

Department of Pure Mathematics

Graduate Courses

Fall 2009

Course #	Course Title	Instructor	Meet Days/ Time	Room
PMath 632/432	First Order Logic and Computability	B. Csima	M,W,F 10:30 - 11:20 am	MC 4063
PMath 642/442	Fields and Galois Theory	Y.-R. Liu	M,W,F 02:30 - 03:20 pm	MC 2038
PMath 653/453	Functional Analysis	K.R. Davidson	M,W,F 09:30 - 10:20 am	MC 4063
PMath 701	Graduate Algebra	R.D. Willard	M,W,F 01:30 - 02:20 pm	MC 2018B
PMath 702	Introduction to Graduate Analysis	L.W. Marcoux	T, Th 02:00 - 3:30 pm	MC 5046
PMath 822	Topics in Operator Theory: <i>Introduction to K-theory</i>	L.W. Marcoux	T,Th 09:30 - 11:00 am	MC 4062
PMath 900	Topics in Algebra: <i>Representation Theory of Compact Groups</i>	W. Kuo	M,W,F 08:30 - 09:20 am	MC 5046
PMath 944	Topics in Number Theory: <i>Geometry of Numbers</i>	C.L. Stewart	T,Th 09:30 - 11:00 am	MC 5046

Students should discuss their course selection with their Supervisor, the Graduate Officer, or the course Professor.

You will require a "Permission Number" in order to enroll in PMath 822, PMath 900, and PMath 944 using QUEST.

Please obtain your Pure Math Permission Number from Shonn Martin in MC 5064.

**Please enroll in your courses, using Quest, by
Friday, October 23, 2009.**

<http://www.quest.uwaterloo.ca/>

(Revised on September 15, 2009 - new classroom for PM 822)

PMath 632

First Order Logic and Computability

B. Csimá

(held with) **PMath 432**

Instructor: B. Csimá

Office: MC 5171

Phone: (519-888-4567 x 37596

Email: csima@math.uwaterloo.ca

Lectures: MWF, 10:30 - 11:20 am MC 4063

Topics: The concepts of formal provability and logical consequence in first order logic are introduced, and their equivalence is proved in the soundness and completeness theorems. Gödel's incompleteness theorem is discussed; making use of the halting problem of computability theory. Relative computability and the Turing degrees are further studied.

**Required
Textbook:** "Mathematical Logic", 2nd ed. by Ebbinghaus, Flum, Thomas
Published by Springer.

Instructor: Y-R. Liu

Office: MC 5074

Phone: (519) 888-4567 x 35698

Email: yrliu@math.uwaterloo.ca

Lectures: MWF, 2:30 - 3:20 am MC 2038

Topics: This course is designed to introduce to students the Galois theory. The following materials will be covered.

1. Field Extensions: degree of extensions, algebraic and transcendental numbers, simple extensions, splitting fields.
2. The Fundamental Theorem: automorphism groups, separable extensions, normal extensions, Galois extensions, Galois groups, Galois correspondence.
3. Applications: Galois groups of polynomials, cyclotomic extensions, solvability by radicals, geometric constructions.

Textbook: No textbook required

- References:**
- (1) Galois Theory, J.-P. Escofier, Springer.
 - (2) Galois Theory, J. Rotman, Springer.
 - (3) Galois Theory, I. Stewart, Chapman & Hall/CRC.
 - (4) P. Hoffman's class note on Galois Theory. You can also find chapters on Galois Theory in many algebra books, ex. Chapter V of Hungerford's Algebra.

PMath 653
(held with) **PMath 453/**
(and) **AMath 432**

Functional Analysis

Ken Davidson

Instructor: K.R. Davidson

Office: MC 5080

Phone: (519) 885-5467 x 84081

Email: krdauids@uwaterloo.ca

Class Times: MWF, 09:30 - 10:20 am MC 4063

Textbook: "A Course in Functional Analysis, 2d.ed., Graduate Texts in Mathematics, Vol. 96", John B. Conway, Springer, ISBN: 978-0-387-97254-9.

