoff error in computation and to the establishment of convergence rates for algorithms.

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A PROOF OF VAN DER WAERDEN'S CONJECTURE ON DOUBLY STOCHASTIC MATRICES

Let D_n be the set of all doubly stochastic squares matrices of order n i.e. the set of all n x n matrices with nonnegative entries with row and column sums equal to unity. The permanent of an n x n matrix $A=(a_{ij})$ is defined by

$$P(A) = \sum_{\tau \in S_n} \prod_{i=1}^{n} a_{i\,\tau(i)}$$

where S_n is the symmetric group of order n. van der Waerden conjectured that $P(A) \geq n!/n^n$ for all $A \in \mathcal{D}_n$ with equality occuring of and only of $A=E_n$, where E_n is the matrix all of whose entries are equal to 1/n.

While the truth of this conjecture has been shown for a few values of n, the problem is still unsolved. In this paper we give a proof of the validity of the conjecture for general n. The proof follows from a general inequality proved in the paper. The main tool to prove this inequality is through the ideas of 'Signomial Geomtric Programming'.

M. S C H O C H , Bergakademie Freiberg, Freiberg, DDR

EINE MIN-MAX-FIXKOSTENAUFGABE

Gegeben sind die konvexe polyedrische Menge

$$M = \left\{ x \in R^n \mid Ax = a,\ x \geqq 0 \right\},$$

wobei die Matrix A vom Format (m,n) mit $m < n$ ist, und endlich viele reelle Zahlen d_{jv}, $v = 1(1)q_j$, $j = 1(1)n$, für die

$$0 = d_{jo} < d_{j1} < \ldots < d_{jq_j}$$

gilt. Für jedes $j \in N = \{1, 2, .., n\}$ ist durch

$$t_j(x_j) = \begin{cases} t_{jo} & \text{für } x_j = 0 \\ c_{jv}x_j + t_{jv} & \text{für } d_{jv-1} < x_j \leqq d_{jv},\ v = 1(1)q_j, \\ c_{jq_j+1}x_j + t_{jq_j+1} & \text{für } d_{jq_j} < x_j \end{cases}$$

der mit der Variablen x_j verbundene Aufwand gegeben. Dabei setzen wir

$$c_{jv} \geqq 0,\ v = 1(1)q_j,\quad j \in N,$$

und

$$t_{jo} \leqq t_{j1},\ c_{jv}d_{jv} + t_{jv} \leqq c_{jv+1}d_{jv} + t_{jv+1},\ v = 1(1)q_j,\ j \in N,$$

voraus, so daß die Funktionen $t_j(x_j)$ momoton nicht fallend sind. Falls $M \neq \emptyset$ ist, gehören zu jedem $x \in M$ die folgenden eindeutig definierten Indexmengen

$$J_v(x) = \left\{ j \in N \mid d_{jv-1} < x_j \leqq d_{jv} \right\},\ v = 1(1)q = \max_{j \in N} q_j$$

$$J_{q+1}(x) = \left\{ j \in N \mid d_{jq_j} < x_j \right\}.$$

Damit ist für jedes $x \in M$ eindeutig die reelle Zahl

$$f(x) = \max \left[\max_{j \in N} t_{jo},\ \max_{j \in J_{q+1}(x)} (c_{jq_j+1}x_j + t_{jq_j+1}),\ \max_{v=1(1)q} \max_{j \in J_v(x)} (c_{jv}x_j + t_{jv}) \right]$$

definiert. Wir betrachten die folgende Extremalaufgabe

$$f(x^{(o)}) = \min (f(x) \mid x \in M), \qquad (\bar{P})$$

durch die der mit jeweils einer Variablen verbundene Maximalaufwand minimiert wird.

Aufgaben diesen Typs treten beispielsweise in der Netzplantechnik auf, wenn der maximale Ressourcenbedarf minimiert werden soll [3]. Das allgemein bekannte Zeitransportproblem erweist sich als Spezialfall von ($\bar{P}$) mit speziell strukturierter Matrix A und mit $q_j = 1$, $j \in N$, und $c_{j1} = 0$, $t_{jo} = 0$, $j \in N$. In [1] und [4] werden Spezialfälle von ($\bar{P}$) untersucht, während in [2] ausführliche Betrachtungen und ein Algorithmus für ($\bar{P}$) enthalten sind.

