Newton's Law of Cooling

Created by

Barbara Forrest and Brian Forrest

Newton’s Law of Cooling

Newton’s Law of Cooling: Newton’s law of cooling states that an object will cool (or warm) at a rate that is proportional to the difference between the temperature of the object and the ambient temperature T_a of its surroundings. Therefore, if we let $T(t)$ denote the temperature of an object at time t, we get that there is a constant k such that

$$T' = k(T - T_a)$$

Newton's Law of Cooling

Newton's Law of Cooling Cont'd: If we let $D = D(t) = T(t) - T_a$, then

$$D' = T' = k(T - T_a) = kD$$

so D satisfies the equation of exponential growth (or decay).

We know that

$$D = Ce^{kt}$$

and follows that

$$T(t) = Ce^{kt} + T_a$$

where $C = D(0) = T_0 - T_a$ and $T_0 = T(0)$.

Therefore,

$$T(t) = (T_0 - T_a)e^{kt} + T_a.$$

Newton’s Law of Cooling

Newton’s Law of Cooling Cont’d: With

$$T(t) = (T_0 - T_a)e^{kt} + T_a$$

there are three cases:

Case 1: $T_0 > T_a$

Since $T(t)$ is decreasing

$$T' = k(T - T_a) < 0.$$

However, $T > T_a$, so that $k < 0$.

Newton's Law of Cooling

Newton's Law of Cooling Cont'd:

Case 2: $T_0 < T_a$

Since $T(t)$ is increasing

$$T' = k(T - T_a) > 0.$$

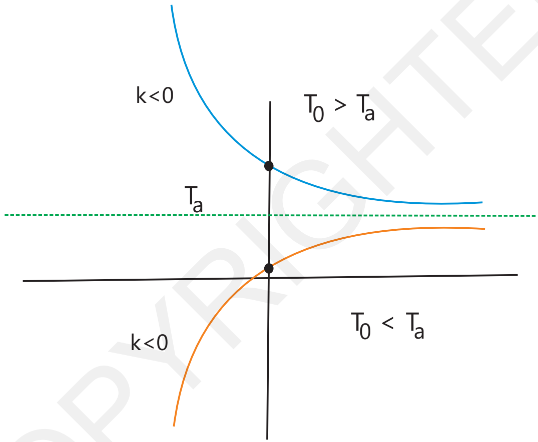
However, $T < T_a$, so that $k < 0$.

Case 3: $T_0 = T_a$

In this case

$$T(t) = T_a$$

Newton's Law of Cooling



Newton's Law of Cooling

$$T(t)=(T_0-T_a)e^{kt}+T_a$$

Newton's Law of Cooling

Problem: A cup of boiling water at $100^\circ C$ is allowed to cool in a room where the ambient temperature is $20^\circ C$. If after 10 minutes the water has cooled to $70^\circ C$, what will be the temperature after the water has cooled for 25 minutes.

Solution: We have $T_0 = 100$ and the ambient temperature is $T_a = 20$ so that

$$\begin{aligned} T(t) &= (T_0 - T_a)e^{kt} + T_a \\ &= (100 - 20)e^{kt} + 20 \\ &= 80e^{kt} + 20 \end{aligned}$$

To find k observe that

$$70 = T(10) = 80e^{k(10)} + 20$$

and hence

$$50 = 80e^{10k} \Rightarrow k = \frac{\ln(\frac{5}{8})}{10}$$

Finally

$$T(25) = 80e^{\frac{\ln(\frac{5}{8})}{10}(25)} + 20 \cong 44.71$$