Alternate Reference: 1. "A Course in Functional Analysis", 2nd edition, John Conway, Springer, 1990.

2. "Analysis Now", G.K. Pedersen, Springer, 1989.

Grading Times: Undergraduates: Assignments 40%, Final Exam 60%.

Grad students: Assignments 40%, Talk 10%, Final Exam 50%.

Outline: 1. Point set topology (brief introduction)

2. Hilbert Spaces

3. Banach Spaces (Hahn-Banach theorem, Open Mapping theorem)

4. Duality (weak topologies, Banach-Alaoglu theorem, Krein-Milman Theorem)

5. Bounded Operators

6. Spectral Theorem for compact operators

Instructor: R.D. Willard

Office: MC 5082

Phone: (519) 885-4567 x 35565

Email: rdwillar@uwaterloo.ca

Textbook: No textbook required.

Outline: Isomorphism theorems, classical structures theorems for finite groups, nilpotent and solvable groups, free groups, presentations, modules over PID's, Hilbert Basis Theorem, Jacobson radical, Artin-Wedderburn Theorem, fields extensions, Jordan and rational Canonical form, tensor products, tensor and exterior algebras.

**Optional
Textbook:** Advanced Modern Algebra
Author: J. Rotman
ISBN: 97801308878687
Publisher: Prentice-Hall

**Meet Day &
Time:** M,W,F 01:30 - 02:20 am

Classroom: MC 2018B

- Phone:** (519) 888-4567 x 35568
- Email:** lwmarcoux@uwaterloo.ca
- Textbook:** No textbook
- Lectures:** T, Th, 02:00 - 03:30 pm in MC 5046
First lecture September 15, 2009
- References:** To be supplied in class.

Zorn's Lemma and the Axiom of Choice, cardinality, introduction to topological spaces, bases, nets, continuous functions and weak topologies, compactness, connectedness, Banach spaces, Contraction Mapping Principal finite-dimensional spaces $C(X)$ and $C_0(X)$, Stone-Weierstrass Theorem, Arzela-Ascoli Theorem, Urysohn's Lemma, ideals in $C_0(x)$.

- Outline:**
1. Basic Set Theory
 - Zorn's Lemma and the Axiom of Choice
 - Cardinal numbers and cardinal arithmetic
 2. Introduction to Topological Spaces
 - Basics notions of topology
 - Bases and subbases
 - Nets
 - Continuous functions and weak topologies
 - Compact and locally compact Hausdorff spaces
 - Compactness in metric spaces
 - Connectedness
 - Baire Category Theorem
 3. Normed linear spaces and Banach spaces
 - Sequence Spaces and \mathbb{R}^n
 - Bounded linear maps
 - Banach Contractive Mapping Theorem and its applications
 - Finite dimensional spaces

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4. $\mathcal{C}(X)$ and $\mathcal{C}_0(X)$

- Completeness and uniform convergence
- Weierstrass approximation theorem
- Stone-Weierstrass theorem for $\mathcal{C}(X)$
- One-point compactifications
- Stone-Weierstrass Theorem for $\mathcal{C}_0(X)$
- Urysohn's Lemma
- Ideals in $\mathcal{C}(X)$ and $\mathcal{C}_0(X)$
- Arzela-Ascoli Theorem and its applications

5. Time permitting, we will study some measure theory on the real line.

Instructor: Professor L.W. Marcoux
Office: MC 5053
Phone: (519) 888-4567 x 35568
Email: lwmarcoux@uwaterloo.ca
Lectures: T,Th 9:30 - 11:00 am, in MC 4062

Basic Outline:

Associated to each C^* -algebra \mathcal{A} are two groups, $K_0(\mathcal{A})$ and $K_1(\mathcal{A})$. The group $K_0(\mathcal{A})$ is constructed by considering equivalence classes of projections in matrix algebras over (the unitization of) \mathcal{A} . Similarly, the group $K_1(\mathcal{A})$ is obtained by considering equivalence classes of unitaries in matrix algebras over (the unitization of) \mathcal{A} . Ordered K -theory has been useful in helping to distinguish between different isomorphism classes of C^* -algebras, and for certain classes of C^* -algebras, has served as a complete isomorphism invariant. In particular, ordered K -theory is at the heart of the *Elliott program*, which seeks to classify simple, nuclear C^* -algebras up to isomorphism. In this course, we shall study the basics of K -theory for C^* -algebras, including the following topics.

