1 Max-Product

Find the Maximum probability that can be achieved by some set of random variables given a set of configurations.

\[
\max_{x_1} P(x_1) = \max_{x_1} \max_{x_2} \max_{x_3} \max_{x_4} \max_{x_5} P(x_1)P(x_2|x_1)P(x_3|x_2)P(x_4|x_2)(P(x_5|x_3)
\]

\[
= \max_{x_1} P(x_1) \max_{x_2} P(x_2|x_1) \max_{x_3} P(x_3|x_4) \max_{x_4} P(x_4|x_2) \max_{x_5} P(x_5|x_3)
\]

\[
m_{ji}(x_i) = \sum_{x_j} \psi_j(x_j) \psi(x_i, x_j) \prod_{k \in N(j) \setminus i} m_{kj}
\]
\[ m_{ji}^{\text{max}}(x_i) = \max_{x_j} \psi^E(x_j) \psi(x_i, x_j) \prod_{k \in N(j) \setminus i} m_{kj} \]

**EXAMPLE:**

\[ m_{53}^{\text{max}}(x_5) = \max_{x_5} \psi^E(x_5) \psi(x_3, x_5) \]
\[ m_{32}^{\text{max}}(x_3) = \max_{x_3} \psi^E(x_3) \psi(x_3, x_5) m_{53}^{\text{max}} \]

## 2 Maximum configuration

Replace max with argmax

\[ m_{53}(x_5) = \arg\max_{x_5} \psi(x_5) \psi(x_3, x_5) \]
\[ \log m_{ji}^{\text{max}}(x_i) = \max_{x_j} \log \psi^E(x_j) + \log \psi(x_i, x_j) + \sum_{k \in N(j) \setminus i} \log m_{kj}^{\text{max}}(x_j) \]

## 3 Basic statistical Problem

- Regression
- Classification
- Clustering
Regression:

\[ P(y|x) = \frac{P(y, x)}{P(x)} = \frac{P(y, x)}{\int_y P(y, x) dy} \]

Classified:

\[ P(y|x) = \frac{P(y, x)}{P(x)} = \frac{P(y, x)}{\sum_y P(y, x) dy} \]
Clustering:

\[ P(y|x) = \frac{P(y, x)}{P(x)} \quad y \text{ unknown} \]

Figure 3:

Classification example: Naive Bayes classifier

\( Y = \{1, 0\}, X_i = \{1, 0\} \)

\[ \begin{array}{c}
\begin{array}{c}
< 01110 > \\
\hline
n
\end{array} \\
\rightarrow_Y Y
\end{array} \]

\[ \begin{array}{c}
< 01110 > \\
\rightarrow 0
\end{array} \]
\[ P(y|x_1, \ldots, x_n) = \frac{P(x_1, \ldots, x_n|y)P(y)}{P(x_1, \ldots, x_n)} = \frac{P(x_1, \ldots, x_n, y)}{P(x_1, \ldots, x_n)} = \frac{P(y) \prod_{i=1,2,\ldots,n} P(x_i|y)}{P(x_1, \ldots, x_n)} \]

Classify

\[ \hat{y} = 1 \iff P(y = 1|x_1, \ldots, x_n) > P(y = 0|x_1, \ldots, x_n) \]

\[ \hat{y} = 1 \iff \frac{P(y=1|x_1, \ldots, x_n)}{P(y=0|x_1, \ldots, x_n)} > 1 \]

\[ \iff \log \frac{P(y=1)}{P(y=0)} + \sum_{i=1\ldots n} \log \frac{P(x_i|y=1)}{P(x_i|y=0)} > 0 \]

\[ P(x_i|y=1) = P_{i1} \]

\[ P(x_i|y=0) = P_{i0} \]

\[
= x_i \log \frac{P_{i1}}{P_{i0}} + (1 - x_i) \log \frac{1 - P_{i1}}{1 - P_{i0}} \\
= \underbrace{x_i \log \frac{P_{i1}(1 - P_{i0})}{P_{i0}(1 - P_{i1})}}_{w} + \underbrace{\log \frac{1 - P_{i1}}{1 - P_{i0}}}_{b} \]
Classify $< x_1 \ldots x_n >$

$(+/)(wx+b)$