2005 GREAT LAKES GEOMETRY CONFERENCE
ABSTRACTS

MICHAEL ANDERSON, SUNY at Stony Brook
"Conformal compactification in Riemannian geometry and general relativity"

Conformal compactifications were introduced by Penrose to study the asymptotic behavior of vacuum spacetimes in general relativity and have since been extensively studied by H. Friedrich, Christodoulou, Klainerman and many others. This idea and related renormalization procedures have also arisen recently in the AdS/CFT correspondence relating gravity in the bulk with gauge theory on the boundary in physics. We will introduce the well-known Fefferman-Graham obstruction tensor and show how it can be used, quite easily, to study conformal compactifications of the vacuum Einstein equations, for all values of the cosmological constant. This gives new and simple proofs of results of H. Friedrich in 3+1 dimensions, and generalizes these results to all even dimensions.

KAI BEHREND, University of British Columbia
"Towards an understanding of Donaldson-Thomas invariants
in terms of Euler characteristics"

Donaldson-Thomas invariants have recently attracted the attention of mathematicians interested in Gromov-Witten theory, because of a new conjecture (the MNOP-conjecture) relating the two theories. Donaldson-Thomas invariants are special, because the corresponding moduli spaces are sets of critical points of certain Chern-Simons type functionals. Because of this, they should be related to Euler characteristics of the corresponding moduli spaces. We explain a conjectural approach to define within algebraic geometry certain Euler characteristics, which are equal to Donaldson-Thomas invariants. As an application, we obtain (conjecturally) Donaldson-Thomas type invariants for non-compact moduli spaces.

JOHN ETNYRE, University of Pennsylvania
"Cusped flow trees and contact homology"

Relative contact homology is a powerful invariant of Legendrian submanifolds in contact manifolds. Recently it has been used to define invariants of embeddings of manifolds in Euclidean space. I will discuss how these invariants are defined and indicate the difficulty in computing them. After that I will discuss an approach that should allow one to compute these invariants. In particular it seems likely this approach can be used to show Ng's new invariants of knots (that is, 1-manifolds in $\mathbb{R}^3$) are precisely these contact homology invariants.

TATYANA FOTH, University of Western Ontario
"Toeplitz operators, automorphic forms, and quantization"

The talk will consist of two parts. The first one is about holomorphic automorphic forms associated to certain Lagrangian tori in compact quotients of the n-dimensional complex hyperbolic space $SU(n,1)/S(U(n) \times U(1))$. In the second part of the talk I will discuss a natural connection in a certain vector bundle over a manifold parametrizing complex structures compatible with a fixed symplectic form on a fixed compact manifold M. In the case when M is a compact surface of genus $g > 1$, holomorphic automorphic forms of weight 4 appear naturally in calculations. Both problems involve discussion of the semi-classical limit.

SERGEI GUKOV, Harvard University
"The Superpolynomial for Knot Homologies"

We start with a brief introduction into knot homology theories and categorification of polynomial knot invariants. In particular, we review the construction and the basic properties of a new homological knot invariant, recently introduced by Khovanov and Rozansky. To every knot diagram, it associates a bigraded chain complex whose graded Euler characteristic is the quantum-group $sl(N)$ invariant. Motivated by the ideas from physics, we then present a reformulation of the $sl(N)$ knot homology in terms of new triply-graded knot invariants. This leads to new conjectures on the structure of the homological knot invariants and suggests a larger theory which unifies the HOMFLY polynomial, the Khovanov-Rozansky homology, and the knot Floer homology of Ozsvath-Szabo-Rasmussen. We also describe the geometric meaning of the new knot invariants in terms of the enumerative geometry of Riemann surfaces with boundaries in a certain Calabi-Yau three-fold.

FRANÇOIS LALONDE, Université de Montréal
"Hamiltonian dynamics and a universal Floer homology"

Floer homology is the main tool of symplectic topology (and the main ingredient on one of the two sides of mirror symmetry). Floer homology is also a powerful tool for deep questions of Hamiltonian dynamics and geometric group theory. I will explain how a natural universal Floer theory can be defined and briefly describe some of its applications.

JOHN LOTT, University of Michigan
"Ricci curvature for metric-measure spaces"

I will describe a notion of a measured length space having a lower Ricci curvature bound. The definition is in terms of the transport of measures on the length space. The lower Ricci bounds are preserved under taking measured Gromov-Hausdorff limits. Various consequences are given, such as Bishop-Gromov inequalities, log Sobolev inequalities and Poincaré inequalities. This is joint work with Cedric Villani.

ECKHARD MEINRENKEN, University of Toronto
"Small models and twisted differentials"

In this talk we will discuss the geometric interpretation of various 'twisted differentials', arising in twisted K-theory, the theory of loop groups, and equivariant cohomology.

ANDRÁS STIPSICZ, Hungarian Academy of Sciences
"Exotic smooth structures on rational surfaces"

Most known smoothable simply connected 4–manifolds admit infinitely many different smooth structures (distinguished, for example, by Seiberg–Witten invariants). There are some 4–manifolds, though, for which the existence of such 'exotic' structures is still open, the most notable examples being the 4–dimensional sphere S^4 and the complex projective plane $\mathbb{CP}^2$. In a recent project with Z. Szabó and J. Park we found constructions of exotic smooth structures on the five- and six-fold blow-up of $\mathbb{CP}^2$. In the lecture we describe the construction of these 4–manifolds and indicate the necessary input from Seiberg–Witten theory for proving their exoticness.