Es werden endlich viele spezielle Elemente von M, die sogenannten Sprungpunkte von f(x) auf M definiert. Mit diesen gilt dann die folgende Aussage.

<u>Satz:</u> Ist $M \neq \emptyset$, dann besitzt ($\bar{P}$) stets eine Optimallösung, und der Optimalwert $f(x^{(o)})$ wird in wenigstens einem Sprungpunkt von f(x) auf M angenommen.

Mit

$$S = \{ x \in M \mid x \text{ Sprungpunkt von } f(x) \text{ auf } M \}$$

kann somit die Aufgabe ($\bar{P}$) durch die kombinatorische Optimierungsaufgabe

$$f(x^{(o)}) = \min (f(x) \mid x \in S) \qquad (P)$$

ersetzt werden.

Für (P) läßt sich auf der Basis des Erweiterungsprinzips [2] ein Algorithmus entwickeln, der in endlich vielen Schnitten entweder eine Optimallösung liefert oder $S = \emptyset$ anzeigt. Dazu wird eine Minorantenfunktion

$$g(x) = \max\left[\max_{j\in N} t_{jo},\ \max_{j\in J_{q+1}(x)} t^{*}_{jq_j+1},\ \max_{v = 1(1)q}\ \max_{j\in J_v(x)} t^{*}_{jv}\right]$$

mit

$$t^{*}_{jv} = c_{jv}d_{jv-1} + t_{jv},\quad v = 1(1)q_j+1,\quad j \in N.$$

gebildet und die Ersatzaufgabe

$$g(x) = \min\ (g(x)\ |\ x\quad S) \qquad (P_E)$$

betrachtet. Zur Lösung von (P_E) ist eine endliche Folge linearer Optimierungsaufgaben zu betrachten. Das im Fall $S \neq \emptyset$ dabei zuerst erhaltene $x \in S$ ist zugleich Optimallösung von (P_E).

Durch Ausnutzen der Aussagen von vier weiteren Sätzen kann dann die Fortsetzung der Bearbeitung von (P) ebenfalls über eine Folge linearer Optimierungsaufgaben erfolgen.

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SIMPLE RESULTS ON COMPACT ACTS IMPLYING SOLVABILITY OF COMPACT DYNAMIC OPTIMIZATION PROBLEMS

In multistage optimization, the feasible region A_k applying at a given period k may change endogenously, as a joint function of the feasible set for that period and the feasible point $x_k \varepsilon A_k$ chosen within it, thus determining the next feasible region A_{k+1}. When all feasible regions A_k belong to the collection A of all nonempty closed subsets of a compact topological semigroup $(X,\circ)$ and the next-feasibility map is given by a continous $\delta : X \times A \to A$ with $\delta(x',\delta(x,A)) = \delta(x'\circ x, A)$, i.e., δ is a left-act, we show that the set of feasible sequences (i.e., the set of sequences $\{x_k\}_{k=1}^{\infty}$ with $x_k \varepsilon A_k$ for k = 1, 2, ...) is compact. This implies the existence of solutions to a wide class of dynamic optimization problems indicated in the text, and extends earlier results of the author [JOTA, 1976]

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SCALED VERSUS UNSCALED VARIABLE METRIC ALGORITHMS : A COMPUTATIONAL EXPERIENCE

Classes of parameter dependent variable metric algorithms for unconstrained nonlinear minimization have been introduced by many authors. Here we are particularly concerned with Broyden one parameter class, to which the well known Davidon-Fletcher-Powell or DFP and Broyden-Fletcher-Shanno-Goldfarb algorithms belong, and with two and three parameter classes introduced by Oren and by Spedicato. As most of the experiments reported in the literature concern functions in few variables, usually less than ten, a set of n-dependent functions has been introduced in order to test the performance of the Broyden-Fletcher-Shanno-Goldfarb methods and of the Oren-Spedicato method for problems of up to 80 variables. A conjecture of Dixon relating to some of the test functions is also explored and conditions are given for its validity. The numerical results show that the first method is preferable for small n while for large n or n greater than 20 the second method is usually better. They confirm moreover that prescaling the initial matrix is usually very useful.

