Instantaneous Velocity

Created by

Barbara Forrest and Brian Forrest

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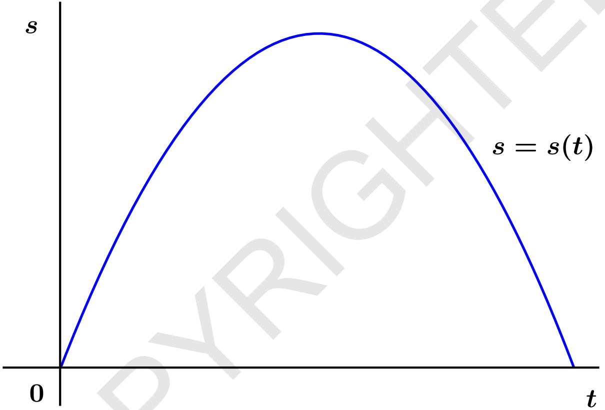
Instantaneous Velocity



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Problem:

A stone is thrown straight upward in the air and eventually falls back to the ground. How can we define the *instantaneous velocity* of the stone at any given time?

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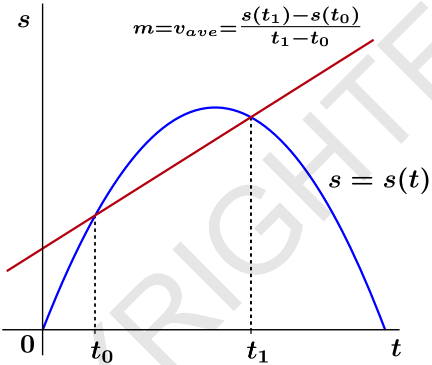

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Recall: The *average velocity* of the stone relative to the ground over the period from time $t = t_0$ to $t = t_1$ is given by the formula

$$V_{\text{ave}} = \frac{\text{displacement (change in position)}}{\text{elapsed time}} = \frac{s(t_1) - s(t_0)}{t_1 - t_0} = \frac{\triangle s}{\triangle t}$$

where

$$\triangle s = s(t_1) - s(t_0) \text{ and } \triangle t = t_1 - t_0.$$

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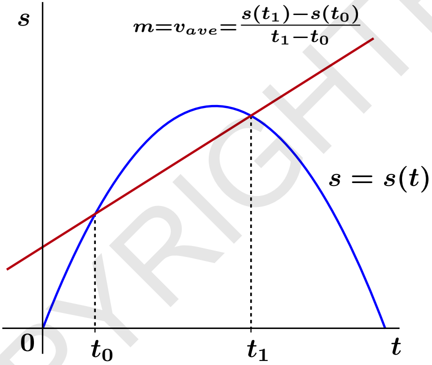

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Geometric Interpretation: V_{ave} is the slope m of the "secant line" to the graph of $s(t)$ through the points $(t_0, s(t_0))$ and $(t_1, s(t_1))$.

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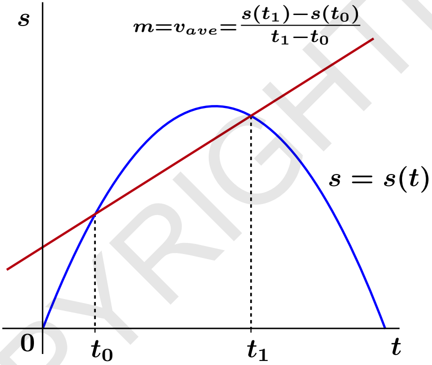

Question: How do we define instantaneous velocity at a point t_0?

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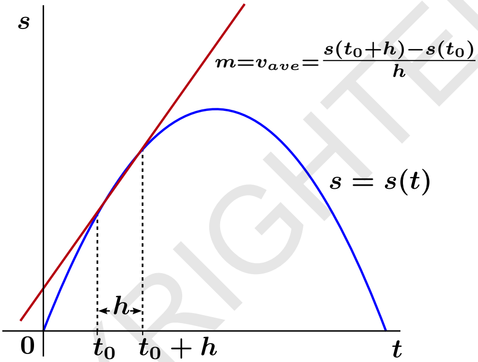

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Key Assumption: The velocity of the stone should not vary too much over very small intervals of time. Therefore, if h is small

$$
\begin{aligned}
v(t_0) &\cong v_{\text{ave}} \\
&= \frac{s(t_0+h)-s(t_0)}{(t_0+h)-t_0} \\
&= \frac{s(t_0+h)-s(t_0)}{h}
\end{aligned}
$$

Instantaneous Velocity

Definition: [Instantaneous Velocity]

The *instantaneous velocity* of an object at time t_0 is given by

$$v(t_0) = \lim_{h \to 0} \frac{s(t_0 + h) - s(t_0)}{h}$$

provided this limit exists.

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