Department of Pure Mathematics
Graduate Courses

Fall 2005

Course #	Course Title	Instructor	Time/ Days	Place
PMath 642/442	Field and Galois Theory	D.Ž. Djoković	1:30 - 2:20 pm, MWF	ES1 350
PMath 653/453	Functional Analysis	N. Spronk	9:30 - 10:20 am, MWF	RCH 204
PMath 664/464	Algebraic Curves	R. Moraru	10:30 -11:20 am, MWF	MC 4063
PMath 701	Graduate Algebra	J.W. Lawrence	8:30 - 10:00 am, T TH	MC 5045
PMath 744	Topics in Number Theory: The Goldbach conjecture and Waring's problem	Y-R. Liu	12:30 - 1:20 pm, MWF	MC 5045
PMath 801	Graduate Analysis	B.E. Forrest	2:30 - 4:00 pm, MW	MC 5045
PMath 822	Topics in Operator Theory: Introduction to Operator Theory	K.R. Davidson	10:30 - 12:00 a.m., T TH	MC 5045

Winter 2006

Course #	Course Title	Instructor	Days/Time	Place
PMath 632/432	Mathematical Logic	TBA	TBA	TBA
PMath 641/441	Algebraic Number Theory	TBA	TBA	TBA
PMath 651/451 AMath 43	Measure and Integration	TBA	TBA	TBA
PMath 667/467	Algebraic Topology	TBA	TBA	TBA
PMath 711	Topics in Mathematical Logic	B.F. Csima	TBA	TBA
PMath 744	Topics in Number Theory	C.L. Stewart	TBA	TBA
PMath 755	Topics in Geometry:	R. Moosa	TBA	TBA
PMath 810	Banach Algebras	TBA	TBA	TBA

Students should discuss their course selection with their Supervisor , the Graduate Officer, or the course Professor.

You will require a "Permission Number" in order to enroll through QUEST.

Please obtain your Permission Number from Shonn Martin in MC 5064

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PMath 642 **Field and Galois Theory** **D.Ž. Djoković**
(held with) **PMath 442**

Normal series, elementary properties of solvable groups and simple groups, algebraic and transcendental extensions of fields, adjoining roots, splitting fields, geometric constructions, separability, normal extensions, Galois groups, fundamental theorem of Galois theory, solvability of radicals, Galois groups of equations, cyclotomic and Kummer extensions.

Required Textbook: No text required

Reference

1. "Galois Theory", 2nd ed., Joseph Rotman, Springer-Verlag, 1990.
2. "Galois Theory", Joseph–Pierre Escofier, Graduate Texts in Mathematics, Springer–Verlag, 2001.
3. "Notes on Galois Theory", Peter Hoffman (recommended)

PMath 653 **Functional Analysis** **N. Spronk**
(held with)
PMath 453/AMath 432

Banach and Hilbert spaces, bounded linear maps, Hahn-Banach theorem, Open Mapping theorem, dual spaces, weak topologies, Tychonoff's theorem, Banach-Alaoglu theorem, reflexive spaces, compact operators.

Required Textbook: Linear Analysis: an Introductory Course, 2nd ed.
by Bela Bollobas, Cambridge University Press, ISBN: 0 521 65577 3

References: "A Course in Functional Analysis", J.B. Conway. Springer-Verlag, 1985.

PMath 664 **Algebraic Curves** **R. Moraru**
(held with) **PMath 464**

An introduction to the geometry of algebraic curves with applications to elliptic curves and computational algebraic geometry. Plane curves, affine varieties, the group law on the cubic, and applications.

Required Textbook: Algebraic Curves
by William Fulton
Out-of-print textbook available from Pixel Planet, MC 2018

PMath 701 **Graduate Algebra** **J.W. Lawrence**

Isomorphism theorems, classical structures theorems for finite groups, nilpotent and solvable groups, free groups, presentations modules over PID's, Hilbert Basis Theorem, Groebner bases, Artin-Wedderburn Theorem, fields extensions, decompositions, calculation of Galois groups.

Course Outline: In this course we will be trying to decide what a given algebraic structure "looks like". The emphasis will be on the use of fundamental theorems to solve problems involving particular algebraic structures. Not every classical theorem in the course will be proved from scratch, but the implications will be discussed.

Required Textbook: No textbook required

Reference: Rotman, J. *Advanced Modern Algebra*,
Prentice Hall, ISBN 0-13-087868-5

Day & Time: Tuesday/Thursday
8:30 - 10:00 am

Where: in MC 5045

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PMATH 744 Topics in Number Theory

The Goldbach conjecture and Waring's problem

An introduction to the circle method

Instructor: Yu-Ru Liu (yrliu@math.uwaterloo.ca)

Course Outline

In 1742, Goldbach conjectured that every positive even integer $n > 2$ is a sum of two primes. In 1770, Waring stated without proof that every positive integer is a sum of four squares, nine cubes, 19 fourth powers and so on. More precisely, for an integer $k \geq 2$, Waring's problem is about the existence of an integer s such that all positive integers are sums of s kth powers.

