Displacement versus Velocity

Created by

Barbara Forrest and Brian Forrest

Displacement versus Velocity

Recall:

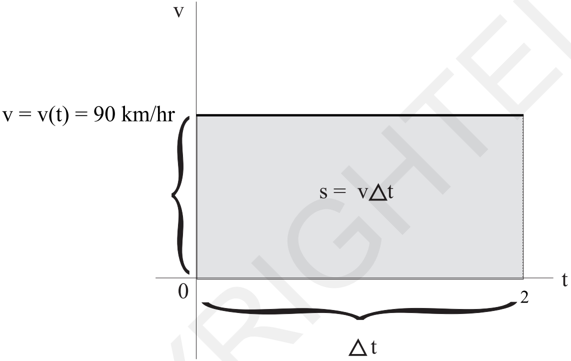
We know that if $s(t)$ represents the displacement of an object and $v(t)$ represents its velocity, then

$$\frac{ds}{dt} = s'(t) = v(t)$$

Problem:

Suppose that a car travels for two hours along a highway. The odometer is broken but the speedometer is working. How can we determine the distance traveled from only the data about the car's velocity?

Displacement versus Velocity



Step 1: Suppose that we always travel forward on the highway at a *constant* velocity of 90 km/hr. If $s = displacement$, $v = velocity$, and $\Delta t = time\ elapsed$, then

$$s = v\Delta t$$

Key Observation: In this case, displacement equals the area under the graph of v from $t = 0$ to $t = 2$.

Displacement versus Velocity

Step 2: Divide the 2 hour duration of the trip into 120 one minute intervals

$$0 = t_0 < t_1 < t_2 < t_3 < \cdots < t_{i-1} < t_i < \cdots < t_{120} = 2 \text{ hours}$$

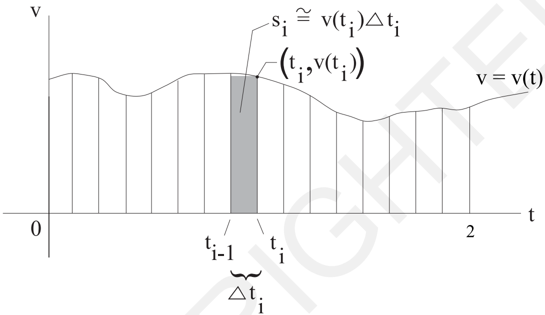
so that $t_i = \frac{2i}{120} = \frac{i}{60}$ hours.

Let s_i be the displacement (distance traveled) during time t_{i-1} until t_i, the i-th minute of the trip.

If s is the total displacement, we have

$$\begin{aligned} s &= s_1 + s_2 + s_3 + \cdots + s_i + \cdots + s_{120} \\ &= \sum_{i=1}^{120} s_i \end{aligned}$$

Displacement versus Velocity



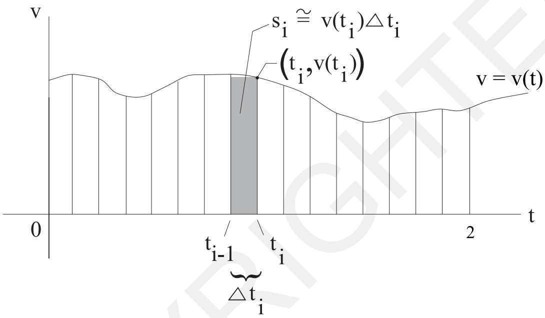
Step 2 (continued): If we assume that the velocity does not vary much over any one minute period, we can assume that the velocity during the interval $[t_{i-1}, t_i]$ was the same as it was at t_i so that

$$s_i \cong v(t_i)\Delta t_i = v(t_i)\frac{2}{120}$$

Then we have the estimate for s:

$$s = \sum_{i=1}^{120} s_i \cong \sum_{i=1}^{120} v(t_i)\Delta t_i = \sum_{i=1}^{120} v(t_i)\frac{2}{120}$$

Displacement versus Velocity



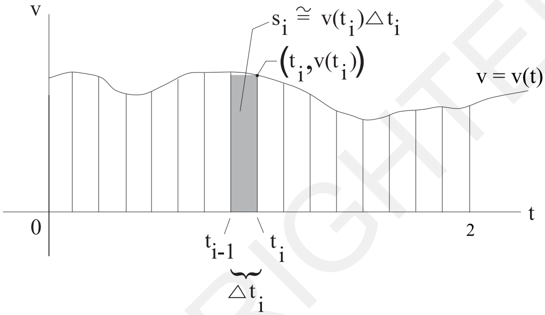
Step 3: Divide the two hour period into 7200 subintervals each of length 1 second to get

$$s_i \cong v(t_i)\Delta t_i = v(t_i)\frac{2}{7200}$$

and

$$s = \sum_{i=1}^{7200} s_i \cong \sum_{i=1}^{7200} v(t_i)\Delta t_i = \sum_{i=1}^{7200} v(t_i)\frac{2}{7200}$$

