Department of Pure Mathematics
Graduate Courses

Winter 2006

Course #	**Course Title**	**Instructor**	**Meet Days/Time**	**Place**
PMath 632/432	Mathematical Logic	R. Moosa	T,Th, 10:00 - 11:20 a.m.	PHY 150
PMath 641/441	Algebraic Number Theory	M. Rubinstein	MWF, 1:30 - 2:20 p.m.	MC 1056
PMath 651/451 AMath 431	Measure and Integration	A. Nica	MWF, 12:30 - 1:20 p.m.	MC 4040
PMath 667/467	Topology	S. New	MWF, 9:30 - 10:20 a.m.	MC 4041
PMath 711	Topics in Mathematical Logic *Computibility Theory*	B. Csima	W,F, 10:30 - 11:50 a.m.	MC 5046
PMath 744	Topics in Number Theory *Diophantine Inequalities*	C.L. Stewart	T,Th, 10:00 to 11:30 a.m.	MC 5046
PMath 801	Graduate Analysis	C.T. Ng	W 2:30 - 3:50 F 2:30 - 3:50	MC 5046 MC 5158A
PMath 810	Banach Algebras and Operator Theory	H. Radjavi	MW 12:00 - 1:20	MC 5046

Students should discuss their course selection with their Supervisor, the Graduate Officer, or the course Professor.

You will require a "Permission Number" in order to enroll through QUEST.

Please obtain your Permission Number from Shonn Martin in MC 5064

Please enroll in your courses by Tuesday, February 14th, 2006.

January 6, 2005

PMath 632 **Mathematical Logic** **R. Moosa**
(held with) **PMath 432**

First-order predicate logic: syntax, semantics, definability, and the completeness, compactness and Löwenheim-Skolem theorems. Examples from and applications to mathematics; especially algebraically closed and real closed fields. Computability and incompleteness: recursive functions, the Church-Turing thesis, Gödel's incompleteness theorems, Church's theorem on the undecidability of validity, Tarski's theorem on the undefinability of truth.

Required Textbook: no textbook required

Reference: "Mathematical Logic" by Joseph R. Shoenfield. Association for Symbolic Logic, 2001.

PMath 641 **Algebraic Number Theory** **M. Rubinstein**
(held with) **PMath 441**

Unique factorization, Dedekind domains, class numbers, Dirichlet's unit theorem, Diophantine equations, Fermat's "last theorem".

Required Textbook: no textbook required

Reference: "Problems in Algebraic Number Theory", Series: Graduate Texts in Mathematics, Vol. 190, Murty, M. Ram, Esmonde, Jody, 2nd ed., 2005, XVI, 352 p., Hardcover ISBN: 0-387-22182-4

PMath 651 | **Measure and Integration** | **A. Nica**

(held with) **PMath 451**
AMath 431

Required Textbook: no textbook required

General measures, measurability, Caratheodory extension theorem and construction of measures, integration theory, convergence theorems, L^p spaces, absolute continuity, Radon-Nikodym theorem, product measures, Fubini's theorem, signed measures, Riesz Representation theorems for L^p and $C(X)$.

Recommended Reference

1. "Real and Complex Analysis', W. Rudin. McGraw-Hill, 3rd edition, 1987.

PMath 667 | **Topology** | **S. New**

(held with) **PMath 467**

Required Textbook: Algebraic Topology
Author: Allen Hatcher
Publisher: Cambridge University Press

Homotopy of spaces, the fundamental group, the classification of two dimensional manifolds, covering spaces, Euler characteristic, homology groups; applications to the fundamental theorem of algebra, the Borsuk-Ulam theorem, and the ham sandwich theorem.

References

1. "Algebraic Topology: An Introduction", W.S. Massey. Springer-Verlag, 1997.
2. "Topology", J.R. Munkres. Prentice-Hall, 2000.
3. "Basic Topology", by M. A. Armstrong. Springer-Verlag, 1997.

PMath 711 **Topics in Mathematical Logic** **Barbara F. Csima**

Computability Theory

Course Title: Topics in Mathematical Logic—Computability Theory

Course Number: PMATH 711

Time: WF 10:30-11:50 AM

Location: MC 5046

Instructor: Barbara F. Csima

Prerequisites: Ability to do proofs.

Text: *Computability Theory and Applications* by Robert I. Soare

This will be an introduction to Computability Theory.

We will begin with a brief introduction (review) of Turing machines to help introduce the computable and computably enumerable sets, and computable and partial computable functions. We discuss Church's Thesis, that formally computable and intuitively computable are the same.

