Squeeze Theorem for Sequences

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Example

Example: Consider the sequence $\{\frac{\sin(n)}{n}\}$.

Does it converge? If so, what does it converge to?

Note that $|\sin(n)| \leq 1$ for all n, so

$$\left|\frac{\sin(n)}{n}\right| \leq \frac{1}{n}.$$

Then

$$-\frac{1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}.$$

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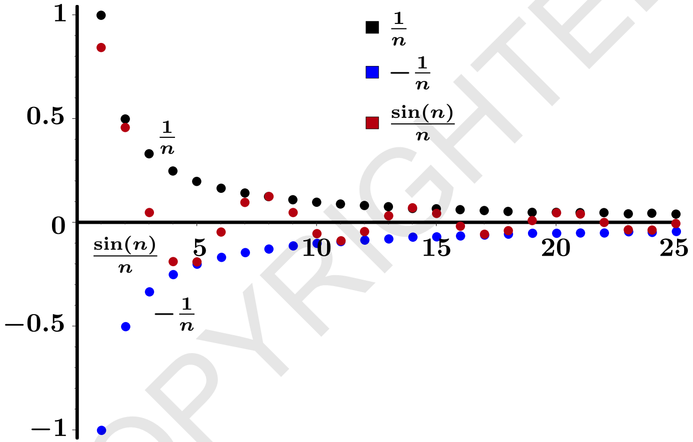
Example (continued)



$$\lim_{n\to\infty} \frac{1}{n} = 0 = \lim_{n\to\infty} -\frac{1}{n} \Rightarrow \lim_{n\to\infty} \frac{\sin(n)}{n} = 0.$$

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Squeeze Theorem for Sequences

Theorem: [Squeeze Theorem for Sequences]

Assume that $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ are such that

$$c_n \leq b_n \leq a_n$$

for all $n \in \mathbb{N}$.

Assume also that

$$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n.$$

Then $\lim_{n\to\infty} b_n$ exists and

$$\lim_{n\to\infty} b_n = L.$$

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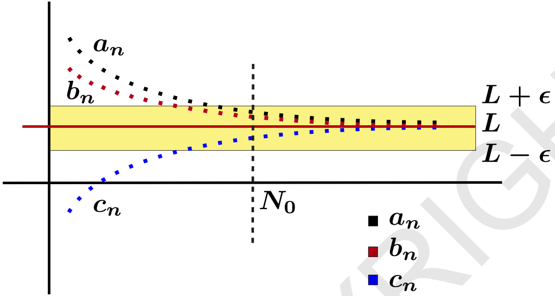
Proof of Squeeze Theorem for Sequences



Proof:

Consider the sequences $\{a_n\}$ and $\{c_n\}$.

Assume that

$$\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} c_n.$$

Let $\epsilon > 0$.

We can find an N_0 such that if $n \geq N_0$, then

$$L - \epsilon < c_n \leq a_n < L + \epsilon.$$

Therefore, if $n \geq N_0$

$$L - \epsilon < c_n \leq b_n \leq a_n < L + \epsilon \Rightarrow | b_n - L | < \epsilon.$$

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