1 Evaluation

$X_i$ is an evidence node whose observed value is $\overline{x}_i$. To show that $X_i$ is fixed at the value $\overline{x}_i$, we define an evidence potential $\delta(x_i, \overline{x}_i)$ whose value is 1 if $x_i = \overline{x}_i$ and 0 otherwise.

So

$$g(\overline{x}_i) = \sum_{x_i} g(x_i) \delta(x_i, \overline{x}_i)$$

When we have more than one variable such as $p(F|\overline{E})$, the total evidence potential:

$$\delta(x_i, \overline{x}_E) = \prod_{i \in E} \delta(x_i, \overline{x}_i)$$
1.1 Elimination and directed graph

Given a graph $G = (V, E)$, an evidence set $E$ and a query node $F$, we first choose an eliminating ordering $I$ such that $F$ appears last in the ordering. For a graph $G = (V, E)$

![Directed Graph](image)

Figure 1: directed graph

$I = \{6, 5, 4, 3, 2, 1\}$ (1 should be last node, ordering is crucial, don’t change the order)

active list: $p(x_1), p(x_2|x_1), p(x_3|x_1), p(x_3|x_2), p(x_5|x_3), p(x_6|x_3), p(x_2|x_5)$

For $i \in V$ put $p(x_i|x_{\pi_i})$ in active list

For $i \in E$ put $p(x_i|x_{\pi_i})$ in active list

We first eliminate node $X_6$. We place $m_6(x_2, x_5)$ on the active list, having removed $X_6$. We now eliminate $X_5$.

$$\frac{p(x_5|x_3) \cdot m_6(x_2, x_5)}{m_5(x_2, x_3)}$$

Likewise, we can also eliminate $X_4, X_3, X_2$ (which yields the unnormalized conditional probability $p(x_1|x_6)$) and $X_1$. Then it yields
\[ m_1 = \sum_{x_1} \phi_1(x_1) \] which is the normalization factor, \( p(x_6) \)

1.2 Elimination and undirected graphs

Now for undirected graph \( G' \)

maximal clique: \( \{x_1, x_2\}, \{x_1, x_3\}, \{x_2, x_4\}, \{x_3, x_5\}, \{x_2, x_5, x_6\} \)

\[
\text{Figure 2: undirected graph } G'
\]

potential functions: \( \varphi(x_1, x_2), \varphi(x_1, x_3), \varphi(x_2, x_4), \varphi(x_3, x_5) \) and \( \varphi(x_2, x_3, x_6) \)

\[
p(x_1 | x_6) = p(x_1, x_6) / p(x_6) \quad \cdots \cdots \cdots \cdots \quad (*)
\]

\[
p(x_1, x_6) = \frac{1}{Z} \sum_{x_2} \sum_{x_3} \sum_{x_4} \sum_{x_5} \sum_{x_6} \varphi(x_1, x_2) \varphi(x_1, x_3) \varphi(x_2, x_4) \varphi(x_3, x_5) \varphi(x_2, x_3, x_6) \delta(x_6, x_6)
\]

\( \frac{1}{Z} \) looks crucial, but no effect cause for \((*)\). It is because both denominator and numerator have \( \frac{1}{Z} \), so we can cancel it.
When we remove a node, we have to connect the parents of the node.

ex. For the graph $G$(figure3)

when we remove $x_1$, $G$ becomes figure4

if we remove $x_2$, $G$ becomes figure5

1.2.1 Moralization

First connect all of the parents of each node because involving all variables $X_{\pi_i}$ will necessarily appear in our calculation. Then drop the orientation of all edges in the graph, converting the graph to an undirected graph. This procedure is moralization.

ex. $I = \{x_6, x_5, x_4, x_3, x_2, x_1\}$
When we moralize the directed graph (figure 6), then it becomes the undirected graph (figure 7).
Figure 5:

Figure 6: directed graph
Figure 7: undirected graph