Continuity on an Interval

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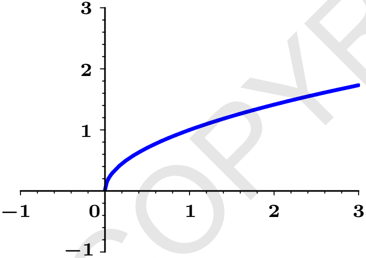
Continuity on an Open Interval

Definition: [Continuity on an Open Interval]

We say that $f(x)$ is continuous on the open interval I if it is continuous at every $a \in I$.

Problem: How does this work for closed intervals?



Is $f(x) = \sqrt{x}$ continuous at 0?

Answer: $\lim_{x \to 0} \sqrt{x}$ does not exist.

One-sided Continuity

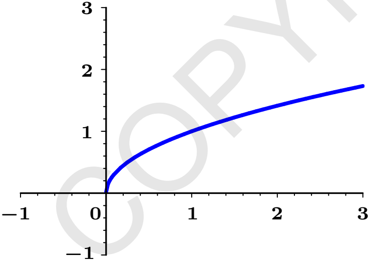
Definition: [One-sided Continuity]

We say that $f(x)$ is continuous from the left at $x = a$ if

$$\lim_{x \to a^-} f(x) = f(a).$$

We say that $f(x)$ is continuous from the right at $x = a$ if

$$\lim_{x \to a^+} f(x) = f(a).$$



Example: $f(x) = \sqrt{x}$ is continuous from the right at 0.

Continuity on a Closed Interval

Definition: [Continuity on a Closed interval]

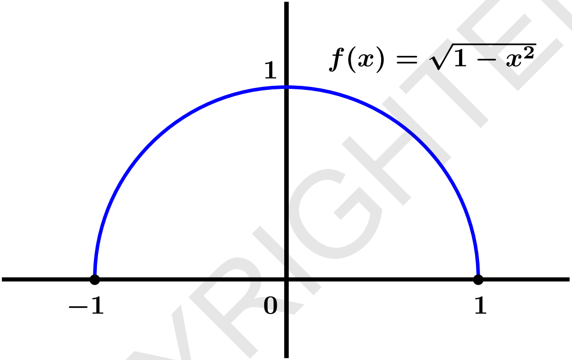
We say that $f(x)$ is continuous on the closed interval $[a, b]$ if:

1) It is continuous on (a, b). That is $f(x)$ is continuous at every $x \in (a, b)$ in the usual sense.

2) $\lim_{x \to a^+} f(x) = f(a)$.

3) $\lim_{x \to b^-} f(x) = f(b)$.

Continuity on a Closed Interval



Example: Let $f(x) = \sqrt{1-x^2}$.

We have that

$$f(-1) = \lim_{x \to -1^+} \sqrt{1-x^2} = 0 = \lim_{x \to 1^-} \sqrt{1-x^2} = f(1)$$

so $f(x)$ is continuous on $[-1, 1]$.

Continuity on an Interval

Observation: The following are equivalent:

1) $f(x)$ is continuous an interval I.

2) If $\{x_n\}$ is a sequence in I with $x_n \to x_0 \in I$, then

$$f(x_n) \to f(x_0).$$