- Projections and unitary elements
- The K_0 -group of a unital C^* -algebra
- K_0 as a functor
- The K_1 -group of a C^* -algebra
- The index map and higher K -functors
- Bott periodicity and (time permitting) the six-term exact sequence

Grading: Grades will be based upon assignments (roughly 4 or 5 of these), as well as lectures.

Textbook: I will mainly be following the text:

An introduction to K -theory for C^ -algebras*, by M. Rørdam, F. Larsen and N.J. Laustsen, London Math. Soc., Student Texts **49**, Cambridge University Press (2000).

Another useful resource will be the book:

K -theory and C^ -algebras*, by N.E. Wegge-Olsen, Oxford Science Publications, Oxford University Press, 1993.

Representation Theory of Compact Groups

Instructor: Professor W. Kuo

Lectures: M,W,F 08:30 - 09:20 am in MC 5046
First lecture September 14, 2009

About the Course: This is a first course in representation theory aimed at graduate and advanced undergraduate students. It only requires basic knowledge on linear algebra and groups theory, such as PMATH 346. One of the main themes will be how representation theory and Lie groups provide examples of interesting phenomena in algebras, geometry, and topology.

We plan to cover the following topics, as many as time allows.

representations of finite groups, Lie algebras, the Peter-Weyl theorem, maximal tori, root systems, the classification of compact Lie groups, the Weyl character formula.

Prerequisite: Group Theory

Textbook: There is no textbook. The instructor will use his own notes. The following are the references.

1. T. Bröcker & T.t. Dieck, "Representations of Compact Lie groups," GTM 98, Springer-Verlag, 1985.
2. W. Fulton & J. Harris, "Representation Theory: A First Course," GTM 128, Springer-Verlag, 1991.
3. B.C. Hall, "Lie groups, Lie algebras, and representations : an elementary introduction," Springer-Verlag, GTM 222, New York, 2003.
4. J.E. Humphreys, " Introduction to Lie Algebras and Representation Theory," Springer-Verlag, 1972.
5. A.W. Knap, " Representation Theory of Semisimple Groups: An Overview Based on Examples," Princeton University Press, 1986.
6. W. Rossmann, "Lie Groups: An Introduction Through Linear Groups," Oxford University Press, 2003.
7. J.-P. Serre, "Linear Representation of finite groups," Springer-Verlag, New York, 1977.

Instructor: Cameron L. Stewart

Email: cstewart@uwaterloo.ca

Lectures: T,Th 09:30 - 11:00 am

Where: MC 5046

**First
Lecture:** September 15, 2009

**About the
Course:** In this course we shall explore the basic properties of lattices in n -dimensional Euclidean space and lattice points in convex bodies. To Minkowski we owe the term Geometry of Numbers, which describes the area, and the following result, which is known as Minkowski's theorem. If a convex body is centrally symmetric with respect to the origin and contains no point of a lattice besides the origin, then its volume is at most $2^n * \det L$. This is one of the basic results in the area and we shall consider some of its consequences. Other topics I hope to cover include covering minima, the Lenstra-Lenstra-Lovasz algorithm, the reduction of binary quadratic and cubic forms, the Leech lattice, sphere packing, the kissing number of spheres, heights of subspaces and Siegel's Lemma.

The course will be self contained and accessible to beginning graduate students.

Textbook: Professor will be using his own notes.

Exams: There will be a seminar talk, essay, and final exam.