Divergence to ∞

Created by

Barbara Forrest and Brian Forrest

Divergence to ∞

Recall: We saw that the sequence

$$a_n = (-1)^{n+1}$$

diverged.

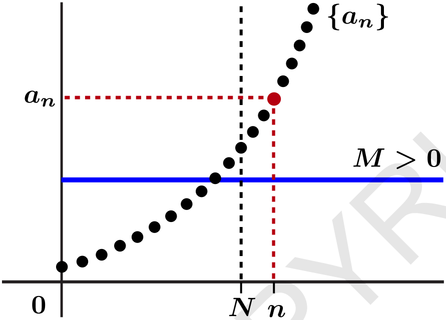
Question: Consider the sequence

$$a_n = n.$$

Does the sequence converge?

Observation: The terms grow without bound!

Divergence to ∞



Definition: [Divergence to ∞]

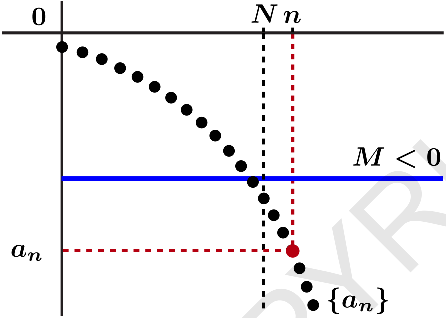
We say that a sequence $\{a_n\}$ *diverges to ∞* if for every $M > 0$ there exists an $N \in \mathbb{N}$ such that if $n \geq N$, then

$$a_n > M.$$

In this case, we write

$$\lim_{n\to\infty} a_n = \infty.$$

Divergence to $-\infty$



Definition: [Divergence to $-\infty$]

We say that a sequence $\{a_n\}$ *diverges to* $-\infty$ if for every $M < 0$ there exists an $N \in \mathbb{N}$ such that if $n \geq N$, then

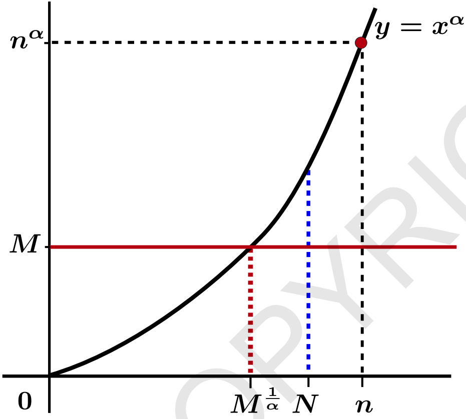
$$a_n < M.$$

In this case, we write

$$\lim_{n\to\infty} a_n = -\infty.$$

Remark: If $\lim_{n\to\infty} a_n = \pm\infty$, **the sequence does not converge.**

Example



Example:

Let $\alpha > 0$. Then

$$\lim_{n\to\infty} n^\alpha = \infty$$

Proof:

Given $M > 0$, choose

$$N > M^{\frac{1}{\alpha}}.$$

If $n > N$, then

$$n^\alpha > (M^{\frac{1}{\alpha}})^\alpha = M.$$

So $\lim_{n\to\infty} n^\alpha = \infty$.