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ON OPTIMISATION OF ADAPTIV NUMERICAL ALGORITHMS

Let X be a separable Banach space, K a convex, compact set in X, Φ a centralsymmetrical, weakly compact subset of the unit sphere of the dual space X, of X. Let f be a linear continuous operator from X to another Banach space, Y. We are interested in getting an optimal estimate of f(x), constructed from the values "of admissible measurements":

$$\varphi_n(x), \quad n=1,\dots, N, \quad \varphi_n \in \Phi, \quad x \in K$$

where N is a fixed natural number and the linear functionals, φ_n are chosen by an adaptiv algorithm

$$\varphi_{n+1} = A_{n+1}(\varphi_j, \varphi_j(x), 1 \le j \le n, N), \quad 0 \le n \le N-1$$

Let

$$C^N = \{c_1, \dots, c_N\}, \quad \varphi^N = \{\varphi_1, \dots, \varphi_N\}$$

$$K(\varphi^N, C^N) = \{x \mid x \in K, \ \varphi_j(x) = c_j, \ j=1,\dots,N\} = K(N)$$

$$\varepsilon(K(N)) = \inf_{z_0 \in X} \sup_{z \in K(N)} \| f(z) - f(z_0) \|_Y = \varepsilon(A^N, x)$$

Our optimisation problem consist in finding (by dynamic programming)

$$\inf_{A^N} \sup_{x \in K} \varepsilon(A^N, x) = \varepsilon(N, K, f)$$

and the corresponding optimal algorithms $\{A_n^*(\cdot)\} = A^{*N}$, (if it exists). We investigate the differences between optimal passiv, (when $\{\varphi_n, n=1,\dots N\}$, are chosen in one step), and optimal adaptiv algorithms. Conditions given in terms of the geometric structure of K (e.g. its decomposability into a <u>sum</u> of two subsets "similar" to K), when there exists an (asymptotic) adaptiv algorithm yielding the same accuracy, over K, as the optimal N step algorithms, for all $N = 2^k - 1$, $k = 1, 2, \dots$. Concrete examples include cases, when K, Φ, f are given as

$$K = \{ x(t): R^n \to R^1, \| \mathrm{grad}_t \, x(t) \|_{L_p} \le M, p = 2, \infty, \ (m \le \dot{x}(t) \le M)$$

$$\text{or when } \operatorname{Spectrum}\left(\frac{\partial^2 x(t)}{\partial t^i \partial t^j}\right) \in [m, M], \quad 0 \le m < M < \infty \}$$

$$\Phi = \{ \varphi \mid \varphi(x) = x(t), \ t = t(\varphi) \in S \text{ compact in } R^n \}$$

$$f(x) = \int_S x(t)\,dt, \ \text{(quadratura)}; \quad f(x) = x, \ \text{(approximation)}.$$

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ALGEBRAIC CHARACTERIZATIONS OF UNIMODULAR AND TOTALLY UNIMODULAR MATRICES

A unifying framework for algebraic characterizations of totally unimodular matrices (i.e., matrices where each square submatrix has determinant 0 or ± 1) is proposed. Three fundamental properties of certain all-integer matrices are shown to give rise to all known characterizations, which are classified into two types according to the type of submatrix specified in each characterization. This approach leads to a sharpening of several known results and yields new characterizations.

Based on the above mentioned results for totally unimodular matrices, several useful algebraic characterizations of unimodular matrices (i.e., matrices where each basis has determinant ± 1) are developed. Essential for these characterizations is a classification of integer matrices as unimodular, non-trivially non-unimodular, trivially non-unimodular based on a simple algebraic condition. Failure of previous attempts to characterize unimodular matrices is shown to be caused by trivially non-unimodular matrices.

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ON NONDIFFERENTIABLE OPTIMIZATION

There are many problems of mathematical programming requiring the minimization of functions which are not everywhere differentiable, and many more which may be, but do not have to be, posed that way. This paper will (i) describe some of these problems; (ii) cite the mathematical background necessary in the study of algorithms for solving them; (iii) review the ideas, and what has so far been learned about the performance, of the principal solution methods that have been so far proposed.

76/4010 MTESZ HNy.-Bp.