Limits of Sequences I

Created by

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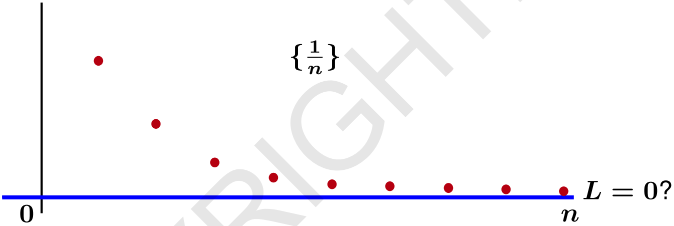
What is the Limit of a Sequence?

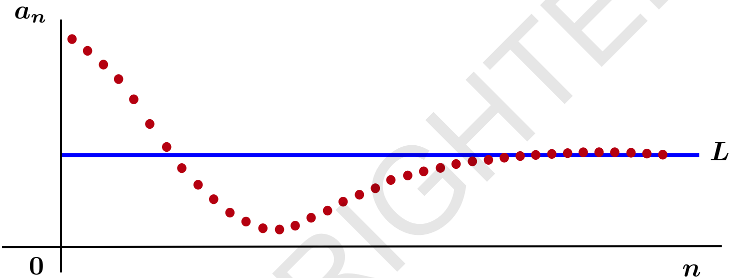


Consider $\{\frac{1}{n}\}$.

As n **gets larger and larger**, the terms get **closer and closer to** 0.

We want to call 0 *the limit of the sequence* $\{\frac{1}{n}\}$ as n goes to ∞.

Heuristic Definition of a Limit of a Sequence

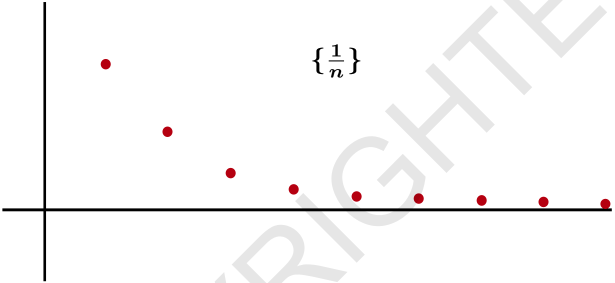


Heuristic Definition:

We say that L is the *limit of the sequence* $\{a_n\}$ as n goes to ∞ if as n **gets larger and larger** the terms of $\{a_n\}$ get **closer and closer** to L.

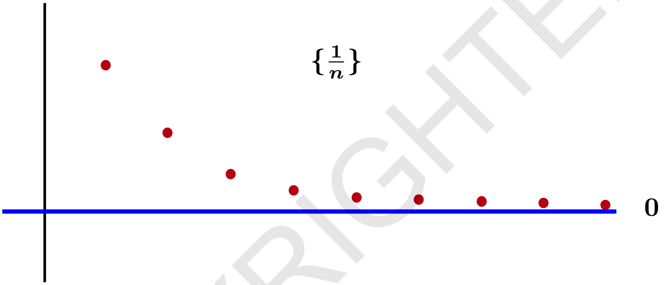
Question: What's wrong with this definition?

Heuristic Definition of a Limit of a Sequence



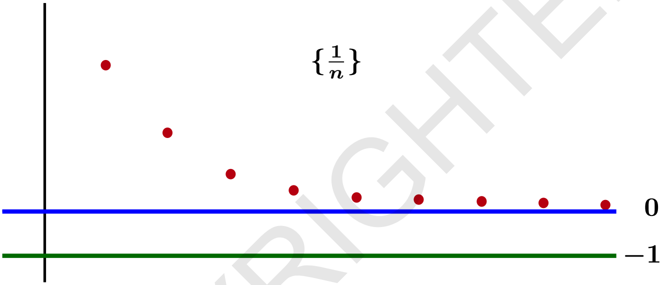
Again, consider $\{\frac{1}{n}\}$.

Heuristic Definition of a Limit of a Sequence



Again, consider $\{\frac{1}{n}\}$. As n *gets larger and larger*, the terms get *closer and closer to* $\mathbf{0}$.

