Arithmetic for Limits of Sequences II

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Examples

Example 1: Evaluate $\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^2+2}$.

Observation: Note that $\lim_{n\to\infty} 3n^2+2n-1 = \infty = \lim_{n\to\infty} 4n^2+2$.

For large n

$$3n^2+2n-1 \approx 3n^2 \qquad \text{and} \qquad 4n^2+2 \approx 4n^2$$

so

$$\frac{3n^2+2n-1}{4n^2+2} \approx \frac{3n^2}{4n^2} = \frac{3}{4}.$$

This might lead us to guess:

$$\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^2+2} = \frac{3}{4}.$$

Examples

Example 1 (revisited): Evaluate $\lim\limits_{n\to\infty} \frac{3n^2+2n-1}{4n^2+2}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^2+2} &= \lim_{n\to\infty} \frac{n^2}{n^2} \cdot \frac{3+\frac{2}{n}-\frac{1}{n^2}}{4+\frac{2}{n^2}} \\
&= \lim_{n\to\infty} \frac{3+\frac{2}{n}-\frac{1}{n^2}}{4+\frac{2}{n^2}} \\
&= \frac{\lim\limits_{n\to\infty} 3 + \lim\limits_{n\to\infty} \frac{2}{n} - \lim\limits_{n\to\infty} \frac{1}{n^2}}{\lim\limits_{n\to\infty} 4 + \lim\limits_{n\to\infty} \frac{2}{n^2}} \\
&= \frac{3+2(0)-0}{4+2(0)} \\
&= \frac{3}{4}.
\end{aligned}$$

Examples

Example 2: Evaluate $\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^3+2}$.

Observation: For large values of n the terms in the sequence behave like

$$\frac{3n^2}{4n^3} = \frac{3}{4n}$$

which converges to 0.

Examples

Example 2 (revisited): Evaluate $\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^3+2}$.

$$\begin{aligned}
\lim_{n\to\infty} \frac{3n^2+2n-1}{4n^3+2} &= \lim_{n\to\infty} \frac{n^2}{n^3} \cdot \frac{3+\frac{2}{n}-\frac{1}{n^2}}{4+\frac{2}{n^3}} \\
&= \lim_{n\to\infty} \frac{1}{n} \cdot \frac{3+\frac{2}{n}-\frac{1}{n^2}}{4+\frac{2}{n^3}} \\
&= \lim_{n\to\infty} \frac{1}{n} \cdot \lim_{n\to\infty} \frac{3+\frac{2}{n}-\frac{1}{n^2}}{4+\frac{2}{n^3}} \\
&= 0 \cdot \frac{3}{4} \\
&= 0.
\end{aligned}$$

Examples

Example 3: Evaluate $\lim_{n\to\infty} \frac{3n^2+5}{n^{3/2}+2}$.

Observation: For large values of n the terms in the sequence behave like

$$\frac{3n^2}{n^{3/2}} = 3n^{\frac{1}{2}}$$

which diverges to ∞.

We have

$$\begin{aligned}
\lim_{n\to\infty} \frac{3n^2+5}{n^{3/2}+2} &= \lim_{n\to\infty} \frac{n^2}{n^{\frac{3}{2}}} \cdot \frac{3+\frac{5}{n^2}}{1+\frac{2}{n^{3/2}}} \\
&= \lim_{n\to\infty} \sqrt{n} \cdot \frac{3+\frac{5}{n^2}}{1+\frac{2}{n^{3/2}}} \\
&= \infty
\end{aligned}$$