1.1.) Moralization

\[ P(X_i) = \prod_i P(X_i | X_{\pi i}) \]

\[ P(X) = \frac{1}{Z(\psi)} \prod_{C_i \in C} \psi(C_{x_i}) \]

An undirected graph must have the same variables and potential as a directed graph. Hence, \( \psi(x_1, x_2, x_3) \)
To convert a graph from directed to undirected, we need to first moralize a graph.

Example 1.)

\[
P(X_6)P(X_6 | X_5) P(X_6 | X_3, X_4)
\]

However, one flaw in elimination algorithm is that ordering is not trivial. Let’s look at another example, we want to find \(x_1, x_2, \ldots, x_4\) given the symptoms \(y_1, y_2 \ldots y_4\):

Example 2.)

Faults (disease)

\[
X_1 \quad X_2 \quad X_3 \quad X_4
\]

Symptoms

\[
y_1 \quad y_2 \quad y_3 \quad y_4
\]

\[
P(X_i | Y)
\]
1.2.) Sum–Product Algorithm

*Advantage:* the time complexity of computing the probability of one node is the same as time complexity of computing all

*Disadvantage:* it works only for trees

Trees are graphs with no cycles.

i.e.)

![Tree example](image1)

This is a tree.

![Non-tree example](image2)

This is NOT a tree.

In undirected models, there is one and only one path between any pair of nodes. As in directed models, if the corresponding moral graph is a tree, then the graph is a tree.

Example 3.)

![Moralized example](image3)

Since there is a cycle in the moralized graph, hence, the graph is not a tree.
Any directed tree can be converted to undirected graphs.

\[ G \in \{V,E\} \]
\[ P(X_v) = \prod_{c_i} \psi(X_{c_i}) / Z(\psi) \]

The following are the cyclic in the above graph:
\{X_5,X_2\}, \{X_4,X_2\}, \{X_2,X_1\}, \{X_3,X_1\}, \{X_2\}, \{X_1\} ...

In general,
\[ P(X_v) = (1/ Z(\psi)) * \prod_{i \in v} \psi(X_i) * \prod_{i \in v} \psi(X_i, X_j) \quad (1) \]

\[ P(X_v) = P(X_1)P(X_2|X_1)P(X_3|X_1)P(X_2|X_4)P(X_2|X_5) \quad (2) \]

If:
\[ \psi(X_Y) = P(X_Y) \]
\[ \psi(X_i) = 1 \quad \text{if } i \text{ is not a root} \]
\[ \psi(X_i, X_j) = P(X_j|X_i) , \quad \{i\} = \pi_j \]

Then:
\[ (1) = (2) \]

Whereas in the undirected graph:
\[ P(X_v) = (1/ Z(\psi)) * \psi(X_1) * \psi(X_2) * \psi(X_3) * \psi(X_1, X_2) * \psi(X_1, X_3) * \psi(X_2, X_1) * \psi(X_2, X_3) \]
\[ = (1/ Z(\psi)) * P(X_1) * P(X_2|X_1) * P(X_3|X_1) * P(X_4|X_2) * P(X_5|X_2) \]
$M_{32}$ denotes: Message from node 3 to 2

\[
M_{32} = \sum_{X_5} \psi(X_5) \psi(X_5, X_3)
\]

\[
M_{42} = \sum_{X_4} \psi(X_4) \psi(X_4, X_2)
\]

\[
M_{32} = \sum_{X_3} \psi(X_3) \psi(X_3, X_2) M_3(X_3)
\]