Department of Pure Mathematics
Graduate Courses

Fall 2007

Course #	Course Title	Instructor	Meet Days/ Time	Room
PMath 632/432	First Order Logic and Computability	B. Csima	MW, 11:30 a.m. - 12:50 p.m.	MC 5045
PMath 642/442	Field and Galois Theory	Y.-R. Liu	MWF, 1:30 - 2:20 p.m.	MC 6007
PMath 653/453	Functional Analysis	N. Spronk	MWF, 9:30 - 10:20 a.m.	PHY 313
PMath 722	Topics in Universal Algebra *Topics in Universal Algebra*	R.D. Willard	TH,F 10:30 - 12:00 noon	MC 5046
PMath 733	Topics in Group Theory *Rep. Theory of Compact Groups*	W. Kuo	M,W,F 8:30 - 9:20 a.m.	MC 4064 *
PMath 801	Graduate Analysis	B.E. Forrest	MW, 2:30 - 3:50 p.m.	MC 5045
PMath 822	Topics in Operator Theory *Intro. to Operator Algebras*	K.R. Davidson	Tuesdays, Sept. 11 - Dec. 4 1:00 - 3:00 p.m. Not on September 18 Not on November 13	Held at the Fields Institute, University of Toronto

Students should discuss their course selection with their Supervisor, the Graduate Officer, or the course Professor.

You will require a "Permission Number" in order to enroll in PMath 822 and PMath 733 using QUEST.

Please obtain your Permission Number from Shonn Martin in MC 5064.

Please enroll in your courses, using Quest, by

Friday, October 19, 2007.

http://www.quest.uwaterloo.ca/

(Revised on September 13, 2007) *

PMath 632 **First Order Logic and Computability** **B. Csima**
(held with) PMath 432

Instructor: **B. Csima**
Office: **MC 5171**
Phone: **(519) 885-1211 x 37596**
Email: **csima@math.uwaterloo.ca**

Topics: **The concepts of formal provability and logical consequence in first order logic are introduced, and their equivalence is proved in the soundness and completeness theorems. Gödel's incompleteness theorem is discussed; making use of the halting problem of computability theory. Relative computability and the Turing degrees are further studied.**

Required Textbook: **"Mathematical Logic", 2nd ed. by Ebbinghaus, Flum, Thomas Published by Springer.**

PMath 642 **Field and Galois Theory** **Y.-R. Liu**
(held with) PMath 442

Instructor: Y-R. Liu
Office: MC 5074
Phone: (519) 888-4567 x 35698
Email: yrliu@math.uwaterloo.ca

Topics: Normal series, elementary properties of solvable groups and simple groups, algebraic and transcendental extensions of fields, adjoining roots, splitting fields, geometric constructions, separability, normal extensions, Galois groups, fundamental theorem of Galois theory, solvability of radicals, Galois groups of equations, cyclotomic and Kummer extensions.

Textbook: No textbook required

References:

1. "Galois Theory", 2nd ed., Joseph Rotman, Springer-Verlag, 1990.
2. "Galois Theory", Joseph–Pierre Escofier, Graduate Texts in Mathematics,
Springer–Verlag, 2001.
3. "Notes on Galois Theory", Peter Hoffman (recommended)

PMath 653 **Functional Analysis** **N. Spronk**
(held with) **PMath 453/**
(and) **AMath 432**

Instructor: N. Spronk
Office: MC 5078
Phone: (519) 885-5467 x 35559
Email: nspronk@math.uwaterloo.ca

Topics: Normed/Banach spaces: topological spaces, $\mathcal{C}_b(X)$; Hölder's & Minkowski's inequalities, ℓ_p spaces; spaces of linear operators. Dual spaces: Hahn-Banach theorem, separation by hyperplanes. Finite dimensional normed spaces. Consequences of Baire category theorem: Banach-Steinhaus theorem, open mapping theorem, closed graph theorem. Weak topologies, product topologies: compactness, Tychonov's theorem, Banach-Alaoglu theorem, metrisation; second dual spaces: Goldstine's theorem, reflexivity; Krein-Milman theorem. Euclidean/Hilbert spaces: Cauchy-Schwarz inequality, Orthogonal decomposition, Riesz representation theorem, orthonormal systems and orthonormal bases. Linear operators on Banach spaces: adjoint operators, Gelfand spectrum. Compact operators: spectral theorem.

Textbook: "Linear Analysis: An Introductory Course", 2nd Edition, Béla Bollobas, Cambridge, 1999.

References:

1. "A Course in Functional Analysis", 2nd edition, John Conway, Springer, 1990.
2. "Analysis Now", G.K. Pedersen, Springer, 1989.

Topics in Universal Algebra

Instructor: R.D. Willard
Office: MC 5082
Phone: (519) 885-1211 x 35565
Email: rdwillar@uwaterloo.ca
Textbook: No textbook required.

Outline: Universal algebra is the study of completely arbitrary algebraic systems, especially those with a finite domain. Universal algebra provides both a context and a language that are occasionally useful to "real" algebraists, engineers and theoretical computer scientists, yet universal algebra is also a rich theory in its own right.

The first part of this course (approximately 8 weeks) will cover the elements of universal algebra. The second part (4 weeks) will consist of one extended application: the algebraic approach to the CSP Dichotomy Conjecture of Feder and Vardi.

Students will be given regular written assignments and an end-of-term project with presentation.

The text for the first part of the course is chapters 1 and 2 of "A Course in Universal Algebra" by S. Burris and H.P. Sankappanavar, Springer Graduate Texts in Mathematics, 1981, freely downloadable from

http://www.math.uwaterloo.ca/ snburris/htdocs/ualg.html

The primary source for the second part of the course is A. Bulatov, P. Jeavons, and A. Krokhin, Classifying the complexity of constraints using finite algebras, SIAM J. Comput., 34 (2005), 720-742.