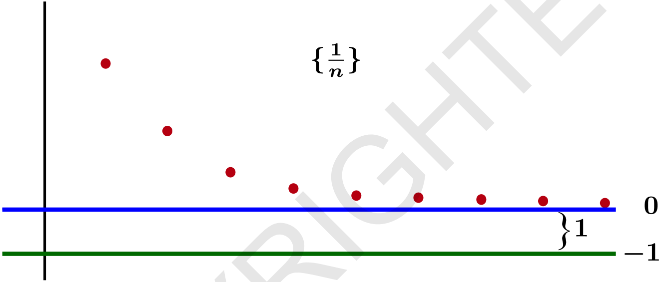
Heuristic Definition of a Limit of a Sequence



Again, consider $\{\frac{1}{n}\}$. As n *gets larger and larger*, the terms get *closer and closer to* 0.

But these terms also get closer and closer to -1.

Heuristic Definition of a Limit of a Sequence



Question: What is special about 0 that makes us choose it as the limit of $\{\frac{1}{n}\}$ instead of -1?

Answer: The terms of $\{\frac{1}{n}\}$ approximate 0 as **closely as we would like** when n is large enough, but these terms are never within 1 unit of -1.

The Limit of a Sequence

New Heuristic Definition:

We say that L is the *limit* of the sequence $\{a_n\}$ as n goes to infinity if no matter what positive tolerance $\epsilon > 0$ we are given, we can find a cutoff $N \in \mathbb{N}$ such that the terms a_n approximate L with an **error** less than ϵ provided that $n \geq N$.

Formal Definition: [Limit of a Sequence]

We say that L is the *limit* of the sequence $\{a_n\}$ as n goes to infinity if for every $\epsilon > 0$, there exists an $N \in \mathbb{N}$ such that if $n \geq N$, then

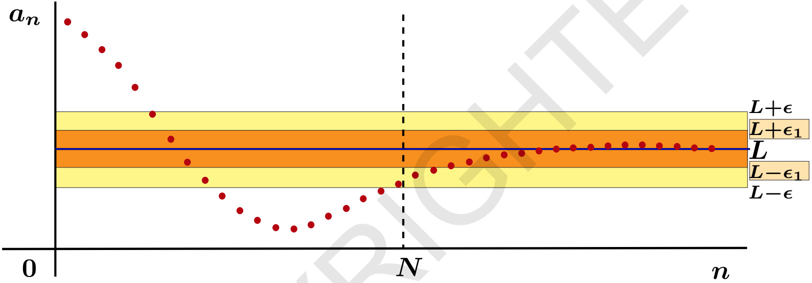
$$| a_n - L | < \epsilon.$$

In this case, we say that $\{a_n\}$ *converges* to L and we write

$$\lim_{n\to\infty} a_n = L.$$

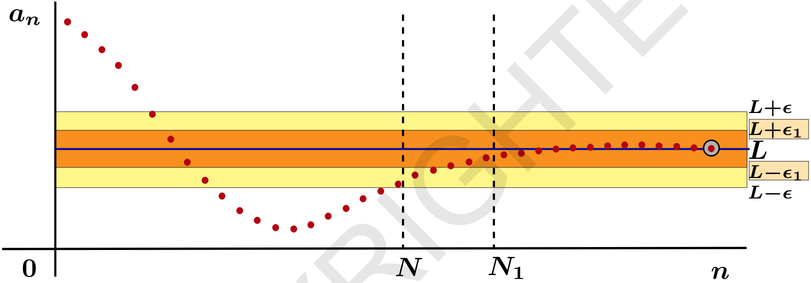
If no such L exists we say that $\{a_n\}$ *diverges*.

The Limit of a Sequence



1. Identify L.
2. Specify the error $\epsilon > 0$.
3. Find the cutoff N.
4. Choose a smaller ϵ_1.

The Limit of a Sequence



1. Identify L.
2. Specify the error $\epsilon > 0$.
3. Find the cutoff N.
4. Choose a smaller ϵ_1.
5. **Repeat Step 3** with a larger N_1.