Department of Pure Mathematics
Graduate Courses

Winter 2010

Course #	Course Title	Instructor	Meet Days/Time	Room
PMath 641/441	Algebraic Number Theory	P. Ingram	MWF 2:30 - 3:20	MC 2035
PMath 644/444	Ring, Module, and Representation	R. Moosa	TTh 2:30 - 3:50	RCH 206
PMath 651/451	Measure and Integration	A. Nica	MWF 12:30 - 01:20	MC 4058
PMath 665/465	Differential Geometry	S. Karigiannis	TTh 1:00 - 2:20	MC 2034
PMath 833	Topics in Harmonic Analysis: *Harmonic Analysis*	Kathryn Hare	TTh 11:30 - 01:00	MC 5046
PMath 900	Topics in Algebra *Infinite Group Theory*	J. Lawrence	TTh 10:00 - 11:30	MC 5046
PMath 944	Topics in Number Theory: *Computational Number Theory*	Kevin Hare	MWF 10:30 - 11:30	MC 5046
PMath 955	Topics in Geometry: *Gauge Theory*	R. Moraru	MW 09:00 - 10:20	MC 5046

- Students should discuss their course selection with their Supervisor, the Graduate Officer, or the course Professor.
- You will require a 'Permission Number' in order to enroll in some courses, using QUEST.
- Please obtain your 'Permission Number' from Shonn Martin in MC 5064

- A 'Drop/Add' form is required if you wish to Audit a course. Please see Shonn in MC 5064.

Please enroll in your courses by
Friday, February 13th, 2010

Revised December 21, 2009

PMath 641 **Algebraic Number Theory** **P. Ingram**
(held with) **PMath 441**

Professor: Patrick Ingram

Office: MC 5154

Email: pingram@math.uwaterloo.ca

Course Outline:

Topics: The course will cover number theory in rings (usually the ring of integers in some number field). We'll look at unique factorization, algebraic integers, the ideal class group, and Dirichlet's unit theorem. If time allows, possible additional topics include solving diophantine equations, or number theory in one-dimensional polynomial rings.

References: The main references will be "*Algebraic Theory of Numbers*", by P. Samuel (this is currently available from Dover), and some additional notes which will be available on the web.

Required Textbook:	Algebraic Theory of Numbers (paperback) by Pierre Samual, ISBN: 978 0 486 4666 8, Dover Publishing

PMath 644 **Rings, Modules, and Representations** **R. Moosa**
(held with) **PMath 444**

Professor: R. Moosa

Office: MC 5049

Email: rmoosa@math.uwaterloo.ca

Textbook: *No textbook is required.* Professor will use his own notes.

Course Outline:
In this course we will focus on the theory of not necessarily commutative rings (with identity) and modules over such rings. Highlights include the characterization of finitely generated modules over principal ideal domains and the Artin-Wedderburn structure theorem for semisimple rings.

In the final third or quarter of the course (depending on how things go) we will apply ring- and module-theoretic techniques to the study of representations of finite groups.
There is no required text for the course, but for the sake of reference the following two books will be placed on reserve in the Davis library:

Algebra, by T.W. Hungerford.

A first course in noncommutative rings, by T.Y. Lam.

The course web page is
http://www.student.math.uwaterloo.ca/~pmat444.

There will be between eight and ten homework assignments over the course of the term, collected typically on Thursdays at the beginning of class.

There will be no midterm exam.
There will be a final exam.
The registrar will probably inform us of the final exam schedule in late February.

Homework	40%
Final Exam	60%

PMath 451/PMath 651/AMath 431

(Measure and Integration)

Winter Term 2010

Room and time: M W F 12:30–1:20 pm, in MC 4058.

Instructor: Alexandru Nica

Material covered: This course provides a basic introduction to measure spaces and to abstract integration theory. Some important "chapters" of the course are:

- Abstract measure spaces and how to construct them.
- Measurable functions and how to integrate them.
- Convergence theorems.
- L^p-spaces.
- Absolute continuity and the Radon-Nikodym Theorem.
- Product measures and Fubini's Theorem.
- Radon integrals and Riesz representation theorems.