Displacement versus Velocity



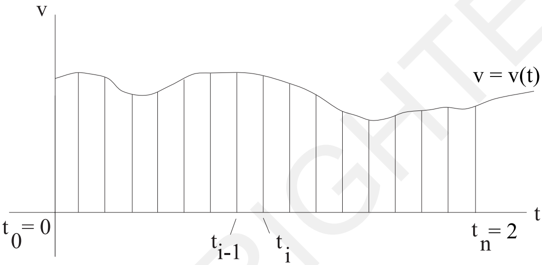
Step 4: Divide the two hour period into n subintervals each of length $\frac{2}{n}$ hours with $t_i = \frac{2i}{n}$ to get

$$s_i \cong v(t_i)\Delta t_i = v(t_i)\frac{2}{n}$$

and

$$s = \sum_{i=1}^{n} s_i \cong S_n = \sum_{i=1}^{n} v(t_i)\Delta t_i = \sum_{i=1}^{n} v(t_i)\frac{2}{n}.$$

Displacement versus Velocity



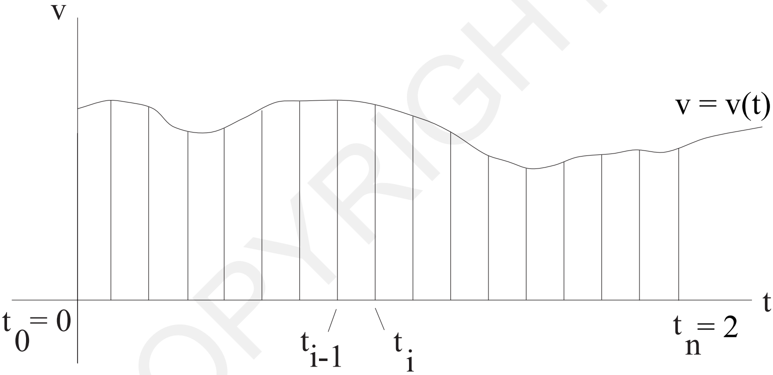
Step 5: With

$$S_n = \sum_{i=1}^{n} v(t_i)\Delta t_i = \sum_{i=1}^{n} v(t_i)\frac{2}{n}$$

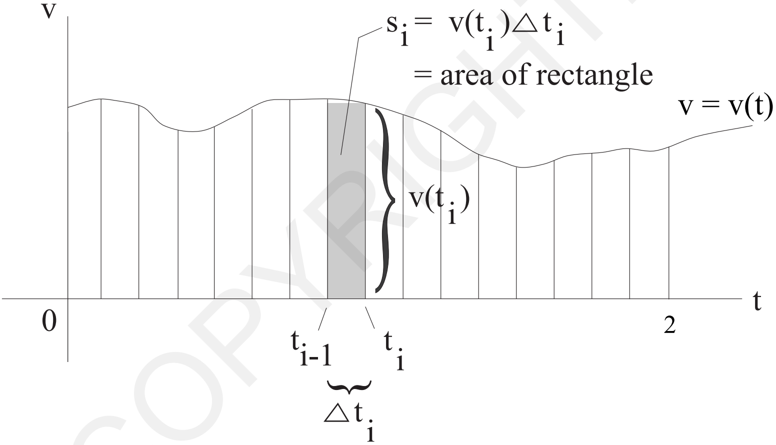
it can be shown that the sequence $\{S_n\}$ converges and that

$$\lim_{n\to\infty}\{S_n\} = s.$$

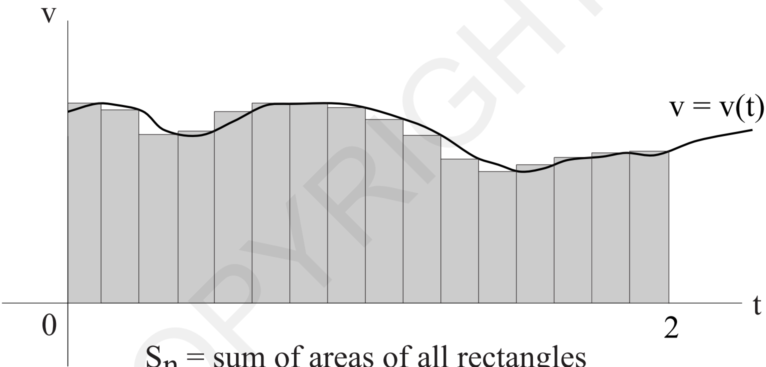
Geometric Interpretation: Displacement versus Velocity



Geometric Interpretation: Displacement versus Velocity

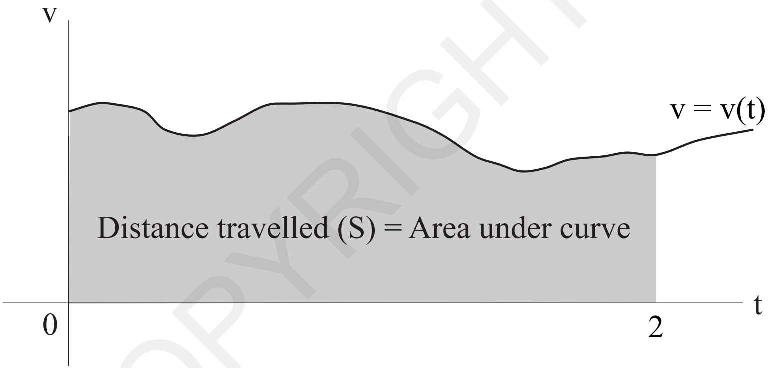


Geometric Interpretation: Displacement versus Velocity



S_n = sum of areas of all rectangles

Geometric Interpretation: Displacement versus Velocity



Conclusion: Displacement is the area under the velocity graph!