Continuity

Created by

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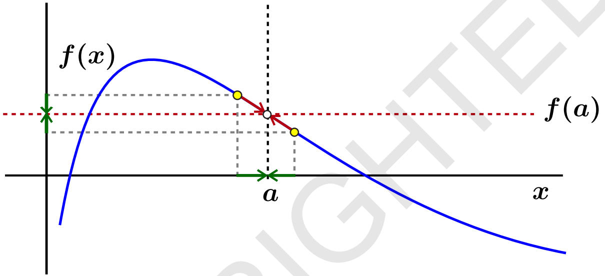
Definition of Continuity



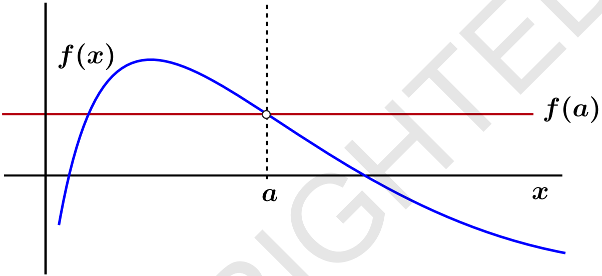
Definition: [Continuity]

We say that $f(x)$ is continuous at $x = a$ if

1) $\lim_{x \to a} f(x)$ exists.
2) $\lim_{x \to a} f(x) = f(a)$.

Otherwise, we say that $f(x)$ is *discontinuous* at a or that a is a *point of discontinuity* for $f(x)$.

Definition of Continuity



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1) $\lim_{x \to a} f(x)$ exists.

2) $\lim_{x \to a} f(x) = f(a)$.

Otherwise, we say that $f(x)$ is *discontinuous* at a or that a is a *point of discontinuity* for $f(x)$.

Note: $2) \Rightarrow 1)$.

Definition of Continuity

Observe: The following are equivalent:

1) $\lim_{x \to a} f(x) = f(a)$.

2) For every $\epsilon > 0$, there exists a $\delta > 0$ such that $0 <| x - a |< \delta \Rightarrow| f(x) - f(a) |< \epsilon$.

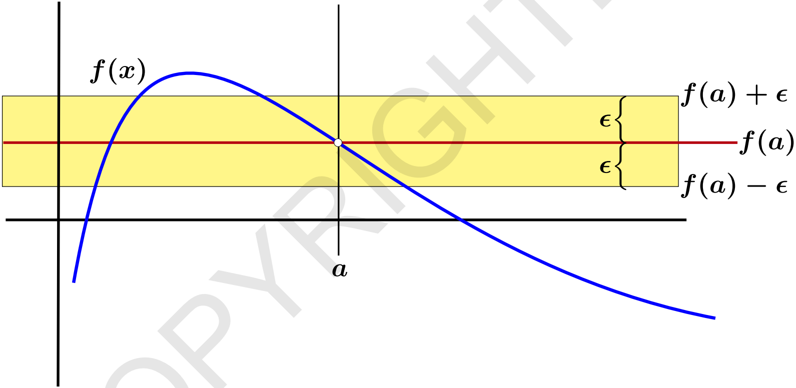
3) For every $\epsilon > 0$, there exists a $\delta > 0$ such that $| x - a |< \delta \Rightarrow| f(x) - f(a) |< \epsilon$.

Alternate Definition: [Continuity]

We say that $f(x)$ is continuous at $x = a$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that if $| x - a |< \delta$, then

$$| f(x) - f(a) |< \epsilon.$$

Definition of Continuity



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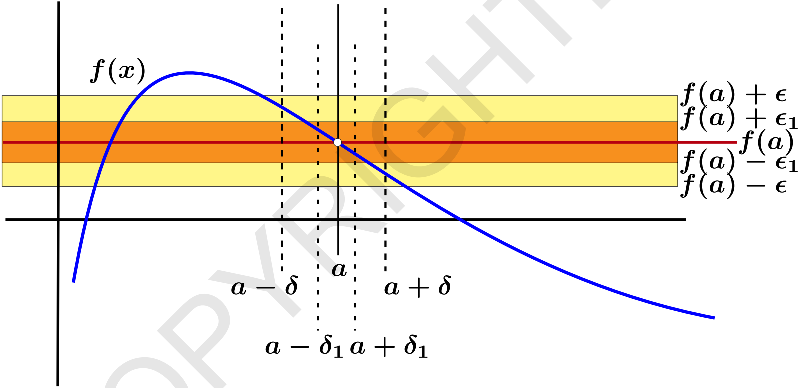

Sequential Characterization of Continuity

Recall: The following are equivalent:

1) $\lim_{x \to a} f(x) = L$.

2) If $\{x_n\}$ is a sequence with $x_n \to a$ and $x_n \neq a$, then $\lim_{n \to \infty} f(x_n) = L$.

Theorem: [Sequential Characterization of Continuity]

The following are equivalent:

1) $f(x)$ is continuous at $x = a$.
2) If $\{x_n\}$ is a sequence with $x_n \to a$, then $\lim_{n \to \infty} f(x_n) = f(a)$.