Required textbook: There is no required textbook. There are many good books which present the material mentioned above, but the lectures will not follow strictly one of them. I will attempt to make the sequence of lectures clear and self-contained, so that your notes themselves can be used as primary reference when you study this material.

Recommended textbook: W. Rudin, Real and complex analysis, McGraw-Hill (several editions). A copy of this textbook will be placed on reserve in the Davis Centre Library, under call number QA300.R82.

Web-site of the course: Updated information about the course will be posted periodically on its web-site, www.student.math.uwaterloo.ca/˜pmat451

PMath 665/465/ AMath 333

Differential Geometry

Winter 2010

- **Instructor:** Spiro Karigiannis **Email:** *karigiannis@math.uwaterloo.ca*
- **Telephone:** 519-888-4567 ext 32810 **Office:** MC 5082

- **Office Hours:** Mon/Wed 14:00 – 15:00, or by appointment.
- **Course Lectures:** Tue/Thu 13:00 – 14:20, in MC 2034.

Course Description: An introduction to differentiable manifolds. The tangent and cotangent bundles. Vector fields and differential forms. The Lie bracket and Lie derivative of vector fields. Exterior differentiation, integration of differential forms, and Stokes's Theorem. Riemannian manifolds, affine connections, and the Riemann curvature tensor.

Detailed list of course topics: I have prepared a very ambitious outline of topics that I would like to cover in this course. There is no way we will be able to cover all these topics in full detail in just 12 weeks. Some topics will be mentioned only briefly in passing, while other topics will be left for the students to work through in the assignment problems. The core material of the course is sections 1.1 to 1.8 and 2.1 to 2.4 below. Anything more that we get to do will be a bonus. Hopefully we can cover at least one or two more sections from the list below.

Part One: Smooth manifold theory

1.1 Differentiable (smooth) manifolds. Examples of manifolds. Smooth functions on manifolds and smooth maps between manifolds. Some topological aspects of smooth manifold theory, including orientability. Partitions of unity.

1. Tangent vectors and cotangent vectors on manifolds. Vector fields and 1-forms. The tangent bundle and cotangent bundle of a manifold and their smooth manifold structures.

2. Tensor bundlles. The exterior bundle of a manifold. Sections of tensor bundles. Differential forms. Characterization of orientability by differential forms.

3. Immersions and embeddings. Submanifolds. Submersions. Introduction to vector bundles.

4. The Lie bracket and Lie derivative of vector fields. The Lie derivative of tensors and forms.

5. Existence and uniqueness theorem for ordinary differential equations. Flows of vector fields.

6. The Frobenius Theorem. Integrability of distributions. Foliations.

7. The exterior derivative of differential forms. Integration of forms over oriented manifolds. Stokes' Theorem. Introduction to deRham cohomology.

8. Introduction to Lie groups and Lie algebras. Applications.

Part Two: Riemannian geometry

2.1 Riemannian metrics. The Levi-Civita connection. Riemann, Ricci, and scalar curvature tensors.

1. Geodesics. The exponential map. The Hopf-Rinow Theorem. Jacobi fields.

2. Flat manifolds. Spaces of constant curvature.

3. Submanifold theory. The second fundamental form. The Cozaddi equations. First and second variational formulas. Minimal submanifolds. Calibrated geometry.
4. Connections on vector bundles. Curvature of connections. Parallel transport. Holonomy.
5. The Laplacian on differential forms. Harmonic forms. The Hodge Theorem. The Bochner formula.
6. Einstein manifolds. Lorentzian metrics. General relativity.
7. Conformal geometry. The Weyl tensor. The Yamabe problem.
8. Other topics in geometric analysis. Ricci curvature and volume comparison. Sectional curvature and distance comparison. Relations between curvature and topology.