We then introduce the notion of Turing reducibility, which gives rise to a partial ordering on sets of natural numbers. We examine this ordering, mainly restricting our attention to the computably enumerable sets. Along the way we introduce common proof techniques used in Computability Theory, such as the finite and infinite injury priority methods and forcing using an oracle.

At the end of the course we give some examples of current research in Computability.

Notes: Aside from the first two weeks (Turing machines / Church's Thesis), none of the material overlaps with other offerings in PMATH or CS. This course would be of interest to graduate and advanced undergraduate students in PMATH and CS.

About the text: This is a greatly updated and modernized version of Soare's text *Recursively Enumerable Sets and Degrees.* The new text will be handed out chapter by chapter as we go through it.

PMath 744 | **Topics in Number Theory:** | **Winter 2006**

Diophantine Inequalities

Instructor: C.L. Stewart

The purpose of this course is to introduce some of the fundamental ideas from the field of Diophantine approximation, the study of rational approximation to real numbers. We shall discuss the notions of height for algebraic numbers and for subspaces, Siegel's lemma on the solutions of systems of linear equations, as well as results from the Geometry of numbers such as Minkowski's theorem. With these ideas at hand I plan to discuss Schmidt's theorem concerning the simultaneous approximation of algebraic number by rationals. The Subspace Theorem of Schmidt has many extraordinary applications and we shall discuss some of them.

Required Textbook: no textbook required

Day & Time: T Th 10:00 - 11:30 a.m.

Where: MC 5046

The first class will be held on
Thursday, January 5, 2006.

PMath 801 **Graduate Analysis** **C.T. Ng**

Zorn's Lemma and the Axiom of Choice, cardinality, introduction to topological spaces, bases, nets, continuous functions and weak topologies, compactness, connectedness, Banach spaces, Contraction Mapping Principal finite–dimensional spaces $C(X)$ and $C_0(X)$, Stone–Weierstrass Theorem, Arzela–ascoli Theorem, Urysohn's Lemma, ideals in $C_0(x)$.

Outline:

1. Basic Set Theory
   - Zorn's Lemma and the Axiom of Choice
   - Cardinal numbers and cardinal arithmetic
2. Introduction to Topological Spaces
   - Basics notions of topology
   - Bases and subbases
   - Nets
   - Continuous functions and weak topologies
   - Compact and locally compact Hausdorff spaces
   - Compactness in metric spaces
   - Connectedness
   - Baire Category Theorem
3. Normed linear spaces and Banach spaces
   - Sequence Spaces and $\mathbb{R}^n$
   - Bounded linear maps
   - Banach Contractive Mapping Theorem and its applications
   - Finite dimensional spaces
4. $\mathcal{C}(X)$ and $\mathcal{C}_0(X)$
   - Completeness and uniform convergence
   - Weierstrass approximation theorem
   - Stone-Weierstrass theorem for $\mathcal{C}(X)$
   - One-point compactifications
   - Stone-Weierstrass Theorem for $\mathcal{C}_0(X)$
   - Urysohn's Lemma
   - Ideals in $\mathcal{C}(X)$ and $\mathcal{C}_0(X)$
   - Arzela-Ascoli Theorem and its applications

continued ...

PMath 801 **Graduate Analysis** **C.T. Ng**

References: Rudin, W., *Real and Complex Analysis, Third Edition, McGraw-Hill Book Co., New York, 1987, xiv + 416pp.*
ISBN: 0-07-054234-1 00A05

Textbook: No textbook

When & Where: 2:30 - 3:50 p.m., Wednesdays in MC 5046
2:30 - 3:50 p.m., Fridays in MC 5158A

PMath 810 **Banach Algebras & Operator Theory** **H. Radjavi**

Background: PM 653 or equivalent is the prerequisite and students feeling they have the equivalent background should approach the Graduate Officer before signing up for PM 810.

Course Description: Banach algebras, functional calculus, Gelfand transform, Jacobson radical, Banach space and Hilbert space operators, Fredholm alternative, spectral theorem for compact normal operators, ideals in $C*$-algebras, linear functionals and states, GNS construction, von Neumann algebras, strong/weak operator topologies, Double Commutant theorem, Kaplansky's Density Theorem, spectral theorem for normal operators.

Textbook: A set of notes will be available for you to photocopy.

When & Where: 12:00 - 13:20 p.m., MW in MC 5046