Although the Goldbach conjecture remains open until today, one can show that almost all even integers can be represented in the prescribed way. Vinogradov also proved that every sufficiently large odd integer is a sum of three primes. In the case of Waring's problem, we have more satisfactory answers. The existence of s was known for a long time. In addition, substantial progress have been made recently to reduce the size of s. For example, Wooley proved that that for large k, we can take $s < (1 + o(1))k \log k$.

The main technique involved in the above work is called the (Hardy-Littlewood) circle method. The purpose of this course is to introduce this method as a valid tool for problems in additive number theory. In particular, we will focus on the Waring-Goldbach type problems.

Prerequisites

Students who know calculus will be able to understand most materials in the course. Some familiarity in analytic number theory will also be helpful.

Textbook

There is no required textbook for this course.

References

(1) H. Davenport, *Analytic methods for Diophantine equations and Diophantine inequalities*', Cambridge Mathematical Library (2005)
(original version: Ann Arbor Publishers (1962)).
(2) M. Nathanson, *Additive number theory*, Springer-Verlag (1996).
(3) R. C. Vaughan, *The Hardy-Littlewood method, second edition*, Cambridge University Press (1997).

Classtime (proposed)

Monday, Wednesday, Friday 12:30-1:20 pm. in MC 5045

PMath 801 **Graduate Analysis** **B.E. Forrest**

Zorn's Lemma and the Axiom of Choice, cardinality, introduction to topological lspaces, bases, nets, continuous functions and weak topologies, compactness, connectedness, Banach spaces, Contraction Mapping Principal finite–dimensional spaces $C(X)$ and $C_0(X)$, Stone–Weierstrass Theorem, Arzela–ascoli Theorem, Urysohn's Lemma, ideals in $C_0(x)$.

Outline:

1. Basic Set Theory
   - Zorn's Lemma and the Axiom of Choice
   - Cardinal numbers and cardinal arithmetic
2. Introduction to Topological Spaces
   - Basics notions of topology
   - Bases and subbases
   - Nets
   - Continuous functions and weak topologies
   - Compact and locally compact Hausdorff spaces
   - Compactness in metric spaces
   - Connectedness
   - Baire Category Theorem
3. Normed linear spaces and Banach spaces
   - Sequence Spaces and $\mathbb{R}^n$
   - Bounded linear maps
   - Banach Contractive Mapping Theorem and its applications
   - Finite dimensional spaces
4. $\mathcal{C}(X)$ and $\mathcal{C}_0(X)$
   - Completeness and uniform convergence
   - Weierstrass approximation theorem
   - Stone-Weierstrass theorem for $\mathcal{C}(X)$
   - One-point compactifications
   - Stone-Weierstrass Theorem for $\mathcal{C}_0(X)$
   - Urysohn's Lemma
   - Ideals in $\mathcal{C}(X)$ and $\mathcal{C}_0(X)$
   - Arzela-Ascoli Theorem and its applications

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PMath 801 **Graduate Analysis** **B.E. Forrest**

References: Rudin, W., *Real and Complex Analysis, Third Edition, McGraw-Hill Book Co., New York, 1987, xiv + 416pp.*
ISBN: 0-07-054234-1 00A05

Textbook: No textbook

Day & Time: 2:30 - 3:50 pm, MW

Where: MC 5045

To find out when an organizational meeting will be held, please contact B.E. Forrest at:

beforrest@math.uwaterloo.ca

PMath 822 **Topics in Operator Theory:** **K.R. Davidson**

Introduction to Operator Theory

This course is an introduction to the study of bounded linear operators on Hilbert space. A course in basic functional analysis is assumed. Knowledge of measure theory and of analytic functions is a definite asset.

Course Topics:

1. Compact and Fredholm operators
   - Fredholm index
   - Invariant subspaces, Lomonosov's Theorem
   - Ringrose's Theorem
2. Normal operators.
   - diagonal operators and multiplication operators
   - the spectral theorem, spectral measures and the functional calculus
   - Weyl-von Neumann Berg Theorem
2. Compact and Fredholm operators
   - Fredholm index
   - Invariant subspaces, Lomonosov's Theorem
   - Ringrose's Theorem
3. The Unilateral Shift
   - Invariant subspaces, Beurling's Theorem
   - Toeplitz operators
   - Fredholm index
4. Nonself-adjoint Operator algebras
   - topics as time permits

Required Textbook: No textbook required

Reference: Rotman, J. *Advanced Modern Algebra*,
Prentice Hall, ISBN 0-13-087868-5

Day & Time: Tuesday/Thursday.
10:30 am - 12:00 noon

Where: MC 5045