Part Three: Possible additional topics

3.1 Symplectic structures on vector spaces. Symplectic manifolds. Examples. Lagrangian submanifolds.

1. Complex structures on vector spaces. Almost complex manifolds. Examples. Integrability of almost complex structures. The Newlander-Nirenberg Theorem.
2. Vector cross product structures on vector spaces. Manifolds with vector cross products. Examples.

Marking Scheme: For graduate students in PMath 665, your course mark will be determined as follows:

- Assignments: 40% (eight assignments, one about every week and a half, worth 5% each)
- FINAL EXAM: 40% (2.5 hours; date and time of the final exam TBA)
- Project and presentation: 20% (a ten to fifteen page project and 30 minute presentation)

Textbooks: There are two required textbooks for this course:

- J.M. Lee; *Introduction to Smooth Manifolds*; Springer-Verlag.
- J.M. Lee; *Riemannian Manifolds: An Introduction to Curvature*; Springer-Verlag.

These books are both softcover, so their combined price should be very reasonable. We will cover about 14 of the 20 chapters in the first book, and about 6 or 7 of the 11 chapters of the second book. (Of course, some sections will be covered in much more detail than other sections.)

Here are some additional references that students might find useful:

- W. Boothby; *Introduction to Differentiable Manifolds and Riemannian Geometry*; Academic Press.
- M.P. do Carmo; *Riemannian Geometry*; Birkhäuser.
- S. Gallot, D. Hulin, and J. Lafontaine; *Riemannian Geometry*; Springer-Verlag.
- J. Jost; *Riemannian Geometry and Geometric Analysis*; Springer-Verglag.
- B. O'Neill; *Semi-Riemannian Geometry with Applications to Relativity*; Academic Press.
- P. Petersen; *Riemannian Geometry*; Springer-Verlag.
- M. Spivak; *A Comprehensive Introduction to Differential Geometry, Vol. 1*; Publish or Perish.
- M. Spivak; *A Comprehensive Introduction to Differential Geometry, Vol. 2*; Publish or Perish.

PMath 833 | **Topics in Harmonic Analysis** | **Kathryn Hare**

Harmonic Analysis

Professor: Kathryn Hare

Office: MC 5072

Email: kehare@math.uwaterloo.ca

Lectures: TTh 11:30 - 1:00 MC 5046

This will be an introductory course on harmonic analysis on locally compact, abelian groups. The course will cover the basic topics of Fourier analysis, such as convolution, characters, the dual group and the Fourier transform.

Major theorems include:

- Parsevals/ Plancherel theorem
- Pontryagin duality theorem
- Inversion theorem
- Bochners theorem

Following the basic material, we will study Sidon sets and Interpolation sets (I0 sets). Hadamard sets are special examples of these sets and we will see that many of the classical theorems about trigonometric series with frequencies lying in a Hadamard set can be proved very elegantly by considering the more abstract situation.

Some of the material will be taken from recent research papers and open problems will be discussed. References for the basic material include:

- Rudin Fourier analysis on groups
- Hewitt and Ross Abstract harmonic analysis
- Katznelson An introduction to harmonic analysis

Prerequisites: Measure theory and basic functional analysis.

PMath 900 **Topics in Algebra** **John Lawrence**

Infinite Group Theory

Instructor: John Lawrence

Office: MC 5057

Email: jwlawrence@math.uwaterloo.ca

Office Hours: TBA

Lectures: TTh 10:00 - 11:20 am MC 5046

Textbook: *There is no textbook.* Professor will use his own course notes.

Topics include:

1) Basics from universal algebra, free objects, varieties, equational logic
2) Free groups, subgroups of free groups, presentations, normal forms, free products, HNN extensions, Baumslag-Solitar groups
3) Nilpotent varieties, $\mathbb{V}(S_3)$, uncountability of varieties
4) Decision problems; word problem, conjugacy problem, equivalence and satisfaction problems; complexity
5) Burnside problems

Grades: Grades will be determined by assignments and a written project and talk.

