Subsequences and Tails of Sequences

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Subsequences: New Sequences from Old

Definition: [Subsequence]

Given a sequence $\{a_n\}$ and a sequence $\{n_k\} \subset \mathbb{N}$ such that

$$n_1 < n_2 < n_3 \ldots < n_k \ldots$$

define

$$b_k = a_{n_k}.$$

The sequence $\{b_k\} = \{a_{n_1}, a_{n_2}, a_{n_3}, \ldots, a_{n_k}, \ldots\}$ is called a *subsequence* of $\{a_n\}$.

Example: Let $a_n = \frac{1}{n}$ and $n_k = 2k$. Then

$$b_k = a_{n_k} = a_{2k} = \frac{1}{2k}.$$

Subsequences: New Sequences from Old

$$\{a_n\} = \{a_1, a_2, a_3, a_4, \ldots, a_{2k}, \ldots\}$$

$$\{b_k\} = \{b_1, b_2, \ldots, b_k, \ldots\}$$

$$= \{a_2, a_4, \ldots, a_{2k}, \ldots\}$$

Subsequences: New Sequences from Old

Example: Given $\{a_n\}$ and $j \in \mathbb{N} \cup \{0\}$, let

$$\{b_k\} = \{a_{k+j}\} = \{a_{1+j}, a_{2+j}, a_{3+j}, \ldots\}.$$

For example if $j = 3$, we get

$$\{a_{k+3}\}_{k=1}^{\infty} = \{a_{1+3}, a_{2+3}, a_{3+3}, \ldots\} = \{a_4, a_5, a_6, \ldots\}.$$

Note: Such a subsequence is called a *tail* of $\{a_n\}$.

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