PMath 733 | Topics in Group Theory | W. Kuo

Representation Theory of compact groups

Instructor: Wentang Kuo
Office: MC 5059
Phone: (519) 885-1211 x 37202
Email: wtkuo@math.uwaterloo.ca
Topics: This is a first course in representation theory aimed at graduate and advanced undergraduate students. It only requires basic knowledge on linear algebra and groups theory. One of the main themes will be how representation theory and Lie groups provide examples of interesting phenomena in algebras, geometry, and topology.

We plan to cover the following topics, as many as time allows:

representations of finite groups, Lie algebras, the Peter-Weyl theorem, maximal tori, root systems, the classification of compact Lie groups, the Weyl character formula.

References

[1] T. Bröcker & T.t. Dieck, "Representations of Compact Lie groups," GTM 98, Springer-Verlag, 1985.

[2] W. Fulton & J. Harris, "Representation Theory: A First Course," GTM 128, Springer-Verlag, 1991.

[3] B.C. Hall, "Lie groups, Lie algebras, and representations : an elementary introduction," Springer-Verlag, GTM 222, New York, 2003.

[4] J.E. Humphreys, " Introduction to Lie Algebras and Representation Theory," Springer-Verlag, 1972.

[5] A.W. Knapp, " Representation Theory of Semisimple Groups: An Overview Based on Examples," Princeton University Press, 1986.

[6] W. Rossmann, "Lie Groups: An Introduction Through Linear Groups," Oxford University Press, 2003.

[7] J.-P. Serre, "Linear Representation of finite groups," Springer-Verlag, New York, 1977.

PMath 801 **<u>Graduate Analysis</u>** **B.E. Forrest**

Zorn's Lemma and the Axiom of Choice, cardinality, introduction to topological spaces, bases, nets, continuous functions and weak topologies, compactness, connectedness, Banach spaces, Contraction Mapping Principal finite–dimensional spaces $C(X)$ and $C_0(X)$, Stone–Weierstrass Theorem, Arzela– ascoli Theorem, Urysohn's Lemma, ideals in $C_0(x)$.

Outline:

1. **Basic Set Theory**
   - **Zorn's Lemma and the Axiom of Choice**
   - **Cardinal numbers and cardinal arithmetic**
2. **Introduction to Topological Spaces**
   - **Basics notions of topology**
   - **Bases and subbases**
   - **Nets**
   - **Continuous functions and weak topologies**
   - **Compact and locally compact Hausdorff spaces**
   - **Compactness in metric spaces**
   - **Connectedness**
   - **Baire Category Theorem**
3. **Normed linear spaces and Banach spaces**
   - **Sequence Spaces and $\mathbb{R}^n$**
   - **Bounded linear maps**
   - **Banach Contractive Mapping Theorem and its applications**
   - **Finite dimensional spaces**
4. $\mathcal{C}(X)$ **and** $\mathcal{C}_0(X)$
   - **Completeness and uniform convergence**
   - **Weierstrass approximation theorem**
   - **Stone-Weierstrass theorem for** $\mathcal{C}(X)$
   - **One-point compactifications**
   - **Stone-Weierstrass Theorem for** $\mathcal{C}_0(X)$
   - **Urysohn's Lemma**
   - **Ideals in** $\mathcal{C}(X)$ **and** $\mathcal{C}_0(X)$
   - **Arzela-Ascoli Theorem and its applications**

References: **To be supplied in class.**

Textbook: **No textbook required**

PMath 822 | **Topics in Operator Theory:** | **Man Duen Choi &**
| **Intro. to Operator Algebras** | **K.R. Davidson**

Instructors: Man Duen Choi and Kenneth R. Davidson

When: 10 2-hour lectures, Tuesdays 1:00 - 3:00 p.m.
September 11 - December 4
Not including September 18th and November 13th

Where: Fields Institute For Research In Mathematical Sciences
222 College Street
Toronto, ON M5T 3J1
(416) 348-9710.
Directions: http://www.fields.utoronto.ca/aboutus/directions.html

Transportation will be arranged after September 1st.

About the Course: This course is an introduction to abstract operator algebras. It has roots in the theory of completely positive maps and dilation theory, and Arveson's approach to studying nonself-adjoint algebras via the minimal enveloping C*-algebra, the C*-envelope.

As the course is divided into ten two hour lectures (September 11 through December 4, not including September 18 and November 13), each lecture will be an overview of the week's topic; but serious students will need to read the omitted proofs and related material on their own.

1. Background on C*-algebras
2. CP maps, Stinespring's Theorem.
3. Sz.Nagy and Ando dilation theorems, Commutant lifting theorem.
4. Arveson extension and dilation theorems.
5. The injective and C*-envelopes
6. CB maps, Wittstock, etc.
7. CB homomorphisms, Paulsen's Thm., Haagerup, Christensen.
8. Polynomially bounded operators, Pisier's counterexample.
9. Abstract operator algebras, Blecher–Ruan–Sinclair Thm.
10. Universal operator algebras, factorization, Pisier's similarity degree.

Textbook: The text is required.
We will take much of the material from Vern Paulsen's book
"*Completely bounded maps and Operator Algebras*",
Cambridge University Press, 2002, ISBN 0-521-81660-6

Exams: There will be no exams.

To Receive a Credit: **Students who wish to receive credit should obtain permission from their home university to count the course there, and we can provide a letter stating your participation and level of performance. If you take it for credit, then you will be asked to hand in a written report including exercise solutions and possibly a write-up of some topic related to the course.**