PMath 944 **Topics in Number Theory:** **Kevin Hare**

Computational Number Theory

Instructor: K. G. Hare

Email: kghare@math.uwaterloo.ca

Phone: (519) 885-1211 x 35556

Office: MC 5086

Lectures: MWF 10:30am-11:30am, Room: MC 5046

Office Hours: Monday 11:30-12:30, Wednesday 1:30-2:30, Friday 11:30-12:30

Syllabus: This course is based around a collection of classical unsolved extremal problems for polynomials. These problems, all of which lend themselves to computational exploration, lie at the interface of analysis, combinatorics, and number theory. Some examples of problems to be looked at include:

- What can be said about, and how do we find, monic integer polynomials with only one root ζ outside the unit circle?
- What can be said about, and how do we find integer polynomials $p(z)$, of degree n such that $|p(z)|$ is "small" on the interval $[0, 1]$?
- Given $\mathcal{P}$ the set of all polynomials with coefficients $\{0, \pm 1\}$, what happens if we evaluate $\mathcal{P}$ at a particular fixed real number? (Say for example 1, $\sqrt{2}$ or $\frac{1+\sqrt{5}}{2}$).
- How big can the coefficients of a cyclotomic polynomial get?
- Consider the base β representation of a number. What happens if $\beta \notin \mathbb{Z}$?
- What computational tools can be used to aid in these investigations?

Recommended Text: *Computational Excursions in Analysis and Number Theory* - Peter Borwein

Grading: Grading will be some combination of Assignments, Paper/Project, Presentation and an Oral Final Exam.

PMath 955 **Topics in Geometry** R. Moraru

Gauge Theory

Instructor: Ruxandra Moraru

Email: moraru@math.uwaterloo.ca

Office: MC 5170

Office Hours: TBA

Course Lectures: MW 09:00 – 10:20, in MC 5046

Textbook: No textbook is required for this course.

Overview:
This course will be an introduction to gauge theory. It will cover material that any graduate student interested in geometry will find useful (such as bundles, connections, characteristic classes, Hodge Theory, and Yang-Mills Theory). The course should be accessible to students who have taken PMATH 465 (Differential Geometry) or an equivalent course. For students who do not have the required background, it is highly recommended that they also attend the lectures of PMATH 465, which is taught on Tuesdays and Thursdays in Winter 2010.

Topics to be covered:

(a) Basics:

- Review of manifolds, vector fields, differential forms.
- Vector bundles and principal bundles: definitions and basic constructions.
- Connections, curvature, and gauge groups.
- Covariant derivatives and holonomy.
- Characteristic classes and Chern Weil Theory.

(b) More specialized topics:

- Flat vector bundles and flat connections.
- Metric connections on vector bundles.
- Some important equations of gauge theory: Yang-Mills, anti-self-duality, Hermitian-Einstein.
- Instanton moduli spaces.

(c) Some important theorems and applications:

- Flat connections and representations of the fundamental group.
- Stable holomorphic bundles and the Kobayashi-Hitchin correspondence.
- Donaldson and Seiberg-Witten invariants (time permitting).

(d) Potential additional topics:

- Berger's classification of Riemannian holonomy groups.
- Generalisations to higher dimensional manifolds of instantons on four-manifolds.

Some useful references:

- "Differential Geometry of Complex Vector Bundles", S. Kobayashi, Princeton, 1987;
- "Complex Geometry: An Introduction", D. Huybrechts, Universitext, Springer, 2004;
- "Characteristic classes", J. Milnor and J. Stasheff, Princeton, 1974;
- "An introduction to gauge theory", J. W. Morgan, in: Gauge theory and the topology of four-manifolds, ed. R. Friedman and J. W. Morgan, IAS/Park City Mathematics Institute, 1994;
- "The Geometry of Four-Manifolds", S. K. Donaldson and P. B. Kronheimer, Oxford Mathematical Monographs, Oxford University Press, 1997.

The first lecture will be held

Monday, January 4, 2010