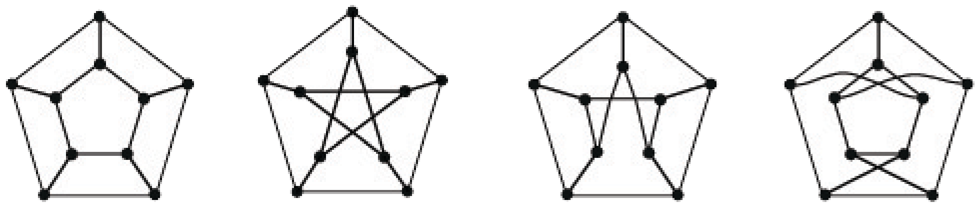
ASSIGNMENT 9 ECE 103 (Spring 2009)

Due in tutorial on Monday, July 20.

1. Consider the graphs G_1, G_2, G_3, H as follows:



G_1 G_2 G_3 H

   (a) Prove that no two of G_1, G_2, or G_3 are isomorphic.

   (b) One of G_1, G_2, G_3 is isomorphic to H. Determine which, with proof.

2. For any $n \in \mathbb{N}$, the *n-grid* G_n is the graph with n^2 vertices (x, y) where the integers x, y satisfy $1 \leq x \leq n$ and $1 \leq y \leq n$. Vertices (x, y) and (x', y') are joined by an edge if and only if $|x - x'| = 1$ and $y = y'$, or $|y - y'| = 1$ and $x = x'$.

   (a) How many edges does G_n have?

   (b) Prove that G_n is bipartite.

3. (a) Given a graph G, a *k-clique* is a subset of k vertices of G such that every pair of vertices in the subset is adjacent in G.

      i. How many k-cliques are there in the complete graph K_n?

      ii. How many k-cliques are there in the complete bipartite graph $K_{n,n}$?

   (b) A *Hamiltonian cycle* in a graph is a cycle that visits every vertex.

      i. How many Hamiltonian cycles are there in the complete graph K_n?

      ii. How many Hamiltonian cycles are there in the complete bipartite graph $K_{n,n}$?

4. Prove that if G is a connected graph, any two longest paths in G have a vertex in common.