Pure Math Graduate Courses
Winter 2004

PMath 641/441	**Algebraic Number Theory**	**W. Kuo**
PMath 653/453 /AMath 432	**Functional Analysis**	**L.W. Marcoux**
PMath 664/464	**Algebraic Curves**	**D.K. McKinnon**
PMath 665/465 /AMath 433	**Differential Geometry**	**B.D. Park**
PMath 667/467	**Topology**	**P.N. Hoffman**
PMath 744	Topics in Number Theory p**-adic Analysis, Trees, Sieves**	**C.L. Stewart**
PMath 822	Topics in Operator Theory **Simultaneous Triangularization**	**Heydar Radjavi**
PMath 844	Topics in Functional Analysis **von Neuman II-1 Factors**	**A.M. Nica**

continued ...

PMath 641 — Algebraic Number Theory — **W. Kuo**

(held with) **PMath 441**

Unique factorization, Dedekind domains, class numbers, Dirichlet's unit theorem, Diophantine equations, Fermat's "last theorem".

Reference

1. "Number Fields", Daniel A. Marcus, Springer-Verlag, 1977..
2. "Algebraic Number Theory", 2nd ed. Serge Lang, Graduate Texts in Mathematics, Springer–Verlag, 1994.
3. "Algebraic Number Theory", Ian Stewart and David Tall, John Wiley & Sons, 1979

PMath 653 — **Functional Analysis** — **L.W. Marcoux**

(held with)
PMath 453/AMath 432

Banach and Hilbert spaces, bounded linear maps, Hahn-Banach theorem, Open Mapping theorem, dual spaces, weak topologies, Tychonoff's theorem, Banach-Alaoglu theorem, reflexive spaces, compact operators.

Textbook: No text required

References

1. "Linear Analysis, an introductory course", B. Bollobas, Cambridge Press, 1990.
2. "Applications of Functional Analysis and Operator Theory", V. Hutson and J.S. Pym, Academic Press, 1980.
3. "A Course in Functional Analysis", J.B. Conway. Springer-Verlag, 1985.
4. "Fundamentals of the theory of Operator Algebras", R.V Kadison and J.Ringrose, Academic Press.

PMath 664 — **Algebraic Curves** — **D.K. McKinnon**

(held with) **PMath 464**

An introduction to the geometry of algebraic curves with applications to elliptic curves and computational algebraic geometry. Plane curves, affine varieties, the group law on the cubic, and applications.

Textbook: "Algebraic Curves" by William Fulton (out of print.)
Copies printed by Coursewares Solutions, sold at the Math Copy Center.)

PMath 665 **Differential Geometry** **B.D. Park**
(held with)
PMath 465/AMath 433

Topics to be covered (time permitting): Curves and surfaces, m-dimensional surfaces in R^n, intrinsic Riemannian geometry, mathematical aspects of General Relativity, Gauss-Bonnet Theorem, geodesics and global geometry.

Textbook: "Riemannian Geometry, A Beginner's Guide", Frank Morgan, A. K. Peters Ltd., 1998. x+156pp.

Background: This is intended to be a first course in differential geometry, aimed at around fourth-year level. Undergraduate students who did not take AMath 333/PMath 365 should see the instructor for override permission.

PMath 667 **Topology** **P.N. Hoffman**
(held with) **PMath 467**

Homotopy of spaces, the fundamental group, the classification of two dimensional manifolds, covering spaces, Euler characteristic, homology groups; applications to the fundamental theorem of algebra, the Borsuk-Ulam theorem, and the ham sandwich theorem.

References

1. "Algebraic Topology: An Introduction", W.S. Massey. Springer-Verlag, 1997.
2. "Topology", J.R. Munkres. Prentice-Hall, 2000.
3. "Basic Topology", by M. A. Armstrong. Springer-Verlag, 1997.

