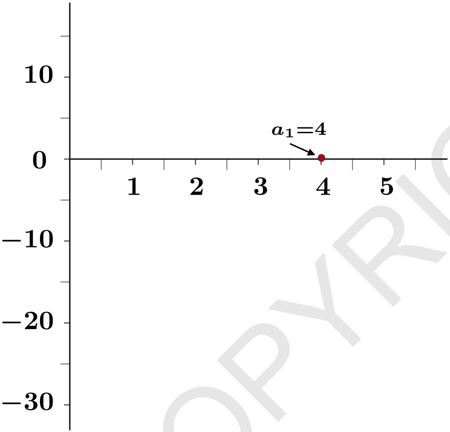
Heron's Algorithm

Created by

Barbara Forrest and Brian Forrest

Heron's Algorithm for Finding Square Roots



Example:

Calculate $\sqrt{17}$. Let $a_1 = 4$ and

$$a_{n+1} = \frac{1}{2}(a_n + \frac{17}{a_n}).$$

n	a_n
1	4

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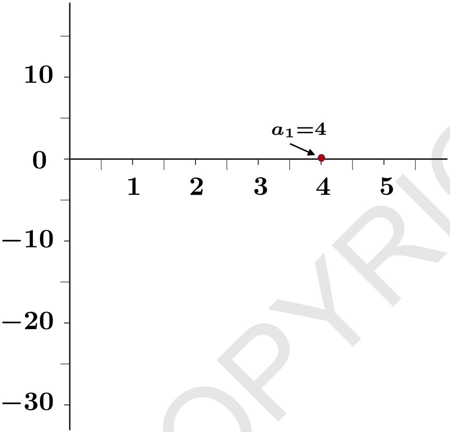

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n	a_n
1	4
2	4.125

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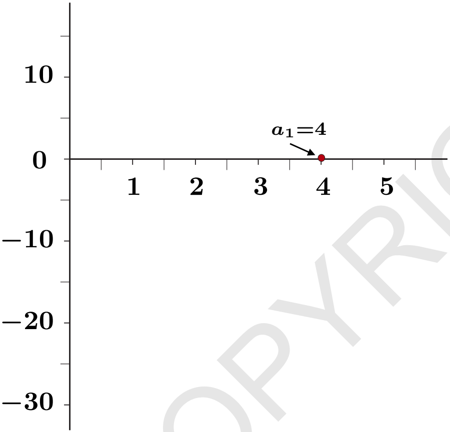

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Calculate $\sqrt{17}$. Let $a_1 = 4$ and

$$a_{n+1} = \frac{1}{2}(a_n + \frac{17}{a_n}).$$

n	a_n
1	4
2	4.125
3	4.1231060606
4	4.1231056256
5	4.1231056256
6	4.1231056256
7	4.1231056256
8	4.1231056256
9	4.1231056256
10	4.1231056256

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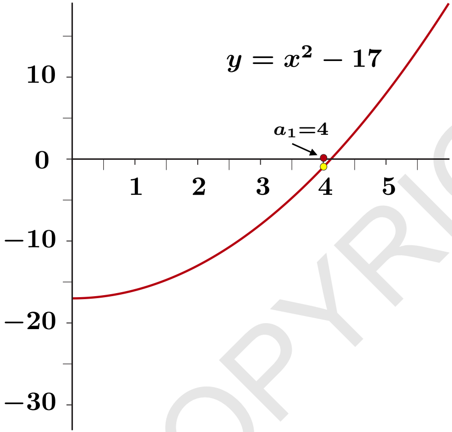

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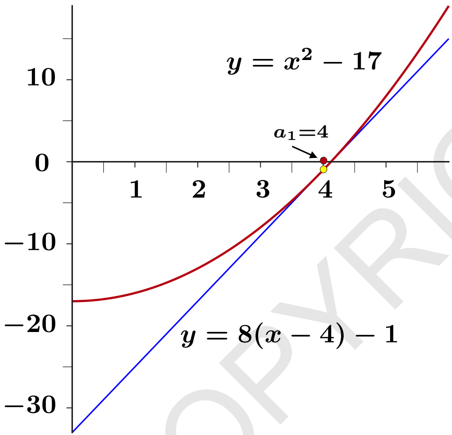

