Derivatives of Exponential Functions

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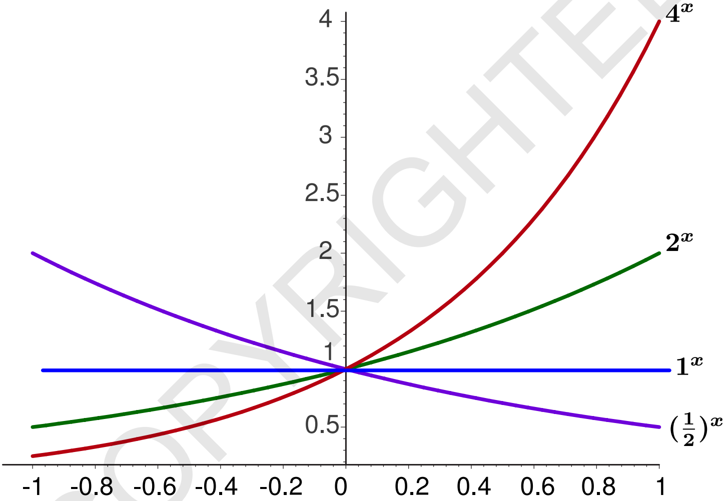
Derivative of a^x



Problem: If $f(x) = a^x$ where $a > 0$, what is $f'(x)$?

Derivative of a^x

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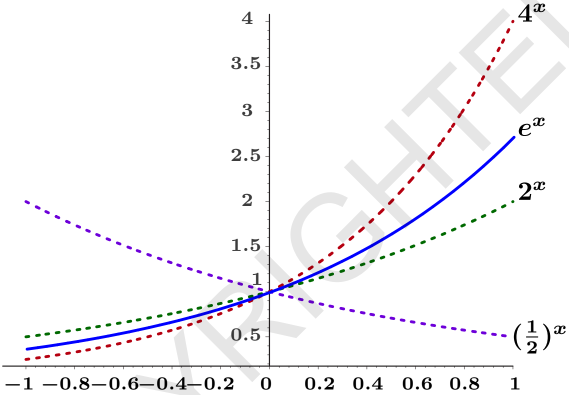
$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} \\ &= \lim_{h\to 0} \frac{a^x(a^h - 1)}{h} \\ &= a^x \cdot \left(\lim_{h\to 0} \frac{a^h - 1}{h} \right) \\ &= a^x \cdot f'(0). \end{aligned}$$

Derivative of a^x



Definition: [Euler's Constant e]

Let $f(x) = a^x$. Of all the possible choices for $a > 0$, e is the unique base such that if $f(x) = e^x$, then $f'(0) = 1$.

$$2 < e < 4 \Rightarrow e \cong 2.718281828$$

Derivative of a^x

Summary: Let $a > 0$. Then:

1) If $f(x) = a^x$, then $f'(x) = f'(0) \cdot a^x$.
2) If $f(x) = e^x$, then $f'(x) = e^x$.

Problem: If $f(x) = a^x$, and $f'(x) = f'(0) \cdot a^x$, what is $f'(0)$?

Answer: $\ln(a)$

Derivative of a^x

Theorem: [Derivative of a^x]

Let $a > 0$. If $f(x) = a^x$, then

$$f'(x) = \ln(a) \cdot a^x$$

for all $x \in \mathbb{R}$.

Derivative of e^x

Special Case: Since $\ln(e) = 1$, we have:

Theorem: [Derivative of e^x]

Let $f(x) = e^x$. Then

$$f'(x) = e^x$$

for all $x \in \mathbb{R}$.