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p-adic Analysis, Trees, Sieves

Let p be a prime number, let k be a positive integer and let f be a polynomial with integer coefficients. We shall give estimates for the number of solutions of the congruence $f(x) \equiv 0 \pmod{p^k}$. In order to do so we introduce the notion of a tree of solutions and show how we may define the product of trees. We also introduce the basics of p-adic analysis in order to relate the structure of solution trees with the original problem. As Koblitz has remarked: "p-adic analysis can be of interest to students for several reasons. First of all, in many areas of mathematical research - such as number theory and representation theory p-adic techniques occupy an important place. More naively, for a student who has just learned calculus, the "brave new world" of non-Archimedean analysis provides an amusing perspective on the world of classical analysis. p-adic analysis, with a foot in classical analysis and a foot in algebra and number theory, provides a valuable point of view for a student interested in any of those areas".

We shall also study sieve methods such as Gallagher's larger sieve and the large sieve. These allow one to estimate the size of sets of integers which are devoid of elements from certain congruence classes modulo p for several primes p.

Textbook: No text required

Day & Time: Tuesday, Thursday, 10:00 - 11:20 a.m.

Where: MC6091A

The first class will be held on

Thursday, January 8, 2004
10:00 - 11:20 a.m.
in MC 6091A

continued ...

Simultaneous Triangularization

Outline: The theme of the course is (simultaneous) reducibility and triangularizability of collections of bounded operators on Banach spaces. Reducibility means having a common nontrivial invariant subspace. Triangularizability means that a maximal subspace chain is contained in the lattice of invariant subspaces of the collection. This generalization of commutativity, the subject of many classical results in finite dimensions, has been studied extensively in recent years. This course will treat several of these results that link the concept to other topics such as spectral mapping theorems, properties of spectral radii and traces, and the structure of semigroups and algebras of operators. The only prerequisites are a solid course in linear algebra and another in functional analysis.

Background: A course in Banach spaces (Hahn Banach Theorems, closed graph theorem, etc.) and a course in Banach Algebras (functional calculus, elements of C*-algebra theory, spectral theorem for normal operators, compact operators) will be adequate background for this course. Not all of this is absolutely essential. Those in doubt are asked to consult the instructor.

Textbook: "Triangularization"
by H. Radjavi & P. Rosenthal
Publisher: Springer
ISBN: 0-387-98467-6 (softcover) or
0-387-98467-4 (hardcover, if you are rich)

Day & Time: To Be Announced

Where: To Be Announced

Organizational meeting will be held on

Tuesday, January 6, 2004

1:30 - 2:20 p.m.

in MC 5045

continued ...

PMath 844 **Topics in Functional Analysis** **A. Nica**

von Neuman II-1 Factors

Outline: This is a continuation of the course of introduction to von Neumann algebras taught by the instructor in the Fall Term 2003. The focus will be on von Neumann factors of type II_1 – i.e. von Neumann algebras with trivial centre which are infinite dimensional but are nevertheless "finite" in a suitable sense (e.g. in the sense that the projections in the algebra have a finite "dimension", which can take any value between 0 and 1). The research work and literature on von Neumann II_1 factors are very extensive; for instance the study of the so-called "hyperfinite II_1 factor" (the first example of II_1 factor, found by Murray and von Neumann in the 40's) could alone provide material for a one-term course. I will not attempt to do a wide survey of this literature, the goal of the course is a lot more modest: I will cover a few general facts about II_1 factors, after which the course will move towards the (comparatively recent) theory of subfactors of Jones. Thus, at least in the latter part of the term, the course will follow parts of the textbook by Jones and Sunder which is recommended below.

Background: I will assume that the students know some basic facts about C^*-algebras and W^*-algebras, as covered for instance in the von Neumann algebras course taught in PMath in the Fall Term 2003.

Textbook: "Introduction to Subfactors"
by V. Jones & V.S. Sunder
Publisher: Cambridge University Press
ISBN: 0 521 58420 5

Day & Time: To Be Announced

Where: To Be Announced

Organizational meeting will be held

Tuesday, January 6, 2004

10:30 - 11:20 a.m.

in MC 5045