Example:

Calculate $\sqrt{17}$. Let $a_1 = 4$ and

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{17}{a_n}\right).$$

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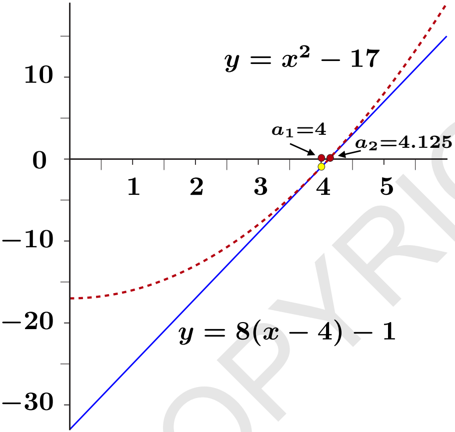

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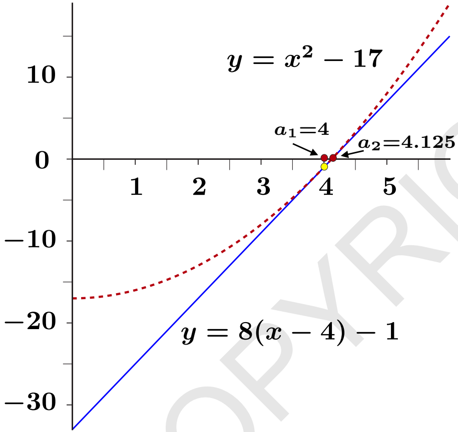

Note: $\sqrt{17} = 4.1231056256\ldots$.

Example:

Calculate $\sqrt{17}$. Let $a_1 = 4$ and

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{17}{a_n}\right).$$

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4	4.1231056256
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Heron's Algorithm for Finding Square Roots

Heron's Algorithm

Consider $\alpha > 0$. Let $a_1 = 1$ and

$$a_{n+1} = \frac{1}{2}(a_n + \frac{\alpha}{a_n}).$$

Then the algorithm generates a sequence of values that rapidly approach $\sqrt{\alpha}$.

Historical Notes:

1) *Heron's Algorithm* is named in honor of the first century Greek mathematician Heron who is attributed with the first explicit description of the method.
2) This process is also called the *Babylonian Square Root Method* since the method was used by Babylonian mathematicians to calculate $\sqrt{2}$.

Heron's Algorithm for Finding Square Roots

Example: Calculate $\sqrt{479678}$.

n	a_n	a_n
1	1	
2	239839.5	
3	119920.75	
4	59962.37498	
5	29985.18731	
6	15000.59224	
7	7516.284755	
8	3790.051626	
9	1958.307009	
10	1101.62613	
11	768.5266734	
12	696.3396879	
13	692.5980076	
14	692.5879006	

Solution:
By using *Heron's Algorithm* with $\alpha = 479678$, $a_1 = 1$ and

$$a_{n+1} = \frac{1}{2}(a_n + \frac{\alpha}{a_n})$$

the sequence of values listed in the table is generated. Thus,

$$\sqrt{479678} \approx 692.5879006$$

Heron's Algorithm for Finding Square Roots

Example: Calculate $\sqrt{479678}$.

n	a_n	a_n
1	1	700
2	239839.5	692.6271429
3	119920.75	692.5879017
4	59962.37498	692.5879006
5	29985.18731	692.5879006
6	15000.59224	692.5879006
7	7516.284755	692.5879006
8	3790.051626	692.5879006
9	1958.307009	692.5879006
10	1101.62613	692.5879006
11	768.5266734	692.5879006
12	696.3396879	692.5879006
13	692.5980076	692.5879006
14	692.5879006	692.5879006

Solution:
By using *Heron's Algorithm* with $\alpha = 479678$, $a_1 = 1$ and

$$a_{n+1} = \frac{1}{2}(a_n + \frac{\alpha}{a_n})$$

the sequence of values listed in the table is generated. Thus,

$$\sqrt{479678} \approx 692.5879006$$

Remark:
The algorithm works with any $a_1 > 0$, but performance improves if a_1 is chosen close to $\sqrt{\alpha}$.